GRB BOOKS

Sameer Bansal

GRB 1000 Challenging Problems in

Mathematics

For **JEE** (Main & Advanced)

& All Other Competitive Entrance Examinations

with Solutions

CONTENTS

Hints & Solutions

CONTENTS

Single Correct Type Questions

1. The value of definite integral $\displaystyle\int_{\frac{-1}{\sqrt{3}}}^{\frac{1}{\sqrt{3}}} \frac{\cos^{-1}\left(\frac{2x}{1+x^2}\right)+\tan^{-1}\left(\frac{2x}{1-x^2}\right)}{e^x+1}\,dx$ is equal to:

(a) $\frac{\pi}{2\sqrt{3}}$ (b) $\frac{\pi}{\sqrt{3}}$ (c) $\frac{\pi}{4\sqrt{3}}$ (d) $\frac{\pi}{3\sqrt{3}}$

2. $\displaystyle\lim_{n\to\infty} \frac{n^2}{\left((n^2+1^2)(n^2+2^2)\ldots\ldots\ldots(n^2+n^2)\right)^{\frac{1}{n}}}$ equals:

(a) $2e^{2+\frac{\pi}{2}}$ (b) $2e^{2-\frac{\pi}{2}}$ (c) $\frac{1}{2}e^{2-\frac{\pi}{2}}$ (d) $\frac{1}{2}e^{2+\frac{\pi}{2}}$

3. The solution of differential equation $\frac{dy}{dx}=\frac{x^2+y^2+1}{2xy}$ satisfying $y(1)=0$ is given by:

(a) a circle (b) $y^2=x^2+x-10$ (c) hyperbola (d) ellipse

4. The value of $\displaystyle\lim_{x\to 0}\left[(1-e^x)\frac{\sin x}{|x|}\right]$ equals:

[Note: $[\ \cdot\ ]$ denotes the greatest integer function.]

(a) 0 (b) -1 (c) 1 (d) does not exist

5. If $\displaystyle\lim_{\alpha\to 0}\frac{e^{\cos(\alpha^n)}-e}{\alpha^m}=\frac{-e}{2}$ where m and n are positive integers greater than 1, then the value of $\frac{m}{n}$ is:

(a) 2 (b) 3 (c) 4 (d) 5

6. Let $f(x)=\begin{cases}\dfrac{x^2\sin\left(\frac{1}{x}\right)+2x}{(1+x)^{\frac{1}{x}}-e}, & x\neq 0\\ \lambda, & x=0\end{cases}$.

If $f(x)$ is continuous at $x=0$, then the value of λ is:

(a) $\frac{-2}{e}$ (b) $\frac{2}{e}$

(c) $\frac{4}{e}$ (d) $\frac{-4}{e}$

7. Let $f: R \to R$ be a continuous function satisfying $f(x) + \int_0^x t f(t)\, dt + x^2 = 0 \;\forall\, x$. Then:

(a) $f(x)$ has more than one point in common with x-axis

(b) $f(x)$ is odd function

(c) $\lim_{x \to \infty} f(x) = 2$

(d) $\lim_{x \to -\infty} f(x) = -2$

8. If the normal at one end of latus rectum of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ passes from one end of minor axis and e is eccentricity of ellipse, then:

(a) $e^2 + e + 1 = 0$ (b) $e^4 - e^2 + 1 = 0$ (c) $e^2 - e + 1 = 0$ (d) $e^4 + e^2 - 1 = 0$

9. If $\lim_{x \to 0} \frac{10 - \sum_{k=1}^{10} (\cos kx)}{x^2} = \frac{a}{b}$ where a and b are co-prime, then the value of $(a + b)$ is equal to:

(a) 384 (b) 385 (c) 386 (d) 387

10. The value of definite integral $\int_{-\pi}^{\pi} \frac{2x(1 + \sin x)}{1 + \cos^2 x} dx$ is:

(a) $\frac{\pi}{2}$ (b) π (c) π^2 (d) $\frac{\pi^2}{2}$

11. Let k be natural number. Defined S_k as the sum of the infinite geometric series with first term $(k^2 - 1)$ and common ratio $\frac{1}{k}$, that is $S_k = \frac{k^2 - 1}{k^0} + \frac{k^2 - 1}{k^1} + \frac{k^2 - 1}{k^2} + \ldots\ldots$. The value of $\sum_{k=1}^{\infty} \frac{S_k}{2^{k-1}}$, is:

(a) 20 (b) 18 (c) 16 (d) 14

12. Let $y = \tan^{-1}\left(\frac{4x}{1 + 5x^2}\right) + \tan^{-1}\left(\frac{2 + 3x}{3 - 2x}\right)$ where $x \in \left(0, \frac{2}{3}\right)$. If $\frac{dy}{dx} = \frac{\alpha}{1 + 25x^2}$, then the value of α is equal to:

(a) 3 (b) 4 (c) 5 (d) 6

13. If $\sum_{r=1}^{100} \sin^{-1}\left(\frac{1}{\sqrt{r^2+1}\sqrt{r^2+2r+2}}\right)$ is equal to $\tan^{-1}\left(\frac{p}{q}\right)$ where p and q are co-prime, then the value of $(p+q)$ is equal to:
(a) 99 (b) 100 (c) 101 (d) 102

14. Through the vertex of the parabola $y^2 = 4ax$, two chords are drawn and the circle on these chords as diameters intersect at a point.
If A and B be the angles made with the x-axis by tangents at the other ends of chords and C be the angle made with the x-axis by the line joining vertex of the parabola and point of intersection of circles, then $\cot(A)+\cot(B)+m\tan(C)=0$ for some constant positive integer m. The value of m, is:
(a) 2 (b) 3 (c) 4 (d) 5

15. The value of $\lim_{n\to\infty}\left(\ln\left(\sqrt[n]{\frac{4}{n^2}}\right)+\ln\left(\sqrt[n]{\frac{16}{n^2}}\right)+\ln\left(\sqrt[n]{\frac{36}{n^2}}\right)+\ldots\ldots+\ln\left(\sqrt[n]{\frac{4n^2}{n^2}}\right)\right)$ equals:
(a) $4\ln(2)$ (b) $2\ln(2)-2$
(c) $2\ln(2)-4\ln(4)-4$ (d) $2\ln(4)-2$

16. Let f be a differentiable function satisfy $x^2 f'(x)+2xf(x)=e^x$ and $f(2)=\frac{e^2}{4}$, then:
(a) $f(x)$ has no local maxima and no local minima.
(b) $f(x)$ has both local maxima and local minima.
(c) $f(x)$ has local maxima but no local minima.
(d) $f(x)$ has no local maxima but local minima.

17. AB is tangent to the circle whose equation is $x^2+y^2=9$. The coordinates of point A are $(-10, 0)$ and point $B(a, b)$ is in the third quadrant. The slope of AB is:
(a) $\frac{-9\sqrt{91}}{91}$ (b) $\frac{-3\sqrt{91}}{91}$ (c) $\frac{-3\sqrt{91}}{10}$ (d) $\frac{-6\sqrt{91}}{10}$

18. The largest value of $\frac{y}{x}$, where (x, y) is real number pair satisfying $(x-3)^2+(y-3)^2=6$, is:
(a) $2\sqrt{3}$ (b) $2+\sqrt{3}$ (c) $3+2\sqrt{2}$ (d) $6+2\sqrt{3}$

19. If $f(x)$ is a real values bijective function satisfying $f'(x)=\sin^2(\sin(x+1))$ and $f(0)=3$, then the value of $(f^{-1})''(3)$ is equal to:
(a) $-\frac{2\sin(\cos)\sin 1}{\sin^5(\cos 1)}$ (b) $-\frac{2\sin(\sin 1)\cos 1}{\sin^5(\sin 1)}$
(c) $-\frac{2\sin(\cos 1)\sin^2 1}{\sin^6(\cos 1)}$ (d) $-\frac{\sin^2(\sin 1)}{\cos^2(\cos 1)}$

20. Consider the circle $x^2 + y^2 = 25$ and a point $A(1, 2)$ lying inside it. Next consider secants of the circle passing through point A. It turns out that the mid-point of the secants, lie on another circle of centre (a, b) and radius r. Then triplet (a, b, r) is:

(a) $(1, 2, 5)$ (b) $\left(\frac{1}{2}, 1, \frac{\sqrt{5}}{2}\right)$ (c) $(0, 0, \sqrt{5})$ (d) $(1, 2, \sqrt{5})$

21. Let A_k be the finite area bounded by the line $y = kx + k$ and the parabola $y = x^2$, where k is a positive real number. The value of $\lim\limits_{k \to \infty} \frac{A_k}{k^3}$ equals:

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{6}$ (d) $\frac{2}{3}$

22. Through a random point (p, q) on the cartesian plane secants are drawn to the circle $x^2 + y^2 = r^2$. If the locus of mid-point of the secants to the circle is $x^2 + 2hxy + y^2 + 2gx + 2fy + c = 0$. Then:

(a) $h = pq$ (b) $g = p$ (c) $f = q$ (d) $c = 0$

23. Suppose that x_1 and x_2 are the positive real solution of $x^2 - bx + c = 0$ provided that $x_1^2 + \sqrt{x_2^2 - 2x_2} = 2x_1 - 1$. The minimum value of $(b + c)$, is:

(a) 2 (b) 3 (c) 4 (d) 5

24. If $I = \int\limits_{e^{\pi/6}}^{e^{\pi/2}} \frac{\sin(\ln(\sin(\ln x)))\cos(\ln x)}{x\sin(\ln x)} dx$, then the value of $\cos^{-1}(I + 1)$ is equal to:

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$ (c) $\ln 2$ (d) $2 \ln 2$

25. If $y = (x + \sqrt{1 + x^2})^n$, then the value of $(1 + x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx}$ is equal to:

(a) $-y$ (b) $-n^2 y$ (c) $n^2 y$ (d) $-ny^2$

26. If the value of definite integral $\int\limits_{\pi/4}^{\pi/3} e^x \left(\frac{2 + \sin 2x}{1 + \cos 2x}\right) dx$ is expressed as $e^{\frac{\pi}{a}}\left(be^{\frac{\pi}{c}} - 1\right)$, then the value of $\frac{b^2 c}{a}$, is:

(a) 3 (b) 6 (c) 9 (d) 12

27. Consider a function $f: R \to R$ such that $f(x) = \begin{cases} \sin(\pi x), & \text{if } x \in Q \\ \tan(\pi\sqrt{|x|}), & \text{if } x \notin Q \end{cases}$. If $\lim\limits_{x \to N} f(x)$ exists, then the sum of all positive integers $N < 100$, is equal to:

(a) 225 (b) 245 (c) 265 (d) 285

28. If the equation $\sum_{n=0}^{10} arc\cot\left(\frac{1+2^{2n+1}}{2^n}\right) = arc\cot\frac{a}{b}$, where a and b are coprime positive integers. The value of $\log_2\left(\frac{b+a}{a-b}\right)$, is:

(a) 9 (b) 10 (c) 11 (d) 12

29. If a chord of the circle $x^2+y^2-4x-2y-c=0$ is trisected at the points (1/3, 1/3) and (8/3, 8/3), then the radius of the circle will be:

(a) 3 (b) 4 (c) 5 (d) 6

30. Let y be an implicit function of x defined by $x^{2x}-2x^x\cot y-1=0$. The value of $y'(1)$, where y' denotes the first derivative of y, is:

(a) $-\ln 2$ (b) $\ln 2$ (c) -1 (d) 1

31. The value of $\lim_{h\to 0}\frac{1}{h}\int_1^{1+2h} e^{\sqrt{x}}\sin\left(\frac{\pi x}{3}\right)dx$ equals:

(a) $\sin\frac{\pi}{3}$ (b) $4e\sin\frac{\pi}{3}$ (c) $e\sin\frac{\pi}{3}$ (d) $2e\sin\frac{\pi}{3}$

32. The range of value of λ for which the expression $\frac{2x^2-5x+3}{4x-\lambda}$ can take all real values for $x\in R-\left\{\frac{\lambda}{4}\right\}$, is:

(a) (4, 6) (b) [4, 6] (c) (4, 6] (d) [4, 6)

33. Suppose that a continuous function $f(x)$ satisfies the relation $\int_x^{x+1} f(t)\,dt = e^x$ for every $x\geq 0$. The value of $f(2)-f(0)$, equals:

(a) 1 (b) $e-1$ (c) $e+1$ (d) e^2+1

34. If the value of $\lim_{n\to\infty}\sum_{k=0}^{n}\frac{{}^nC_k}{n^k(k+3)}$ equals L. Then $[L]$ is equal to:

[Note: Where $[k]$ denotes greatest integer function less than or equal to k.]

(a) 0 (b) 1 (c) 2 (d) 3

35. If $f(x)$ is a differentiable function defined for all positive real numbers such that $xf(x) = x + \int_1^x f(t)\,dt$, then the value of $\sum_{k=1}^{10} f(e^k)$ is:

(a) 45 (b) 55 (c) 65 (d) 75

36. If the line $2px + y\sqrt{1-p^2} = 1$ always touches the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \ \forall \ p \in (-1, 1) - \{0\}$.

The eccentricity of this ellipse, is

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{\sqrt{7}}{3}$ (c) $\frac{\sqrt{7}}{4}$ (d) $\frac{\sqrt{3}}{2}$

37. The value of $\sum_{m=1}^{\infty}\left(\tan^{-1}\left(\frac{3m^2-3m+1}{m^6-3m^5+3m^4-m^3+1}\right)\right)$ equals:

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

38. If the solution of inequality $\frac{(\pi^x - 7^x)\log_{10}(x-4)}{(x^2-9x+18)(x^2-x)} < 0$ is in the form $(a, b) \cup (c, \infty)$

	List-I		List-II
(P)	The value of 'a' is	(1)	2
(Q)	The value of 'b' is	(2)	3
(R)	The value of 'c' is	(3)	4
(S)	The value of $(a+b-c)$, is	(4)	5
		(5)	6

Code:	P	Q	R	S		P	Q	R	S
(a)	3	4	5	2	(b)	3	4	2	5
(c)	3	4	2	2	(d)	3	4	5	5

39. Let the equation $x^3 + y^3 + 3xy = 1$ represents the coordinate of one vertex A and the equation of side BC of the triangle ABC. If B is the orthocentre of the triangle ABC, then the equation of side AB is $y = mx + c$. Then absolute value of $(4 - m - c)$, is:

(a) 2 (b) 3 (c) 4 (d) 5

40. Let $f(x)$ and $g(x)$ are functions defined in the real domain and co-domain, such that $\sqrt{1-f^2(x)} = g(x)$, then which of the following statements are necessarily true?

(a) If $g(x)$ is periodic with period 1, then $f(x)$ is periodic with period half.

(b) If $f'(c) = -f(c) = 0.5$, then $\frac{g'(c)}{g(c)} = \frac{1}{3}$.

(c) If $g(x)$ is an even function, then $f(x)$ is odd.

(d) If $g(x)$ is continuous function then $f(x)$ is also continuous in their respective domains.

41. Tangent is drawn at any point (p, q) on the parabola $y^2 = 4ax$. Tangents are drawn from any point on this tangent to the circle $x^2 + y^2 = a^2$, such that the chords of contact pass through a fixed point (r, s). Then p, q, r, s hold which of the given relation?

(a) $rq^2 = 4ps^2$ (b) $r^2q = 4p^2s$ (c) $rq^2 = -4ps^2$ (d) $r^2q = -4p^2s$

42. Let function $f(x) = \sqrt{e^x + x - a}$ for $a \in R$. If there exists $x_0 \in [-1,1]$ such that $f(f(x_0)) = x_0$, then the range of 'a' is:

(a) $[1, e+1]$ (b) $[1, e]$ (c) $\left[\frac{1}{e}-1, 1\right]$ (d) $\left[\frac{1}{e}-1, e+1\right]$

43. The value of the $\lim\limits_{x\to\infty} \dfrac{e^x\left(\left(2^{x^n}\right)^{1/e^x} - \left(e^{x^n}\right)^{1/e^x}\right)}{x^n}$ where n is positive integer, is:

(a) $\ln 2 - \ln 3$ (b) $\ln 3 - \ln 2$ (c) 0 (d) none of these

44. The minimum value of the function $f(x) = x^{\frac{3}{2}} + x^{\frac{-3}{2}} - 4\left(x + \frac{1}{x}\right)$ for all permissible real x, is:

(a) -7 (b) -10 (c) -8 (d) -6

45. If $a = \log_{12}(18)$ and $b = \log_{24}(54)$, the value of $(a+b)^2 + a(10-a) - b(10+b)$, is:

(a) 1/2 (b) 1 (c) 2 (d) 3

46. Let a_n be sequence is geometric progression with first term16 and common ratio of $\frac{1}{4}$. Let P_n be the product of first n terms of the given geometric progression. The value of $\sum\limits_{n=1}^{\infty} P_n^{1/n}$, is:

(a) 16 (b) 32 (c) 64 (d) 68

47. The system of linear equation

$$\begin{aligned} x + \mu y - z &= 0 \\ \mu x - y - z &= 0 \\ x + y - \mu z &= 0 \end{aligned}$$

has a non-trivial solution for:

(a) exactly three values of μ (b) infinitely many values of μ

(c) exactly one value of μ (d) exactly two values of μ

48. If $f(0) = 1$ and $\lim\limits_{t\to x} \dfrac{\sec x f(t) - f(x)\sec t}{t-1} = \sec^2 x$. The value of $\dfrac{f(0)}{f'(0)}$, is:

(a) -1 (b) 0 (c) 1 (d) 2

49. Consider the function $f(x)=|x^2-7x+12|(x^2-7x+10)(x^2-4x+3)$.

Then Rolle's theorem for $f(x)$ is not applicable to which of the following range?

(a) $[1, 3]$ (b) $[2, 4]$ (c) $[3, 5]$ (d) none of these

50. Let f be an invertible function from $R \to R$ satisfying the equation

$$f^3(x)-(x^3+2)f^2(x)+(2x^3+1)f(x)-x^3=0.$$

Then the value of $f'(8)\times(f^{-1})'(8)$, is:

(a) 12 (b) 16 (c) 20 (d) 32

51. If A and B are two equivalence relations defined on set C, then which of the following is always true?

(a) $A \cap B$ is an equivalence relation (b) $A \cap B$ is not an equivalence relation

(c) $A \cup B$ is an equivalence relation (d) $A \cup B$ is not an equivalence relation

52. Let $x_1, x_2, \ldots\ldots, x_{10}$ be the roots of the polynomial equation $x^{10}+x^9+\ldots\ldots\ldots+x+1=0$.

Then the value of $\sum_{n=1}^{10}\left(\frac{1}{1-x_n}\right)$:

(a) 3 (b) 4 (c) 5 (d) 6

53. If $f'(x)+(f'(x))^2+(f'(x))^3+(f'(x))^4+\ldots\ldots\infty=e^x$, where $f'(x)\in(-1,1)$ and $f(0)=0$ then the value of $\lim_{x\to 0}(1+f(x))^{\frac{1}{x}}$ is equal to:

(a) e (b) $\sqrt{e}$ (c) $\frac{1}{\sqrt{e}}$ (d) 1

54. Let α_n, β_n be the distinct roots of the equation $x^2+(n+1)x+n^2=0$. If $\sum_{n=2}^{2021}\frac{1}{(\alpha_n+1)(\beta_n+1)}$ can be expressed in the form $\frac{a}{b}$, where a and b are positive integer, the value of $(b-a)$, is:

(a) 1 (b) 3 (c) 6 (d) 9

55. If $f(x)=\begin{cases}-e^{-x}+k, & x\le 0\\ e^x+1, & 0<x<1\\ ex^2+\lambda, & x\ge 1\end{cases}$ is one-one and monotonically increasing for all $x\in R$, then difference of maximum value of k and minimum value of λ is:

(a) 0 (b) 1 (c) 2 (d) 3

56. The value of $\lim_{n\to\infty}\sum_{r=1}^{n}\frac{\pi}{n}\frac{1}{\sin\left(\frac{(n+r)\pi}{4n}\right)}$ is equal to:

(a) $2\ln(\sqrt{2}-1)$ (b) $4\ln(\sqrt{2}-1)$ (c) $4\ln(\sqrt{2}+1)$ (d) $\ln\sqrt{2}$

57. Let $f(x)=\lim\limits_{n\to\infty}\dfrac{\cos x}{1+(\tan^{-1}x)^n}$, then the value of $\int\limits_0^{\infty} f(x)\,dx$ is equal to:

(a) $\cos(\tan 1)$ (b) $\sin(\tan 1)$ (c) $\tan(\tan 1)$ (d) none of these

58. Let $f:[0,5]\to R$ be such that $f''(x)=f''(5-x)$, $\forall\, x\in[0,5]$, $f'(0)=1$ and $f'(5)=7$, then the value of $\int\limits_1^4 f'(x)\,dx$ is:

(a) 4 (b) 6 (c) 8 (d) 12

59. If $\int \dfrac{3\tan\left(x-\dfrac{\pi}{4}\right)}{\cos^2 x\sqrt{\tan^3 x+\tan^2 x+\tan x}}\,dx = k\tan^{-1}(\sqrt{\tan x+1+\cot x})+C$, then the value of k is: [where C is constant of integration.]

(a) 2 (b) 3 (c) 6 (d) 8

60. If $\int x^{26}(x-1)^{17}(5x-3)\,dx=\dfrac{x^{27}(x-1)^{18}}{k}+C$, where C is constant of integration, then the value of k is:

(a) 3 (b) 6 (c) 9 (d) 12

61. Let $f(x)=\begin{cases}\sqrt{x^2-1}, & x\le\sqrt{10}\\ (\sqrt{10}x-7), & \sqrt{10}<x<5\\ \sin \pi x, & 5\le x<6\\ \{x\} & 6\le x\le 7\end{cases}$, then the number of points where $f(x)$ is discontinuous in [1, 7] is: (where $\{\cdot\}$ denotes fractional part of x.)

(a) 0 (b) 1 (c) 2 (d) 3

62. Let $f(x)=\int\limits_0^x x\ln(1+t^2)\,dt$, then $f''(0)$ is:

(a) 0 (b) 1 (c) 2 (d) 3

63. Let $f(x)=x^3+x+1$ and $g(x)$ be its inverse then equation of tangent to $y=g(x)$ at $x=3$ is:

(a) $x-4y+1=0$ (b) $x+4y-1=0$ (c) $4x-y+1=0$ (d) $4x+y-1=0$

64. The length of sub-tangent to the hyperbola $x^2-4y^2=4$ corresponding to the normal having slope unity is $\dfrac{1}{\sqrt{k}}$, then the value of k is:

(a) 1 (b) 2 (c) 3 (d) 4

65. A stone is dropped into a quiet lake and waves move in circles at the speed of 5 cm/sec. At that instant, when the radius of circular wave is 8 cm, the rate of increase of enclosed area is:

(a) 6π cm^2/sec (b) 8π cm^2/sec (c) $\dfrac{8\pi}{3}$ cm^2/sec (d) 80π cm^2/sec

66. The points (2, 5) and (5, 1) are two opposite vertices of a rectangle. If other two vertices are points on the straight line $y = 2x + k$, then the value of k is:
(a) 4 (b) 3 (c) -4 (d) -3

67. The distance of the point $(1, 2)$ from the line $x + y + 5 = 0$ measured along the line parallel to $3x - y = 7$ is equal to:
(a) $4\sqrt{10}$ (b) 40 (c) $\sqrt{40}$ (d) $10\sqrt{2}$

68. The maximum value of the function $f(x) = 2x^3 - 15x^2 + 36x - 48$ on the set $A = \{x \mid x^2 + 20 \le 9|x|\}$ is:
(a) 5 (b) 7 (c) 9 (d) 11

69. The least non-negative integral value of λ for which the equation $2x^2 - 2(2\lambda + 1)x + \lambda(\lambda + 1) = 0$ has one root less than λ and other root greater than λ, is equal to:
(a) 0 (b) 1 (c) 2 (d) 4

70. If $a_1, a_2, a_3, \ldots\ldots,$ are in arithmetic progression, then $S = a_1^2 - a_2^2 + a_3^2 - a_4^2 + \ldots\ldots - a_{2k}^2$ is equal to:
(a) $\dfrac{k}{2k-1}(a_1^2 - a_{2k}^2)$ (b) $\dfrac{2k}{k-1}(a_{2k}^2 - a_1^2)$
(c) $\dfrac{k}{k+1}(a_1^2 - a_{2k}^2)$ (d) none of these

71. The maximum value of $f(x) = \cos x(1 + \cos x)$ is greater than its minimum value by:
(a) 1 (b) $\dfrac{3}{2}$ (c) 2 (d) $\dfrac{9}{4}$

72. The number of values of x, for which $\tan^{-1}\left(\dfrac{1}{x}\right) = \pi + \tan^{-1} x,\ 0 < x < 1$ is:
(a) 0 (b) 1 (c) 2 (d) ∞

73. If $g(x) = (x^2 + 2x + 3) f(x)$, $f(0) = 5$ and $\lim\limits_{x \to 0}\left(\dfrac{f(x) - 5}{x}\right) = 4$, then $g'(0)$ is equal to:
(a) 22 (b) 18 (c) 23 (d) 25

74. If $f(x)$ is continuous and derivable on $[-2, 5]$ and $-4 \le f'(x) \le 3\ \forall\, x \in (-2, 5)$, then difference of maximum and minimum value of $f(5)$ is equal to:
(a) 7 (b) 21 (c) 28 (d) 49

75. Let $f(x)$ be a cubic polynomial on R which increases in the interval $(-\infty, 0) \cup (1, \infty)$ and decreases in the interval $(0, 1)$. If $f'(2) = 6$ and $f(2) = 2$, then the value of $\tan^{-1}(f(1)) + \tan^{-1}\left(f\left(\dfrac{3}{2}\right)\right) + \tan^{-1}(f(0))$ is equal to:
(a) $\tan^{-1} 2$ (b) $\cot^{-1} 2$ (c) $-\tan^{-1} 2$ (d) $-\cot^{-1} 2$

76. The set of values of p for which $f(x)=p^2x-\int 2^{4-x^2}dx$ is increasing for all $x \in R$, is:

(a) $[-4, 4]$ (b) $(-\infty,-16]\cup[16,\infty)$
(c) $(-\infty,-4]\cup[4,\infty)$ (d) $[-16, 16]$

77. The value of definite integral $\int_1^{\sqrt{3}}\left(x^{2x^2+1}+\ln\left(x^{\left(2x^{2x^2+1}\right)}\right)\right)dx$ is equal to:

(a) 2 (b) 3 (c) 8 (d) 13

78. If $\int_0^x f(t)\,dt = e^x - ae^{2x}\int_0^1 f(t)e^{-t}\,dt$, then $f(1)+2f(2)$ is equal to:

(a) $e-4e^4$ (b) $e-2e^4$ (c) $e-2e^2$ (d) $2e^2-e^4$

79. If $f(x)=\int_2^x \frac{dt}{1+t^4}$, then:

(a) $f(3)<\frac{1}{17}$ (b) $f(3)>\frac{1}{17}$ (c) $f(3)=\frac{1}{17}$ (d) $f(3)>1$

80. If $f(x)=g(x)\,|(x-1)(x-2)\ldots\ldots(x-10)|-2$ is derivable for all $x \in R$, where $g(x)=ax^9+bx^6+cx^3+d,\ a, b, c, d \in R$, then $f'(-1)$ is equal to:

(a) -2 (b) 0 (c) 2 (d) 4

81. A strictly increasing continuous function $f(x)$ intersects with its inverse $f^{-1}(x)$ at $x=\alpha$ and $x=\beta$. If $\int_\alpha^\beta (f(x)+f^{-1}(x))\,dx=13$ where $\alpha, \beta \in N$, then the value of $|\alpha\beta|$ equals:

(a) 25 (b) 36 (c) 42 (d) 56

82. Let $f(x)$ be a continuous and differentiable function such that
$\lim_{h\to 0}\frac{f(3+7h)-f(3+4h)}{h}=4$. Then the value of $f'(3)$ equals:

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{4}{3}$ (d) 1

83. The 1st, 2nd and 3rd terms of an arithmetic series are a, b and a^2, where a is negative then sum of an infinite geometric series whose first three terms are $a,\ a^2$ and b respectively, is:

(a) $\frac{-1}{2}$ (b) $\frac{-3}{2}$ (c) $\frac{-1}{3}$ (d) none of these

84. If $f(x)$ is a differentiable function defined for all positive real numbers such that $xf(x)=x+\int_1^x f(t)\,dt$, then the value of $\sum_{k=1}^{10} f(e^k)$ is:

(a) 45 (b) 55 (c) 65 (d) 75

85. The quadratic equation $x^2+bx+c=0$ has distinct roots. If 2 is subtract from each root then result are the reciprocal of the original root. The value of (b^2+c^2) is:

(a) 2 (b) 3 (c) 4 (d) 5

86. Let $f(x)$ and $g(x)$ are two function defined from $R^+ \quad R$ such that

$$f(x)=\begin{cases}1-\sqrt{x}, & \text{if } x \text{ is rational}\\ x^2, & \text{if } x \text{ is irrational}\end{cases} \quad \text{and} \quad g(x)=\begin{cases}x, & \text{if } x \text{ is rational}\\ 1-x, & \text{if } x \text{ is irrational}\end{cases}$$

The composite function $f(g(x))$ is:

(a) one-one onto (b) many one into
(c) one-one into (d) many one onto

87. A function $f: R \to R$ satisfies the equation $f(x)\,f(y)-f(xy)=x+y,\ \forall\, x, y \in R$ and $f(1)>0$, then:

(a) $f(x)f^{-1}(x)=x^2-4$ (b) $f(x)f^{-1}(x)=x^2-6$
(c) $f(x)f^{-1}(x)=x^2-1$ (d) $f(x)f^{-1}(x)=x^2$

88. $\lim\limits_{x\to 0}\dfrac{\tan(\pi\sin^2 x)+(|x|-\sin(x[x]))^2}{x^2}$ is equal to:

(where [ ] denotes greatest integer function)

(a) π (b) $\pi+1$ (c) 0 (d) does not exist

89. Let S denote the sum of an infinite geometric sequence with $S>0$. If the second terms of this sequence is 1,Then the minimum possible value of S, is:

(a) 2 (b) 4 (c) 6 (d) 8

90. Let $a=\sum\limits_{r=1}^{\infty}\dfrac{1}{r^2}$ and $b=\sum\limits_{r=1}^{\infty}\dfrac{1}{(2r-1)^2}$. Then the value of $\dfrac{3a}{b}$ is equal to:

(a) 2 (b) 3 (c) 4 (d) 6

91. The equations $(\lambda-1)x+(3\lambda+1)y+2\lambda z=0$, $(\lambda-1)x+(4\lambda-2)y+(\lambda+3)z=0$ and $2x+(3\lambda+1)y+3(\lambda-1)z=0$ give non-trivial solution for some values of λ then the ratio $x:y:z$, when λ has smallest of these values is:

(a) $3:2:1$ (b) $3:3:2$ (c) $1:3:1$ (d) $1:1:1$

92. If matrix $A=[a_{ij}]_{3\times 3}$, matrix $B=[b_{ij}]_{3\times 3}$, where $a_{ij}+a_{ji}=0$ and $b_{ij} \quad b_{ji}=0\ \forall\, i, j$, then A^4B^3 is:

(a) singular (b) zero matrix
(c) symmetric (d) skew symmetric

93. If A is an idempotent matrix satisfying $(I-0.4A)^{-1}=I \quad \alpha A$,where I is unit matrix of the same order as that of A, then the value of α is:

(a) $\dfrac{-1}{3}$ (b) $\dfrac{1}{3}$ (c) $\dfrac{-2}{3}$ (d) $\dfrac{2}{3}$

94. Let A and B are square matrices of same order satisfying $AB = A$ and $BA = B$, then $(A^{2019} + B^{2019})^{2020}$ is equal to:

(a) $A+B$ (b) $2020(A+B)$ (c) $2^{2019}(A+B)$ (d) $2^{2020}(A+B)$

95. Let a function $f(x)$ be defined in $[-2, 2]$ as $f(x) = \begin{cases} \{x\}, & -2 \le x < -1 \\ |\operatorname{sgn} x|, & -1 \le x \le 1 \\ \{-x\}, & 1 < x \le 2 \end{cases}$, where $\{x\}$ denotes fractional part, then area bounded by graph of $f(x)$ and x-axis is:

(a) 2 (b) 3 (c) 4 (d) 5

96. Area bounded by the curve $f(x) = \dfrac{x^2 - 1}{x^2 + 1}$ and the line $y = 1$ is:

(a) π (b) 2π (c) $\dfrac{\pi}{2}$ (d) none of these

97. The general solution of the differential equation $(1 + \tan y)(dx - dy) + 2x\,dy = 0$ is:

(a) $x(\sin y + \cos y) - \sin y + Ce^{y}$ (b) $x(\sin y + \cos y) = \sin y + Ce^{-y}$

(c) $y(\sin x + \cos x) = \sin x + Ce^{x}$ (d) none of these

[Note: Where C is constant of integration.]

98. The solution of the differential equation $y^2 dx + (x^2 - xy + y^2)dy = 0$, is:

(a) $\tan^{-1}\left(\dfrac{x}{y}\right) + \ln y + C = 0$ (b) $2\tan^{-1}\left(\dfrac{x}{y}\right) + \ln y + C = 0$

(c) $\ln(y + \sqrt{x^2 + y^2}) + \ln y + C = 0$ (d) $\ln(y + \sqrt{x^2 + y^2}) + C = 0$

[Note: Where C is constant of integration.]

99. The solution of the differential equation $e^{-x}(y+1)dy + (\cos^2 x - \sin 2x)y\,dx = 0$ subjected to condition that $y = 1$ when $x = 0$, is:

(a) $(y+1) + e^x \cos^2 x = 2$ (b) $y + \ln y = e^x \cos^2 x$

(c) $\ln(y+1) + e^x \cos^2 x = 1$ (d) $y + \ln y + e^x \cos^2 x = 2$

100. If $f(x) = x^3 - 3x + 1$, then minimum number of real roots of $f(f(x)) = 0$ is:

(a) 2 (b) 4 (c) 5 (d) 7

101. Slope of tangent to the curve $y = 2e^x \sin\left(\dfrac{\pi}{4} - \dfrac{x}{2}\right)\cos\left(\dfrac{\pi}{4} - \dfrac{x}{2}\right)$ where $0 \le x \le 2\pi$, is minimum at x is equal to:

(a) 0 (b) π (c) 2π (d) none of these

102. The tangent to the curve $y = xe^{x^2}$ at the point $(1, e)$, also passes through the point:

(a) $\left(\frac{5}{3}, 2e\right)$ (b) $\left(\frac{4}{3}, 2e\right)$ (c) $(3, 6e)$ (d) $(2, 3e)$

103. If $f(x) = \begin{cases} \left(\sin\left(\frac{2x^2}{\blacksquare}\right) + \cos\left(\frac{3x}{b}\right)\right)^{\frac{ab}{x^2}}, & x \neq 0 \\ e^{x^2-2x+3}, & x = 0 \end{cases}$ is continuous at $x = 0$, where $b \in R$, then the minimum value of a is:

(a) $\frac{-1}{8}$ (b) $\frac{-1}{4}$ (c) $\frac{-1}{2}$ (d) 0

104. Let S_n denote the sum of first n terms of the arithmetic sequence $\{a_n\}$. If $S_6 > S_7 > S_5$, then the value of integral value of n which satisfy $S_n S_{n+1} < 0$, is:

(a) 10 (b) 11 (c) 12 (d) 13

105. Assume that f is continuous on $[a, b]$, $a > 0$ and differentiable on (a, b). If $\frac{f(a)}{a} = \frac{f(b)}{b}$, then there exists $x_0 \in (a, b)$ such that:

(a) $x_0 f'(x_0) = f(x_0)$ (b) $f'(x_0) + x_0 f(x_0) = 0$
(c) $x_0 f'(x_0) + f(x_0) = 0$ (d) $f'(x_0) = x_0^2 f(x_0)$

106. Let $f : R \to R$ be continuous function and $f(x) = f(2x)$ is true $\forall\, x \in R$ and $f(1) = 3$, then the value of $\int_{-1}^{1} f(f(x))\, dx$ is equal to:

(a) 0 (b) 2 (c) 6 (d) 12

107. From a given solid cone of height H, another inverted cone is carved whose height is h, such that its volume is maximum, then the ratio $\frac{H}{h}$ is equal to:

(a) 2 (b) 3 (c) 4 (d) 6

108. If $f(x) = \int \frac{(3x^4 - 1)}{(x^4 + x + 1)^2}\, dx$ and $f(0) = 0$, then $f(-1)$ is equal to:

(a) $\frac{1}{3}$ (b) $\frac{2}{9}$ (c) 1 (d) 3

109. If $f(x) = x^4 \tan x^3 - x \ln(1 + x^2)$, then the value of $\frac{d^4 f(x)}{dx^4}$ at $x = 0$, is:

(a) 0 (b) 1 (c) $\frac{1}{5}$ (d) $\frac{1}{15}$

110. The number of integers n such that the equation $nx^2+(n+1)x+(n+1)=0$ has only rational roots, is equal to:

(a) 0 (b) 1 (c) 2 (d) more than 2

111. If $p=\cos 55°$, $q=\cos 65°$ and $r=\cos 175°$, then the value of $\frac{1}{p}+\frac{1}{q}+\frac{r}{pq}$ is equal to:

(a) 0 (b) -1 (c) 1 (d) 2

112. The sum of solutions in $(0, 2\pi)$ of the equation $\cos x\cos\left(\frac{\pi}{3}-x\right)\cos\left(\frac{\pi}{3}+x\right)=\frac{1}{4}$ is:

(a) 4π (b) π (c) 2π (d) 3π

113. The L denotes the value of the definite integral $\int_0^1 \frac{1}{1+x^8}dx$, then which one of the following must be true?

(a) $\frac{\pi}{4}<L<1$ (b) $L=\frac{\pi}{4}$ (c) $L>1$ (d) $0<L<\frac{\pi}{4}$

114. Let $f:R\to R$ defined by $f(x)=x^3+3x+1$ and g be the inverse of f, then the value of $g''(5)$ equals:

(a) $\frac{1}{6}$ (b) $\frac{-1}{6}$ (c) $\frac{1}{36}$ (d) $\frac{-1}{36}$

115. If $\lim_{x\to 0}\left(\frac{\sin 3x}{x^3}+\frac{a}{x^2}+b\right)=0$, then the value of $(a+b)$ equals:

(a) 0 (b) $\frac{1}{2}$ (c) $\frac{3}{2}$ (d) 3

116. Let $f(x)=\begin{cases}\left(\frac{2^x+3^x+5^x}{3}\right)^{\frac{3}{x}}, & x\neq 0\\ k, & x=0\end{cases}$.

If $f(x)$ is continuous then the value of k is equal to:

(a) 10 (b) 15 (c) 20 (d) 30

117. Let $f(x)=\sin\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right)$ for all $x\in R$. Then the range of $f(x)$, is:

(a) $(-0.25, 0.5)$ (b) $(-1, 1)$ (c) $[-0.5, 0.5]$ (d) $(-0.25, 0.25)$

118. Let $f(x)$ be a polynomial function satisfying $f'(x)+f(x)=x$. Then the value of $f(4)$ is equal to:

(a) 1 (b) 2 (c) 3 (d) 4

119. The value of $\dfrac{\int_0^{\pi/2}(5\cos^2 x+3\sin^2 x)\,dx}{\int_0^{\pi/2}\sin\theta\cos\theta\sqrt{25\sin^2\theta+9\cos^2\theta}\,d\theta}$ is equal to:

(a) $\frac{9\pi}{25}$ (b) $\frac{48\pi}{49}$ (c) $\frac{8\pi}{17}$ (d) $\frac{24\pi}{40}$

120. The value of $\lim_{x\to\frac{\pi}{2}}\dfrac{4(x-\pi)\cos^2 x}{\pi(\pi-2x)\tan\left(x-\frac{\pi}{2}\right)}$ is equal to:

(a) 1 (b) -1 (c) 0 (d) -2

121. If $\lim_{n\to\infty}\sum_{k=1}^{n}\dfrac{e^{\frac{k}{n}}+e^{\frac{-k}{n}}}{n\sqrt{11-e^{\frac{2k}{n}}-e^{\frac{-2k}{n}}}}=\sin^{-1}\left(\dfrac{e^a-e^{-a}}{b}\right)$ where a and b are positive integers, then the value of $a+b$ is:

(a) 2 (b) 3 (c) 4 (d) 5

122. Let $y=y(x)$ satisfies the differential equation $y'=\ln(xy'-y)$. If $y(1)=-1$ where y is twice differentiable and $y''(x)\neq 0$, then $y(e)$ equals:

(a) 0 (b) 1 (c) e (d) π

123. If the curves $y=\frac{1}{a}e^x$ and $y=\ln(ax)$, (where a is positive) has only one point is common, then the value of $[a]$ is:

[Note: $[\cdot]$ denotes the greatest integer function.]

(a) 1 (b) 2 (c) 3 (d) 4

124. Let $y=f(x)$ be a differentiable function satisfying $f(x)+f'(x)=xe^{-x}$ for all values of real x. If $f(0)=0$, then the value of $f(1)$ equals:

(a) $\frac{1}{2e}$ (b) $\frac{7}{8e}$ (c) $\frac{1}{8e}$ (d) $\frac{3}{4e}$

125. Curve is parametrically represented by $\begin{cases}x=\cos t+\ln\left(\tan\frac{t}{2}\right)\\ y=\sin t\end{cases}$ where t is a parameter. The length of the tangent drawn to the curve at the point where its x-coordinates is equal to its y-coordinates is:

(a) 1 (b) 2 (c) 3 (d) 4

126. If $\alpha_1, \alpha_2, \alpha_3$ and α_4 are the roots of the equation $x^4 + (2-\sqrt{3})x^2 + (2+\sqrt{3}) = 0$, then the value of $(1-\alpha_1)(1-\alpha_2)(1-\alpha_3)(1-\alpha_4)$ is equal to:

(a) 1 (b) 4 (c) $2+\sqrt{3}$ (d) 5

127. The function $f:[0,3] \to [1,29]$, defined by $f(x) = 2x^3 \quad 15x^2 + 36x + 1$, is:

(a) one-one and onto
(b) one-one but not onto
(c) onto but not one-one
(d) neither one-one nor onto

128. If S is the set of all real numbers. A relation R has been defined on S by $aRb \Longleftrightarrow |a-b| \le 1$, then R is:

(a) symmetric and transitive but not reflexive
(b) reflexive and transitive but not symmetric
(c) reflexive and symmetric but not transitive
(d) an equivalence relation

129. If f is a function with domain $[-3, 5]$ and $g(x) = |3x+4|$, then the domain of $(fog)(x)$ is:

(a) $\left(-3, \frac{1}{3}\right)$ (b) $\left[-3, \frac{1}{3}\right)$ (c) $\left[-3, \frac{1}{3}\right]$ (d) $\left[-3, \frac{-1}{3}\right]$

130. The range of the function $f(x) = x^2 + \dfrac{1}{x^2+1}$ is:

(a) $[1, \infty)$ (b) $[2, \infty)$ (c) $\left[\frac{3}{2}, \infty\right)$ (d) $[5, \infty)$

131. Let $f(x) = (x+2)^2 - 2$, $x \ge -2$. If $g(x)$ is a function whose graph is reflection of the graph of $y = f(x)$ in the line $y = x$, then $g(x)$ is equal to:

(a) $-\sqrt{2+x} - 2$ (b) $\sqrt{2+x} + 2$ (c) $\sqrt{2+x} - 2$ (d) $-\sqrt{2+x} + 2$

132. The function $f(x)$ satisfies the functional equation $3f(x) + 2f\left(\dfrac{x+59}{x-1}\right) = 10x + 30$ for all real $x \ne 1$. The value of $f(7)$ is:

(a) 8 (b) 4 (c) -8 (d) 11

133. Which of the following is inverse to itself?

(a) $f(x) = \dfrac{1-x}{1+x}$ (b) $f(x) = e^{\log x}$ (c) $f(x) = 3^{x(x+1)}$ (d) none of these

134. If $f: R \to R$ is defined as $f(x) = \begin{cases} x+4, & x < -4 \\ 3x+2, & -4 \le x < 4 \\ x-4, & x \ge 4 \end{cases}$ then the value of $f(f(f(f(0)))) + 1$ is equal to:

(a) 0 (b) 1 (c) 2 (d) 4

135. The function $f:[0,\infty)\rightarrow[0,\infty)$ defined by $f(x)=\dfrac{2x}{1+2x}$ is:

(a) one-one onto
(b) one-one but not onto
(c) onto but not one-one
(d) neither one-one nor onto

136. Let $f:R\rightarrow R$ be given as $f(x)=\begin{cases}2x+\alpha^2, & x\geq 2\\ \dfrac{\alpha x}{2}+10, & x<2\end{cases}$. If $f(x)$ is into function then least integral positive value of α is:

(a) 1
(b) 2
(c) 3
(d) 4

137. If $g(x)$ and $h(x)$ are invertible function and $h(x)=3g(x)+7$, then $h^{-1}(x)$ is equal to:

(a) $3g^{-1}(x)-7$
(b) $\dfrac{1}{3g^{-1}(x)+7}$
(c) $\frac{1}{3}g^{-1}(x)+7$
(d) $g^{-1}\left(\dfrac{x-7}{3}\right)$

138. Let a polynomial $P(x)$, when divided by $x-1, x-2, x-3$ leaves the remainder 4, 5, 6 respectively. When $P(x)$ is divided by $(x-1)(x-2)(x-3)$, the remainder is ax^2+bx+c then $3a+2b+c$ is equal to:

(a) 3
(b) 4
(c) 5
(d) 6

139. If T_n denotes the n^{th} term of an arithmetic progression such that $T_p=\dfrac{1}{q}$ and $T_q=\dfrac{1}{p}$, then which of the given option is necessarily a root to the equation $(p+2q-3r)x^2+(q+2r-3p)x+(r+2p-3q)=0$, given that $p+2q-3r\neq 0$?

(a) T_{pq}
(b) T_p
(c) T_q
(d) T_{p+q}

140. Let $y=\log_2 x+\log_4 x+\log_{16} x+\ldots\ldots+\infty$ and $4\log_4 x=\dfrac{5+9+13+\ldots\ldots+(4y+1)}{1+3+5+\ldots\ldots+(2y-1)}$, then the value of x^2y equals:

(a) 20
(b) 22
(c) 24
(d) 28

141. If the range of $f(x)=\dfrac{1}{2\{-x\}}-\{x\}$ is $[a, b)$ for real x, then the value of 'a' is:

[Note: $\{k\}$ denotes fraction part function of k.]

(a) $\tan\dfrac{\pi}{8}$
(b) $\cot\dfrac{\pi}{8}$
(c) $\sin\dfrac{\pi}{10}$
(d) $\cos\dfrac{\pi}{5}$

142. If $x=\sin^{-1}(\sin 10)$ and $y=\cos^{-1}(\cos 10)$, then $y-x$ is equal to:

(a) π
(b) 0
(c) 10
(d) 7π

143. The value of λ for which the sum of squares of the roots of the quadratic equation $x^2+(3-\lambda)x+2=\lambda$ has the least value, is:

(a) 1 (b) 2 (c) $\frac{15}{8}$ (d) $\frac{4}{9}$

144. If $\lim\limits_{x\to 0}\frac{(1-\cos x)(\sin x-x)(e^x+e^{-x}-2)}{x^n}$ is finite and non-zero, then the minimum integral value of n, is:

(a) 5 (b) 6 (c) 7 (d) 8

145. If $\tan\alpha=2$, then the value of $\frac{\sin\alpha+\cos\alpha}{3\sin\alpha-2\cos\alpha}$ is equal to:

(a) $\frac{1}{4}$ (b) $\frac{3}{4}$ (c) $\frac{5}{4}$ (d) 2

146. If $\lim\limits_{x\to\infty}\left(\frac{x^3+1}{x^2+1}-(ax+b)\right)=2$, then:

(a) $a=1, b=1$ (b) $a=1, b=2$ (c) $a=1, b=\ 2$ (d) $a=-1, b=-2$

147. If the equations $x^2+2\lambda x+\lambda^2+1=0$, $\lambda\in R$ and $ax^2+bx+c=0$, where a, b, c are lengths of sides of triangle have a common root then the possible range of λ is:

(a) $(0, 2)$ (b) $(\sqrt{3}, 3)$ (c) $(2\sqrt{2}, 3\sqrt{2})$ (d) $(0, \infty)$

148. The value of $\lim\limits_{x\to 2}\left(\frac{1}{x-2}-\frac{2(2x-3)}{x^3-3x^2+2x}\right)$ equals:

(a) $\frac{3}{4}$ (b) $\frac{1}{2}$ (c) $\frac{-1}{2}$ (d) $\frac{-3}{4}$

149. The function $f(x)=\log(x\ \ 1)\quad\log(x-2)$ and $g(x)=\log\left(\frac{x-1}{x-2}\right)$ are identical when x lies in the interval:

(a) $[1, 2]$ (b) $[2, \infty)$ (c) $(2, \infty)$ (d) $(-\infty, \infty)$

150. $\lim\limits_{x\to 0}\frac{5^{3x}-5^x-5^{2x}+1}{\sqrt{1+\cos x}-\sqrt{2}}$ equals:

(a) $-8\sqrt{2}\ln^2 5$ (b) $8\sqrt{2}\ln^2 5$ (c) $-4\sqrt{2}\ln^2 5$ (d) $4\sqrt{2}\ln^2 5$

151. If range of $\cot^{-1}x$ is taken as $\left(0, \frac{\pi}{2}\right]$ then the domain of the function $f(x)=\frac{1}{\sqrt{\ln(\cot^{-1}x)}}$, is:

(a) $(-\infty, \cot 1)$ (b) $(\cot 1, \infty)$ (c) $[0, \cot 1)$ (d) $(0, \cot 1)$

152. The minimum value of the expression $\dfrac{\sin^3\alpha+6\sin^2\alpha+\sin\alpha+2\cos^2\alpha-8}{\sin\alpha-1}$ is equal to:

(a) $\dfrac{-1}{4}$ (b) 2 (c) $\dfrac{3}{4}$ (d) -2

153. Let $f(x)$ be a polynomial satisfying $f(x)f\left(\dfrac{1}{x}\right)+5-3f(x)-3f\left(\dfrac{1}{x}\right)=0,\ \forall x\in R-\{0\}$

and $f(2)=11$, then $f(3)$ is equal to:

(a) 21 (b) 12 (c) 20 (d) 11

154. Let $f:R\to\left(0,\dfrac{2\pi}{3}\right]$ defined as $f(x)=\cot^{-1}(x^2-4x+\alpha)$. The smallest integral value of α such that $f(x)$ is into function, is equal to:

(a) 2 (b) 4 (c) 6 (d) 8

155. The value of $\lim\limits_{x\to 0}\dfrac{\dfrac{x^2}{2}+1-\sqrt{1+x^2}}{\left(\cos x-e^{x^2}\right)\sin(x^2)}$ is equal to:

(a) $\dfrac{1}{12}$ (b) $\dfrac{-1}{12}$ (c) $\dfrac{1}{6}$ (d) $\dfrac{-1}{6}$

156. If $f(x)=3x+|x|,\ g(x)=\dfrac{3x}{4}-\dfrac{|x|}{4}$, then:

(a) $(fog)(x)=3x$ (b) $(fog)(x)=4x$ (c) $(fog)(x)=5x$ (d) $(fog)(x)=2x$

157. The value of x for which the equation $|x^2+6x+6|=|x^2+4x+9|+|2x-3|$ holds, is equal to:

(a) $\left[\dfrac{3}{2},\infty\right)$ (b) $\left(-\infty,\dfrac{3}{2}\right]$ (c) $(-\infty,1]\cup\left[\dfrac{3}{2},\infty\right)$ (d) none of these

158. If $(\sin^{-1}x)^2+(\sin^{-1}y)^2+2\sin^{-1}x\sin^{-1}y=\pi^2$, then x^2+y^2 is equal to:

(a) 1 (b) $\dfrac{3}{2}$ (c) 2 (d) $\dfrac{1}{2}$

159. A continuous even periodic function f with period 8 is such that $f(0)=0,\ f(1)=-2,\ f(2)=1,\ f(3)=2,\ f(4)=3$ then the value of $\tan^{-1}(\tan(f(-5)+f(20)+\cos^{-1}(f(-10)+f(17))))$ is equal to:

(a) $2\pi-5$ (b) $5-2\pi$ (c) $3+\pi$ (d) $3-\pi$

160. If $\cos^{-1}\left(\dfrac{2}{3x}\right)+\cos^{-1}\left(\dfrac{3}{4x}\right)=\dfrac{\pi}{2}\left(x>\dfrac{3}{4}\right)$, then x is equal to:

(a) $\dfrac{\sqrt{146}}{12}$ (b) $\dfrac{\sqrt{145}}{11}$ (c) $\dfrac{\sqrt{145}}{10}$ (d) $\dfrac{\sqrt{145}}{12}$

161. $\lim_{x\to\infty} x\left(\left(\frac{x}{x+1}\right)^x - \frac{1}{e}\right)$ is equal to:

(a) $\frac{-1}{2e}$ (b) $\frac{1}{2e}$ (c) $\frac{-1}{e}$ (d) $\frac{1}{e}$

162. If the first, fifth and last terms of an A.P. are l, m, p respectively and the sum of A.P. is $\frac{(l+p)(4p+m-5l)}{k(m-l)}$, then k is:

(a) 2 (b) 3 (c) 4 (d) 5

163. If $a, b, c \in R$ and $a^2+b^2+c^2+4 = ab+bc+2c+2a$, then roots of $ax^2+bx+c=0$ are:

(a) real and distinct (b) real and equal

(c) imaginary (d) none of these

164. If the roots of $x^4+qx^2+kx+225=0$ are in arithmetic progression, then the value of q is:

(a) 15 (b) -25 (c) 35 (d) -50

165. The sum of the infinite series $\frac{1}{9}+\frac{1}{18}+\frac{1}{30}+\frac{1}{45}+\frac{1}{63}+\ldots\ldots$

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{1}{5}$ (d) $\frac{2}{3}$

166. If $a+c, a+b, b+c$ are in G.P. and a, c, b are in H.P. where $a, b, c>0$, then the value of $\frac{a+b}{c}$ is:

(a) 3 (b) 2 (c) $\frac{3}{2}$ (d) 4

167. The number of values of k for which the equation $(x^2+(2k-6)x+7-3k)(x^2+(2k-2)x+3k-5)=0$ has two different pairs of equal roots, is equal to:

(a) 0 (b) 1 (c) 2 (d) more than 2

168. Let x_1 and x_2 $(x_1 > x_2)$ are the roots of the equation $9^{\log_9(x^2-4x+5)} = x-1$, then the value of $\tan(x_1)\pi + \sec(x_2)\pi$ is:

(a) -1 (b) 0 (c) 1 (d) not defined

169. If $\sin\alpha+\sin\beta+\sin\gamma=-3$, $\alpha, \beta, \gamma \in (0, 2\pi)$, then $\cos 2\alpha+\cos 4\beta+\cos 6\gamma$ is equal to:

(a) -1 (b) 0 (c) 1 (d) 2

170. Number of real solution(s) of the equation $|x-3|^{3x^2-10x+3}=1$ is:

(a) exactly four (b) exactly three (c) exactly two (d) exactly one

171. The least positive value of x satisfying the equation $\frac{\sin x}{\cos 3x}+\frac{\sin 3x}{\cos 9x}+\frac{\sin 9x}{\cos 27x}=0$ is:

(a) $\frac{\pi}{26}$ (b) $\frac{\pi}{27}$ (c) $\frac{\pi}{9}$ (d) $\frac{\pi}{3}$

172. The value of $\cos\left(\log_5\left(\frac{\sin^2 A+\cos^2 A+\tan^2 A-\sec^2 A\cdot\sin^2 A}{(1+\tan^2 A)(1-\sin^2 A)}\right)\right)$ is equal to:

(a) 0 (b) $\frac{1}{2}$ (c) $\cos 1^\circ$ (d) 1

173. If $\cos x+\cos^2 x-1$. Let $E=\sin^{12}x+3\sin^{10}x+3\sin^8 x+\sin^6 x+2$, then the value of $\log_{\tan\frac{\pi}{3}} E$ is:

(a) 1 (b) 2 (c) $\frac{1}{2}$ (d) $\frac{-1}{2}$

174. Solution set of the equation $\sqrt{4^x-2^{x+1}+1}+\sqrt{4^x-2^{x+3}+16}=3$ is:

(a) $x\in(0,2)$ (b) $x\in(0,2]$ (c) $x\in[0,2]$ (d) $x=\{0,2\}$

175. If A lies in the fourth quadrant and $3\tan A+4=0$, then $5\sin 2A+2\sin A+4\cos A$ is equal to:

(a) -1 (b) -2 (c) -3 (d) -4

176. If $\alpha=\sin\theta\,|\sin\theta|$ and $\beta=\cos\theta\,|\cos\theta|$ where $\theta\in\left[\frac{199\pi}{2},100\pi\right]$, then:

(a) $\alpha+\beta=1$ (b) $\alpha+\beta=-1$ (c) $\beta-\alpha=-1$ (d) $\alpha-\beta=1$

177. The polynomials $P(x)=kx^3+3x^2-3$ and $Q(x)=2x^3-5x+k$, when divided by $(x-4)$ leave the same remainder, then k is equal to:

(a) 2 (b) 1 (c) 0 (d) -1

178. Let $a=\log 25$ and $b=\log 225$, then $\log\left(\frac{1}{81}\right)+\log\left(\frac{1}{2250}\right)$ is equal to:

(a) $2a+3b+1$ (b) $2a-3b+1$ (c) $2a-3b-1$ (d) $2a+3b$

179. Let $a, b\in R^+$, such that $\log_{27}a+\log_9 b=\frac{7}{2}$ and $\log_{27}b+\log_9 a=\frac{2}{3}$, then ab is equal to:

(a) 32 (b) 243 (c) 1024 (d) 125

180. If $\sqrt{\left(\frac{1}{\sqrt{27}}\right)^{2-\log_5 13+(2\log_5 9)}}=\left(\frac{\sqrt[a]{b}}{c}\right)^{3/2}$ where a, b, c are co-prime, then:

(a) $b>a+c$ (b) $a>b+c$ (c) $c>a+b$ (d) none of these

181. Total number of solution of $\sin x \cdot \tan 4x = \cos x$ in $x \in (0, \pi)$ is/are:
(a) 7 (b) 6 (c) 5 (d) 4

182. If $\log_b a \cdot \log_c a + \log_a b \cdot \log_c b + \log_a c \cdot \log_b c = 3$, where a, b, c are positive and different real numbers $\neq 1$, then abc is equal to:
(a) 0 (b) 1 (c) 2 (d) -1

183. Value of $\sin\frac{\pi}{n} + \sin\frac{3\pi}{n} + \sin\frac{5\pi}{n} + \ldots\ldots$ to n terms is:
(a) $\frac{-1}{2}$ (b) 0 (c) $\frac{1}{2}$ (d) 1

184. Let $S = \sum_{\alpha=1}^{17} \sin^2(5\alpha)^\circ$, then $[S]$ is equal to:
[**Note:** $[y]$ denote greatest integer function less than or equal to y.]
(a) 9 (b) 8 (c) 17 (d) 18

185. If $2\cos\theta - \sin\theta + 2 = 0$, then the value of $10\cos\theta + 15\sin\theta$ is equal to:
(a) 2 (b) 6 (c) 25 (d) 5

186. If $N = \sqrt{9\cos^2\theta + 16\sin^2\theta} + \sqrt{16\cos^2\theta + 9\sin^2\theta}$, then the sum of the maximum and minimum value of N^2 is:
(a) 49 (b) 50 (c) 99 (d) 100

187. For three concentric circle C_1, C_2 and C_3 with radius 1, r and 9 respectively. If from a point A on C_3 a pair of tangents to circle C_2 are drawn to touch at B and C such that BC is tangent to circle C_1, then the value of $10r$, is [where $1 < r < 9$]:
(a) 30 (b) 40 (c) 50 (d) 60

188. Let $f(x)$ be continuous function satisfying $f(x) = \int_0^x e^{x-y} f'(y)\,dy - (x^2 - x + 1)e^x$.
The value of x for which $f(x) = 0$, is:
(a) $\frac{1}{2}$ (b) 1 (c) $\frac{1}{4}$ (d) $\frac{1}{8}$

189. The value of the expression $\sec^2(\tan^{-1}2) + \operatorname{cosec}^2(\cot^{-1}3) + \operatorname{cosec}\left(2\cot^{-1}2 + \cos^{-1}\frac{3}{5}\right)$ is equal to:
(a) $15 + \frac{24}{25}$ (b) $24 + \frac{15}{25}$ (c) $25 + \frac{15}{24}$ (d) $15 + \frac{25}{24}$

190. Let $f : (-1, 1) \to R$ defined by $f(x) = \ln\left(\frac{1+x}{1-x}\right)$ and $g : R \to (-1, 1)$ defined by $g(x) = \frac{3x + x^3}{1 + 3x^2}$, then $f(g(x))$ is equal to:
(a) $f(x)$ (b) $f^2(x)$ (c) $3f(x)$ (d) $-f(x)$

191. Let $f(x)=\cot^{-1}\left(\operatorname{sgn}\left(\frac{[x]}{2x-[x]}\right)\right)$:

Statement-1: $f(x)$ is discontinuous at $x=1$.

Statement-2: $f(x)$ is non-differentiable at $x=1$.

Which of the following option is correct?

(a) Statement-1 and statement-2 are incorrect.

(b) Statement-1 and statement-2 are correct.

(c) Statement-1 is correct and statement-2 is incorrect.

(d) Statement-1 is incorrect and statement-2 is correct.

[**Note:** $[k]$ denotes greatest integer function less than or equal to k.]

192. Let $f(x)$ be a continuous function $\forall x \in R$ such that $\lim\limits_{x\to\pi/4}\dfrac{\int_2^{\sec^2 x} f(t)dt}{x^2-\frac{\pi^2}{16}} = \frac{k}{\pi}f(a)$ where $a, k \in N$, then the value of k^a is equal to:

(a) 4 (b) 16 (c) 64 (d) 256

193. Let $p(x)=51x^2+mx+c$ and $q(x)=3x^2+bx+a$ are two quadratic polynomial with integer coefficients such that $p(r)=q(r)=0$. If r is an irrational number, then the value of $\frac{c}{a}$ is:

(a) 15 (b) 17 (c) 51 (d) 153

194. If the function $f(x)$ on the domain $\left[\frac{1}{2},\infty\right)$ is defined by $f(x)=2^{x(x-1)}$ then $f^{-1}(x)$ equals:

(a) $\frac{1}{2}(1+\sqrt{1+4\log_2 x})$

(b) $\frac{1}{2}(1-\sqrt{1+4\log_2 x})$

(c) $\sqrt{1+4\log_2 x}$

(d) $\sqrt{1-4\log_2 x}$

195. The range of the function $f(x)=\sin^{-1}\left(\log_2\frac{x^2}{2}\right)$, is:

(a) $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$ (b) $[0,\pi]$ (c) $[-\pi,\pi]$ (d) $\left[0,\frac{\pi}{2}\right]$

196. Let f and g be defined such that $f'(x) = f^2(x) + g^2(x)$ and $g'(x) = 2f(x)g(x) + 1$.

If $f(0) = \frac{1}{5}$, $g(0) = \frac{4}{5}$, then the value of $f\left(\frac{\pi}{12}\right) + g\left(\frac{\pi}{12}\right)$ equals:

(a) 0 (b) $\frac{\sqrt{3}}{2}$ (c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$

197. Let $g(x) = \frac{1}{f^{-1}(x)}$. Given the following data

x	0	1	2	3	4
$f(x)$	-2	-1	2	4	6
$f'(x)$	1/2	2/3	1	4/3	5/3

The value of $g'(4)$, is:

(a) $\frac{-1}{12}$ (b) $\frac{-1}{15}$ (c) $\frac{1}{12}$ (d) $\frac{1}{15}$

198. If $y = \sqrt{\dfrac{x}{\sqrt{\dfrac{x}{\sqrt{\dfrac{x}{\sqrt{\ldots}}}}}}}$, then the value of $\frac{dy}{dx}$ at $x = 8$, is:

(a) $\frac{-4}{3}$ (b) $\frac{1}{12}$ (c) $\frac{4}{3}$ (d) $\frac{-1}{12}$

199. Let $f(x)$ be a function defined by $f(x) = (k \quad x^{10})^{1/10}$ where $k = 1025$ and $f'(2) = \frac{1}{f'(a)}$

where $a \in N$, then 'a' equals:

(a) 1 (b) 2 (c) 3 (d) 4

200. If $y = 2\tan^{-1}\left(\frac{\sqrt{1+x^2} - 1}{x}\right)$ then the value of $\frac{d^2y}{dx^2}$ at $x - 2$, is:

(a) $\frac{2}{36}$ (b) $\frac{-4}{25}$ (c) $\frac{-4}{5}$ (d) $\frac{4}{25}$

201. Let $f: R \to R$ be a function such that for all $x, y \in R$, $f(x) - f(y)| \le 6|x - y|^2$, if $f(3) = 6$ then $f(6)$ is equal to:

(a) 0 (b) 1 (c) 3 (d) 6

202. If $y=1+\dfrac{c_1}{x-c_1}+\dfrac{c_2x}{(x-c_1)(x-c_2)}+\dfrac{c_3x^2}{(x-c_1)(x-c_2)(x-c_3)}$ then $\dfrac{dy}{dx}$ is equal to:

(a) $\dfrac{-y}{x}\left[\dfrac{c_1}{c_1-x}+\dfrac{c_2}{x}+\dfrac{c_1}{c_3-x}\right]$ (b) $\dfrac{-y}{x}\left[\dfrac{c_1}{x}+\dfrac{c_2}{x}+\dfrac{c_3}{c_3-x}\right]$

(c) $\dfrac{y}{x}\left[\dfrac{c_1}{c_1-x}+\dfrac{c_2}{c_2-x}+\left(\dfrac{c_3}{-x}\right)\right]$ (d) $\dfrac{y}{x}\left[\dfrac{c_1}{c_1-x}+\dfrac{c_2}{c_2-x}+\dfrac{c_3}{c_3-x}\right]$

203. Let $f:[-1,0]\to R$ be a function differentiable within the domain and that $\int_{-1}^{0}(f(x))^2dx=10$ and $f(-1)=2$. The value of the integral $\int_{-1}^{0}xf'(x)f(x)\,dx$, is:

(a) -1 (b) -2 (c) -3 (d) -4

204. The value of the definite integral $\int_{-\pi}^{\pi}\dfrac{2x(1-\sin x)}{1+\cos^2 x}dx$ is equal to:

(a) $-\pi^2$ (b) $\dfrac{\pi^2}{2}$ (c) $\dfrac{\pi^2}{4}$ (d) π^2

205. Let a_n be an infinite geometric sequence with a convergent and negative sum. The common ratio of the sequence is r and the first term is a_1 then which one of the following is always true?

(a) $a_1<0$ (b) $a_1>0$ or $r<0$ (c) $a_1<0$ and $r<0$ (d) $r<0$

206. The angle between the curves $x^3-3xy^2+2=0$ and $3x^2y-y^3\quad 2=0$ is:

(a) $\pi/2$ (b) $\pi/4$ (c) $\pi/3$ (d) $\pi/6$

207. Let f be a function defined by $y=f(x)$ where $x=2t-|t|$ and $y=t^2+t|t|$ for $t\in R$, then:

(a) $f(x)$ is both continuous and differentiable at $x=0$.

(b) $f(x)$ is non differentiable at $x=0$.

(c) $f(x)$ is discontinuous at $x=0$.

(d) $f(x)$ is neither continuous nor differentiable at $x=0$.

208. If $f(x)$ and $g(x)$ are both continuous function then the value of

$$\int_{\ln\lambda}^{\ln(1/\lambda)}\frac{f\left(\frac{x^2}{4}\right)(f(x)-f(-x))}{g\left(\frac{x^2}{4}\right)(g(x)+g(-x))}dx \text{ is equal to:}$$

(a) λ (b) 2λ (c) 3λ (d) 0

209. Let $f(x)$ be a continuous function in (0, 1) satisfying $\int_0^1 x\sqrt{x}\, f(x)(1-\sqrt{x} f(x))\,dx = \frac{1}{8}$.

Number of solutions of the equation $f(x) = e^x$ is:

(a) 0 (b) 1 (c) 2 (d) 3

210. The value of $\lim_{x\to 0^+} \frac{\int_0^{\arctan x} (\sin t^2)\,dt}{x\cos x - x}$ is equal to:

(a) $\frac{1}{3}$ (b) $\frac{-1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{-2}{3}$

211. The value of the definite integral $\int_0^{\pi/4} \frac{\sin^3 x \cos^3 x}{(\sin^4 x + \cos^4 x)^2}\,dx$ is equal to:

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{1}{6}$ (d) $\frac{1}{8}$

212. Let $f(x) = x - \frac{1}{x+1}$ and $g(x) = x^2 - 2ax + 4$, where 'a' is a parameter. If $\forall\, x_1 \in [0, 1]$ there exists some $x_2 \in [1, 2]$, such that $f(x_1) \ge g(x_2)$. Then the minimum value of 'a' is:

(a) 1 (b) $\frac{11}{4}$ (c) 3 (d) $\frac{9}{4}$

213. Let $f(x) = \begin{cases} x^2 - 2|x| + a, & x \le 1 \\ 6 + x, & x > 1 \end{cases}$, then number of positive integral value(s) of 'a' for which $f(x)$ has local minima at $x = 1$, is/are:

(a) 6 (b) 7 (c) 8 (d) 9

214. If the function $f(x) = (x^2 + ax + 2a)e^x$ is a strictly increasing function in $(-\infty, \infty)$. Then the number of integral values of 'a' is:

(a) 5 (b) 6 (c) 7 (d) 8

215. In triangle ABC if $\frac{[\Delta ABC]}{R} = 4$, then the value of $a\cos A + b\cos B + c\cos C$, is:

[**Note:** R is the circumradius of triangle ABC and $[\Delta ABC]$ is the area of ΔABC]

(a) 4 (b) 6 (c) 8 (d) 12

216. In a non constant arithmetic progression having odd number of terms, having positive integral common difference, the ratio of the sum of the 1^{st}, 3^{rd}, 5^{th}, 7^{th}, ... terms to the sum of remaining terms is 13 : 12, then the number of terms in the arithmetic progression, is:

(a) 21 (b) 23 (c) 25 (d) 27

217. If the angles A, B and C of triangle ABC are in arithmetic progression and a, b, c represents length of sides opposite to angles A, B and C respectively, then the value of $\frac{a+c}{\sqrt{(a^2-ac+c^2)}}$, is:

(a) $2\cos\frac{A+C}{2}$ (b) $2\sin\frac{A-C}{2}$ (c) $2\sin\frac{A+C}{2}$ (d) $2\cos\frac{A-C}{2}$

218. Let a, b and c be non-negative real numbers satisfying $a+b+c=9$. If the maximum value of the expression $a^2b^3c^4$ can be expressed as 2^x3^y, where x and y are natural numbers, then the value of $\log_{10}(x^y)$, is:

(a) 2 (b) 3 (c) 4 (d) 6

219. If α and β are the roots of the equation $x^2 - mx + 2 = 0$ and $\alpha + \frac{1}{\beta}$, $\beta + \frac{1}{\alpha}$ are the roots of the equation $x^2 - px + q = 0$, then the value of $2q$ equals:

(a) 1 (b) 3 (c) 6 (d) 9

220. Let $f(x) = \ln(x^2 + ax + 1)$. If $f(x)$ is defined $\forall\, x \in R$, then the number of integers in the range of 'a' is:

(a) 1 (b) 3 (c) 6 (d) 9

221. Let $a, b, c \in N$ such that $a < b < c$ satisfying the relation

$$abc + 2bc + 2ac + 2ab + 4a + 4b + 4c = 200.$$

The number of possible values of $a+b+c$ is:

(a) 3 (b) 4 (c) 5 (d) 6

222. For a constant k, the two roots of the quadratic equation $3x^2 - x + k = 0$ are $\sin\theta$ and $\cos\theta$. The value of $54(\sin^3\theta + \cos 3\theta)$, is:

(a) 25 (b) 26 (c) 27 (d) 28

223. Suppose in ΔABC with sides a, b, c the following equation holds true

$$\frac{\cos A}{a} + k_1 = \frac{\cos B}{b} + k_2 = \frac{\cos C}{c} + k_3 = \frac{a^2+b^2+c^2}{8}$$

If $abc = 4$, then the value of $k_1k_2k_3$ is:

(a) 2 (b) 4 (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

224. Let $ABCD$ be a parallelogram, the equations of whose diagonals are $AC: x + 2y - 3 = 0$ and $BD: 2x + y - 3 = 0$. If the length of the diagonal $AC = 4$ units and the area of the parallelogram $[ABCD] = 8$ square units. The length of side BD, is:

(a) $\frac{20}{3}$ (b) 5 (c) $\frac{10}{3}$ (d) 2

225. If $\tan(\alpha-\beta)=\dfrac{\sin(2\beta)}{3-\cos(2\beta)}$, then $\tan\alpha - f(\beta)$. The value of $f\left(\dfrac{\pi}{3}\right)$ equals:

(a) $\sqrt{2}$ (b) $\sqrt{3}$ (c) $2\sqrt{3}$ (d) $3\sqrt{2}$

226. If $a_n=\sum_{r=0}^{n}\dfrac{1}{{}^nC_r}=b_n=\sum_{r=0}^{n}\dfrac{1}{{}^nC_r}$, then the number of ordered pairs (p, q) such that $c_p+c_q=1$, where $c_p=\dfrac{a_p}{b_p}$, is:

(a) 0 (b) 1 (c) 2 (d) 3

227. If $(x+1)$ is factor of $x^3+kx^2-3x+k+2$, then k is equal to:

(a) 0 (b) 1 (c) -2 (d) -1

228. Value of $\log_6(\sqrt{2-\sqrt{3}}+\sqrt{2+\sqrt{3}})$ is:

(a) negative integer (b) rational but not integer
(c) irrational (d) prime

229. Number of value(s) of 'x' satisfying the equation $\log_2(\log_3(x^2))=1$ is/are:

(a) 0 (b) 1 (c) 2 (d) 3

230. Let $a=7^{\frac{1}{\log_8\sqrt{343}}}$ and $b=11^{\frac{1}{\log_5\sqrt{11}}}$, then:

(a) both a and b are prime (b) a and b are relatively prime
(c) both a and b are irrational (d) a and b both are rational but not integer

231. Value of $\dfrac{(x-1)^3+(2x-1)^3-(3x-2)^3}{(x-1)(2x-1)(3x-2)}$ is equal to:

(a) -3 (b) 0 (c) 1 (d) 3

232. If $f(x)=\sin^{-1}\left(\dfrac{2x}{1+x^2}\right)-2\tan^{-1}x$ and $g(x)=\sin^{-1}\left(\dfrac{1-x^2}{1+x^2}\right)+4\tan^{-1}x$ then range of $(f(x)-g(x))$ for $x\in(-\infty,-1]$, is:

(a) $\left[0,\dfrac{3\pi}{2}\right)$ (b) $\left[\dfrac{-3\pi}{2},\pi\right)$ (c) $\left[-\pi,\dfrac{-\pi}{2}\right)$ (d) $\left[\pi,\dfrac{7\pi}{2}\right)$

233. Let $A=\{1, 2, 3, 4, 5, 6, 7\}$. The number of surjective functions defined from A to A such that $f(i)=i$ for atleast four values of i from $i=1, 2, \ldots\ldots, 7$, is:

(a) 7! (b) 92 (c) 126 (d) 407

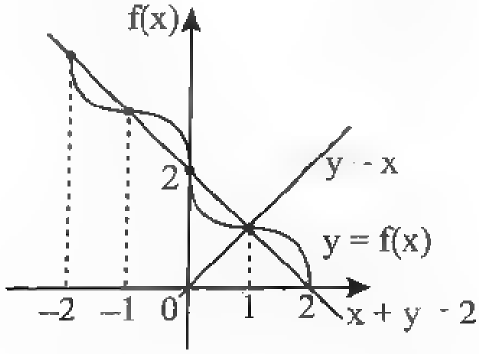
234. If α is the root of the equation $x^2 \quad x+2=0$ then the value of $\dfrac{6(-\alpha^3+2\alpha^2-\alpha)}{\alpha^5-3\alpha^4+3\alpha^3-\alpha^2}$ is equal to:

(a) 3 (b) 6 (c) 9 (d) 12

235. If terms independent of x in the expansion of $\left(3x-\frac{1}{x}\right)^{20}$ and $\left(x+\frac{\sqrt[9]{3^{10}}}{x}\right)^{18}$ are A and B respectively then $\left(\frac{9}{38}A+B\right)$ equals:

(a) $3^{10}\cdot{}^{19}C_8$ (b) $3^{10}\cdot{}^{19}C_9$ (c) $3^9\cdot{}^{20}C_8$ (d) $3^9\cdot{}^{19}C_{40}$

236. If graph of $f(x)$ which is defined in $[-2, 2]$ is shown in the adjacent figure, then number of solution(s) of the equation $f(x)=f^{-1}(x)$ is (are):



(a) 1 (b) 3 (c) 5 (d) 7

237. Let m be a positive integer. If $\lim\limits_{x\to 0}|\cos x+\sin 2x+\sin 3x|^{\cot x}=e^m$, then the value of m is:

(a) 2 (b) 3 (c) 4 (d) 5

238. From an unlimited number of red, white, blue and green balls, a selection of 18 balls is to be made so that there are at least two of each colour. If the number of slection is k, then k is equal to:

(a) 1001 (b) 286 (c) 680 (d) 455

239. Let $a\in\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$ such that $\tan^{-1}\left(\frac{\tan\alpha}{3+2\tan^2\alpha}\right)+\tan^{-1}\left(\frac{2\tan\alpha}{3}\right)=\frac{\pi}{12}$, then α equals:

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{12}$

240. If $\frac{p-1}{2p+3}=\sin^2\theta+2\cos\theta+1\ \forall\,\theta\in R$, then p must lie in the interval:

(a) $(-\infty,-2]\cup\left[\frac{-2}{3},\infty\right)$ (b) $\left(\frac{-3}{2},\frac{-2}{3}\right]$

(c) $\left(-\infty,\frac{-3}{2}\right)\cup\left[\frac{2}{3},\infty\right)$ (d) $\left[-2,\frac{-3}{2}\right)$

241. If $\frac{\cos x+\cos y+\cos z}{\cos(x+y+z)}=2$ and $\frac{\sin x+\sin y+\sin z}{\sin(x+y+z)}=2$, then the value of $\cos(x+y)+\cos(y+z)+\cos(z+x)$ is equal to: (where $x, y, z\in R$)

(a) 3 (b) 1 (c) 2 (d) -1

242. If $\alpha = \frac{1}{3}\sin^{-1}\left(\frac{2x}{1+x^2}\right) + \frac{1}{3}\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ where $x \geq \frac{4}{3}$, then the value of $\frac{\cos 2\alpha + \sec\alpha + 3\sqrt{3}}{\sqrt{3}}$ is equal to:

(a) 3 (b) $2+\sqrt{3}$ (c) $\frac{3(\sqrt{3}+1)}{\sqrt{3}}$ (d) $\left(\frac{\sqrt{3}}{2}+3\right)$

243. In ΔABC, if incircle touches the sides AB, BC and CA at P, Q and R respectively and $s-a=3$, $s-b=5$ and $s-c=7$, then area of the quadrilateral $QCRI$ is, where I is incentre of ΔABC:

[**Note:** Symbols used have usual meaning in ΔABC.]

(a) $\sqrt{7}$ (b) $5\sqrt{7}$ (c) $3\sqrt{7}$ (d) $7\sqrt{7}$

244. If $f(\ln(1+|x|)) = (1-\ln(1+|x|))^{\frac{1}{7}}$, then $f(f(\cos x))$ is equal to:

(a) $\cos(\ln(1+|x|))$ (b) $\cos^7(\ln(1+|x|))$ (c) $\cos x$ (d) $\cos^7 x$

245. If roots of the equation $\frac{1}{x-a}+\frac{1}{x-b}+\frac{1}{x-c}+\frac{1}{x-d}+\frac{(x-2)(x^2+2x+4)}{(x-a)(x-b)(x-c)(x-d)}=0$ are α, β and γ, then sum of the roots of the equation $5(x-\alpha)(x-\beta)(x-\gamma)+8-x^3=0$ is:

(a) $a+b+c+d$ (b) $\frac{3}{4}(abc+bcd+cda+dab)$

(c) $abc+bcd+cda+dab$ (d) $\frac{3}{4}(a+b+c+d)$

246. Let f be a differentiable function such that

$$\lim_{x\to 1}\frac{f(1+x^3-x)-f(x)}{\sin(x-1)} = \lim_{x\to 0}\frac{f(1-x)-f(1)}{x}+10$$ then $f'(1)$ is equal to:

(a) 5 (b) 4 (c) 2 (d) 1

247. The product of all positive integral values of p for which $\log_p 5^{42}$ is an integer, is:

(a) 5^4 (b) 5^8 (c) 5^{48} (d) 5^{96}

248. If $P(-1,2,-3)$ and $Q(3,0,3)$ are two points on the $P_1: 2x+y-z=3$ and $R(x_0, y_0, z_0)$ be a point such that $x_0-2y_0+3z_0+1=0$ and $|PR-QR|$ is maximum, then $(x_0+y_0+z_0)$ is equal to:

(a) 2 (b) -5 (c) 7 (d) 3

249. If $y=f(x)$ satisfies the differential equation $\sin x\frac{dy}{dx}+2y\cos x=8$ with $f(\pi/2)=8$, then the minimum value of $f(x)$ is:

(a) 4 (b) 6 (c) 8 (d) 16

250. If α and β are the roots of the equation $x^2 - x\sin 2\theta + 2\cos^2\theta = 0$, $\theta \in R$ and the maximum value of $(2-\alpha)(2-\beta)$ is $(a+\sqrt{a})$, then a is equal to:

(a) 2 (b) 3 (c) 4 (d) 5

251. Let $f(x, y)$ be a locus of a point $P(x, y)$ satisfying $\alpha(2x - y + 1) + \beta(3x - y) + \gamma(2x + y - 5) = 0 \;\forall\, \alpha, \beta, \gamma \in R$. The least distance between the curve $f(x, y)$ and straight line $3x - 4y + 19 = 0$ is:

(a) 3 (b) 2 (c) $\frac{7}{5}$ (d) $\frac{26}{5}$

252. The number of points where $f(x) = |x + [x]| - 3[2x] + 4[3x]$ is discontinuous in $[-1, 1]$, is:

[**Note:** $[k]$ denotes greatest integer less than or equal to k.]

(a) 9 (b) 8 (c) 7 (d) 6

253. Let $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ be three non zero vectors satisfying $\vec{\mathbf{a}} = \vec{\mathbf{b}} \times \vec{\mathbf{c}} + 2\vec{\mathbf{b}}$ where $|\vec{\mathbf{b}}| = |\vec{\mathbf{c}}| = 2$ and $|\vec{\mathbf{a}}| \le 4$. The sum of possible value(s) of $|2\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}}|$ is:

(a) 8 (b) 12 (c) 20 (d) 32

254. The number of words which can be formed using all the letters of the word "NARENDRABHAI" such that no two non-repeated letters occur together is:

(a) 49 (5!) (b) $48(5!)^2$ (c) 98 (5!) (d) $98(5!)^2$

255. Let $A = [a_{ij}]_{2\times 2}$ be a matrix where $a_{ij} \in \{2, 3\}$. If determinant of matrix A is non-negative, then probability that it is invertible is:

(a) $\frac{1}{2}$ (b) $\frac{5}{11}$ (c) $\frac{5}{16}$ (d) $\frac{3}{16}$

256. $\int_0^{10} [x]^3 \{x\}\, dx$ is equal to:

(a) 2025 (b) $\frac{2025}{2}$ (c) $\frac{2025}{4}$ (d) $\frac{2025}{8}$

[**Note:** Where [ ] and { } denotes greatest integer and fractional part functions respectively]

257. In a parabola $y^2 = 4ax$, two points P and Q are taken such that the tangents drawn to parabola at these points meet at directrix in R. Focus of locus of circumcentre of ΔPQR will be:

(a) $\left(\frac{a}{2}, 0\right)$ (b) $(a, 0)$ (c) $\left(\frac{3a}{2}, 0\right)$ (d) $\left(\frac{5a}{2}, 0\right)$

258. If $A=\begin{bmatrix} a & x & y \\ x & b & z \\ y & z & c \end{bmatrix}$ where $a, b, c, x, y, z \in \{1, 2, 3, 4, 5, 6\}$ and also a, b, c, x, y, z are distinct then number of matrices in A with trace equal to 10 are:

(a) $3(3!)^2$ (b) $2(3!)^2$ (c) $(3!)^2$ (d) $(3!)^3$

259. Given 2019 vectors on a plane. Sum of every 2018 vectors is a scalar multiple of other vector. Not all vectors are scalar multiple of each other.The magnitude of sum of all these vectors is:

(a) 0 (b) $\sqrt{2019}$ (c) 2019 (d) $(2019)^2$

260. The probability of occurrence of a multiple of 2 on one dice and a multiple of 3 on the other dice if both are thrown together, is:

(a) $\frac{7}{36}$ (b) $\frac{1}{3}$ (c) $\frac{1}{6}$ (d) $\frac{11}{36}$

261. Let z be the complex number satisfying $|z+16|=4|z+1|$, then:

(a) $|z|=4$ (b) $|z|=5$ (c) $|z|=6$ (d) $4<|z|<64$

262. If $\sec^{-1}(x)+\tan^{-1}\sqrt{9y^2-1}+\sin^{-1}(x^2+y^2)=\lambda$ has no solution, then exhaustive set of values of λ is equal to:

(a) R (b) $(-1, 1)$ (c) $(0, 2)$ (d) ϕ

263. F_1, F_2 are left and right focus points of the hyperbola $C: \frac{x^2}{a^2}-\frac{y^2}{b^2}=1 (a>0, b>0)$. Point O is the origin of the coordinate, M is an arbitrary point on C and above the x-axis. H is a point of MF_1. Given that $MF_2 \perp F_1F_2$, $MF_1 \perp OH$, $|OH|=\lambda|OF_2|$, where $\lambda \in \left(\frac{1}{3}, \frac{1}{2}\right)$.

Find the range of the eccentricity of the hyperbola C.

(a) $(1, \sqrt{3})$ (b) $(1, \sqrt{2})$ (c) $(\sqrt{2}, \sqrt{3})$ (d) $(\sqrt{2}, 2)$

264. Given that $m, n, s, t \in (0, +\infty)$, $m+n=3, \frac{m}{s}+\frac{n}{t}=1$, m, n are constants and $m<n$. If the minimum value of $s+t$ is $3+2\sqrt{2}$, point (m, n) is the mid-point of a chord of the ellipse $\frac{x^2}{4}+\frac{y^2}{16}=1$. Find the equation of the line where the chord lies:

(a) $x+y-3=0$ (b) $x-2y+3=0$ (c) $2x+y-4=0$ (d) $4x+2y-3=0$

265. Number of 4 digit numbers of the form $N=abcd$ which satisfy following three conditions:

(i) $4000 \le N < 6000$

(ii) N is a multiple of 5

(iii) $3 \le b < c \le 6$

is equal to:

(a) 12 (b) 18 (c) 24 (d) 48

266. Let $f(x)=\frac{1}{\cos^2 x}+\frac{4}{\sin^2 x}$. The minimum value of $f(x)$ for $0<x<\frac{\pi}{2}$ is:

(a) 5 (b) 7 (c) 9 (d) 11

267. Let $g(x)=6x^2-18x+8$, $f_1(x)=|g(x)|$, $f_2(x)=|f_1(x)-P_1|$, $f_3(x)=|f_2(x)-P_2|$ and if P_1-7, then the range of P_2 such that $f_3(x)$ has exactly 10 points of non-differentiability is:

(a) $(1,5,7)$ (B) $[2,5,8]$ (c) $[2,9]$ (d) $(1,8)$

268. $S=\{1,2,3\}$, $f:S\to S$ satisfies the property : $\forall x\in S, f(f(x))=f(x)$ How many different functions are there for $f(x)$?

[**Note:** Can you generalize the result for $S=\{1,2,3,\ldots\ldots,n\}$?]

(a) 8 (b) 10 (c) 1 (d) 4

269. Variable pairs of chords at right angles and drawn through a point P (with eccentric angle $\frac{x}{4}$) on the ellipse $\frac{x^2}{4}+y^2=1$ to meet the ellipse at two points, say A and B. If the line joining A and B passes through a fixed point $Q=(a,b)$ and the line value of a^2+b^2 can be expressed as $\frac{m}{n}$, where m and n are co-prime positive integer, submit your answer as $n-m$.

(a) 1 (b) 2 (c) 3 (d) 4

270. The value of $\sum_{\omega-1}^{\infty}\sin^{-1}\left[\frac{2\omega+1}{\omega(\omega+1)(\sqrt{\omega^2+2\omega}+\sqrt{\omega^2-1})}\right]$ is equal to:

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$ (c) $\frac{3\pi}{4}$ (d) $\frac{\pi}{2}$

271. If α,β and γ are roots of $x^3\quad 2x^2+6x\quad 1=0$, find the value of the following expression

$$\alpha\left(\frac{\alpha^2+\alpha+1}{\alpha^2-\alpha+1}\right)+\beta\left(\frac{\beta^2+\beta+1}{\beta^2-\beta+1}\right)+\gamma\left(\frac{\gamma^2+\gamma+1}{\gamma^2-\gamma+1}\right)$$

(a) 1 (b) 6 (c) 7 (d) 8

272. $\{a_n\}$ and $\{b_n\}$ are arithmetic progressions and their sums of the first n terms are A_n and B_n respectively.

If $\frac{A_n}{B_n}=\frac{n-1}{2n}$ for all positive integers n, then the value of $\frac{a_3+a_5+a_7}{3(b_3+b_9)}+\frac{a_4+a_{10}}{2(b_2+b_{10})}$ is

(a) $\frac{10}{21}$ (b) $\frac{7}{15}$ (c) $\frac{10}{23}$ (d) $\frac{5}{11}$

273. If $I_1 = \int_0^1 \frac{x^{7/2}(1-x)^{9/2}}{30}dx$ and $I_2 = \int_0^1 \frac{x^{7/2}(1-x)^{9/2}}{(x+5)^{10}}dx$ and $\frac{I_1}{I_2} = 5a^3\sqrt{a}$, where $a \in N$, then the value of a is:

(a) 24 (b) 26 (c) 28 (d) 30

274. $y-\cos^{-1}\cos\left(\log_2 2^{\ln e^{\sin^{-1}\sin x}}\right)$. For y as defined above, the value of $\frac{dy}{dx}$ at $x=\frac{\pi}{4}$ is:

(a) 0 (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) 1

275. $ABCD$ is a rectangle with vertices $A(0, 0)$, $B(m, 0)$, $C(m, n)$ and $D(0, n)$ with $m, n \in N$.
Points are chosen starting from $A_0(0,0) \to A_1(x_1, y_1) \to A_2(x_2, y_2)$ and so on.
Such that for $A_k(x_k, y_k) \to A_{k+1}(x_{k+1}, y_{k+1})$, exactly one of the following result holds:
(i) $x_{k+1} = x_k + a$ and $y_{k+1} = y_k$
(ii) $x_{k+1} = x_k$ and $y_{k+1} = y_k + b$
for some $a, b, k \in N$
If $m = n = 6$, number of paths from A_0 to (m, n) consisting of exactly two perpendicular path with $a, b \in \{1, 2\}$.

(a) 50 (b) 128 (c) 338 (d) 882

276. If $\frac{1}{2}\sin^{1}\left(\frac{3\sin 2\alpha}{5+4\cos 2\alpha}\right) = \tan^{1} x$, then the possible value of x is:

(a) $\frac{1}{2}\tan\alpha$ (b) $2\tan\alpha$ (c) $\frac{1}{3}\tan\alpha$ (d) $3\tan\alpha$

277. On the coordinate plane, point A is on the ellipse : $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\,(a>b>0)$. Point F is the right focus point of the ellipse. Point A, B are symmetry about the origin point O, and $AF \perp BF$. e is the eccentricity of the ellipse. If $\angle ABF$ ranges from $\left[\frac{\pi}{6}, \frac{\pi}{4}\right]$, then find the range of e.

(a) $\left[\frac{\sqrt{2}}{2}, 1\right]$ (b) $\left[\frac{\sqrt{2}}{2}, \sqrt{3}-1\right]$ (c) $\left[\frac{\sqrt{3}}{3}, \frac{\sqrt{6}}{3}\right]$ (d) $\left[\frac{\sqrt{2}}{2}, \frac{\sqrt{3}}{2}\right]$

278. On a coordinate plane, ellipse $C_1 : \frac{x^2}{a_1^2}+\frac{y^2}{b_1^2}=1\,(a_1>b_1>0)$ and hyperbola $C_2 : \frac{x^2}{a_2^2}+\frac{y^2}{b_2^2}=1\,(a_2, b_2>0)$ has the same focus point F_1, F_2. Point P is the intersection point of C_1 and C_2 in the first quadrant and $|F_1F_2| = 2|PF_2| \cdot e_1$ is the eccentricity of C_1 and e_2 is the eccentricity of C_2. Find the range of $e_2 - e_1$.

(a) $\left(\frac{1}{3}, \infty\right)$ (b) $\left(\frac{1}{2}, \infty\right)$ (c) $\left[\frac{1}{3}, \infty\right)$ (d) $\left[\frac{1}{2}, \infty\right)$

279. Let $f(x)=\dfrac{x\ln x-\ln x}{9x^2-2e^x x-9x+2e^x}+2$ and $g(x)=\sin^2\left(\dfrac{\pi x^2}{2}\right)$, then the value of $\lim\limits_{x\to 1}\dfrac{f(x)}{g(x)}$ is:

(a) 2 (b) $\dfrac{1}{3}$ (c) 3 (d) $\dfrac{2}{3}$

280. The value of the definite integral $\displaystyle\int_{1/3}^{1}\frac{\pi\cos\left(\frac{2\pi}{3}x\right)+\pi\cos\left(\frac{\pi}{3}x\right)}{\sin\left(\frac{\pi}{2}x\right)\sin\left(\frac{2\pi}{3}x\right)+2\sin\left(\frac{\pi}{2}x\right)\sin\left(\frac{\pi}{3}x\right)}dx$ is equal to:

(a) 1 (b) 2 (c) 3 (d) 4

281. Given $S_A=2m+(2m+1)+(2m+2)+\ldots\ldots+4m$ and $S_B=(2m+1)+(2m+3)+(2m+5)+\ldots\ldots+(4m-1)$. If $\dfrac{S_A}{S_B}=k+\dfrac{1}{l}$, find the value of $k+l$.

(a) $2+m$ (b) $3+m$ (c) $1+m$ (d) $2m$

282. Suppose that $f:R\to R$ is a continuous function and satisfies the equation $f(x)\ f(f(x))=1$ for all $x\in R$. Further, if $f(1000)=999$, then which of the following options are necessarily true?

1. $f(500)=\dfrac{1}{500}$ 2. $f(199)=\dfrac{1}{199}$ 3. $f(2000)=\dfrac{1}{2000}$

4. $f(235)=\dfrac{1}{235}$ 5. $f(1099)=\dfrac{1}{1099}$ 6. $f(x)=\dfrac{1}{x}\ \forall x\in R-\{0,1000\}$

7. No such function exists

Enter the product of the number of all correct options. For example, if correct options are 2 and 3, then enter 6.

(a) 2 (b) 4 (c) 6 (d) 8

283. If inequality $\left(\dfrac{1}{x}\right)^{\lambda/x}\le\dfrac{1}{9}$ has + ve integer solution for then the minimum value of λ (using $\ln 9=2.197$) is:

(a) 3 (b) 4 (c) 5 (d) 6

284. $L=\lim\limits_{n\to\infty}\sqrt{n}\displaystyle\int_0^1\frac{dx}{(1+x^2)^n}$

Suppose that the above limit exists, then choose the correct option.

(a) $\dfrac{1}{2}<L<2$ (b) $4<L<5$ (c) $2<L\le 3$ (d) $L\ge 5$

285. Let $f(x)$ and $g(x)$ be continous, positive function such that $f(\;x)-g(x)\;\;1$,

$f(x)=\dfrac{g(x)}{g(-x)}$ and $\int\limits_{-20}^{20} f(x)\,dx=2020$, then the value of $\int\limits_{-20}^{20} \dfrac{f(x)}{g(x)}\,dx$ is:

(a) 1010 (b) 1050 (c) 2020 (d) 2050

286. The value of the expression $\dfrac{\sin^2 \frac{2\pi}{7}}{\sin^2 \frac{\pi}{7}}+\dfrac{\sin^2 \frac{4\pi}{7}}{\sin^2 \frac{2\pi}{7}}+\dfrac{\sin^2 \frac{\pi}{7}}{\sin^2 \frac{4\pi}{7}}$ is equal to:

(a) 3 (b) 4 (c) 5 (d) 6

287. Let x_n be positive root of the equation $x^n = x^2 + x + 1$. Then the value of $e^{\left(\lim\limits_{n\to\infty} n(x_n-1)\right)}$ is:

(a) 1 (b) 2 (c) 3 (d) 4

288. Let $\log_2 n$ is an integer. If $\prod\limits_{k=1}^{\log_2 n}\left(x^{\frac{n}{2^k}}+1\right)=\dfrac{x^A-B}{x-C}$, where A, B and C are positive integers. Then the value of $(B+C+\log_2 A)$ for $n=2^{92}$ is :

(a) 90 (b) 92 (c) 94 (d) 100

289. Let $0<a<b<\dfrac{\pi}{2}$. If $f(x)=\begin{vmatrix} \sin x & \sin a & \sin b \\ \cos x & \cos a & \cos b \\ \tan x & \tan a & \tan b \end{vmatrix}$, then minimum possible number of roots of $f'(x)=$ lying in (a, b) is:

(a) 0 (b) 1 (c) 2 (d) 3

290. Let $f(x)=\sin x - \cos x + \ln x$. Number of roots of $f(x)=0$ in $(0, \infty)$ is:

(a) 1 (b) 2 (c) 3 (d) 4

291. If a and b are chosen randomly by throwing a pair of fair dice, then the probability that $\lim\limits_{x\to 0}\left(\dfrac{a^x+b^x}{2}\right)^{\frac{2}{x}}=6$ equals:

(a) $\dfrac{4}{9}$ (b) $\dfrac{2}{9}$ (c) $\dfrac{3}{9}$ (d) $\dfrac{1}{9}$

292. Lot A consists of 5 good and 3 defective articles. Lot B consists of 3 good and 5 defective articles. A new lot C is formed by taking 3 articles from A and 4 articles from B. The probability that an article chosen at random from C is defective, is:

(a) $\dfrac{1}{3}$ (b) $\dfrac{2}{5}$ (c) $\dfrac{29}{56}$ (d) none of these

293. In throwing a dice thrice, getting numbers in order denoted by a, b, c, satisfying $a^2+4b^2+4c^2-2ab-4bc-2ac=0$. If probability such that point (a, b, c) lies inside the tetrahedron formed by the plane $x+y+z=10$ and co-ordinate planes is $\frac{6}{\lambda}$, where $\lambda \in N$, then λ is:

(a) 9 (b) 12 (c) 25 (d) 27

294. Let the set of complex numbers $(a_1, b_1), (a_2, b_2), (a_3, b_3)$ .... denoting the points on the complex plane satisfying $(a_{n+1}, b_{n+1}) = (\sqrt{3}\,a_n - b_n, \sqrt{3}\,b_n + a_n)$ for $n = 1, 2, 3, ...$ Suppose $(a_{100}, b_{100}) = (2, 4)$, then the value of $(a_1 + b_1)$ is equal to:

(a) $\frac{1}{2^{96}}$ (b) $\frac{1}{2^{97}}$ (c) $\frac{1}{2^{98}}$ (d) $\frac{1}{2^{99}}$

295. Let M denote the matrix $\begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$, where $i^2=-1$, and let I denote the identity matrix $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. Then the matrix $I + M + M^2 + M^3 + M^4 + ... + M^{2010}$ is equal to:

(a) $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ (b) $\begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$ (c) $\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$ (d) $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

296. If ω is a non-real cube root of unity, then the value of $\frac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2} + \frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2}$ is equal to:

(a) 1 (b) 2 (c) 0 (d) -1

297. The point of intersection of the plane $\vec{r}\cdot(3\hat{i}-5\hat{j}+2\hat{k})=6$ with the straight line passing through the origin and perpendicular to the plane $2x-y-z=4$, is (x_0, y_0, z_0). The value of $(2x_0 - 3y_0 + z_0)$, is:

(a) 0 (b) 2 (c) 3 (d) 4

298. If the equation of the plane passing through the point $(-1, 2, 0)$ and parallel to the lines $\frac{x}{3} = \frac{y+1}{0} = \frac{z-2}{-1}$ and $\frac{x-1}{1} = \frac{y+1}{2} = \frac{z+1}{-1}$ is $ax + by + cz = 1$, then the value of $(a+b+c)$, is:

(a) 3 (b) 4 (c) 5 (d) 10

299. Let $\vec{a}, \vec{b}, \vec{c}$ be three vectors of magnitude 2, 3, 5 respectively, satisfying $|[\vec{a}\ \vec{b}\ \vec{c}]| = 30$. If $(2\vec{a}+\vec{b}+\vec{c})\cdot((\vec{a}\times\vec{c})\times(\vec{a}-\vec{c})+\vec{b}) = k$, then the value of $\left(\frac{k}{103}\right)$ is:

(a) 1 (b) 2 (c) 3 (d) 4

300. Let $f(x) = \lim\limits_{n\to\infty} \frac{x^{2n-1}+ax^3+bx^2}{x^{2n}+1}$ is continuous for all $x \in R$. If points $A(-a, 3)$ and $B((b+1), -1)$ are points of relative maximum and minimum of a cubic polynomial $y=g(x)$, then the value of $g(2)$ is:

(a) 1 (b) 2 (c) 3 (d) 4

MORE THAN ONE CORRECT TYPE QUESTIONS

301. Let $f(x)=e^{\frac{-1}{x^2}}+\int_0^{\frac{\pi x}{2}}\sqrt{1+\sin t}\,dt\ \forall x\in(0,\infty)$, then:

(a) $f'(x)$ exist and is continuous $\forall x\in(0,\infty)$

(b) $f''(x)$ exist $\forall x\in(0,\infty)$

(c) $f'(x)$ is bounded

(d) there exist $\alpha>0$ such that $|f(x)|>|f'(x)|\ \forall x\in(\alpha,\infty)$

302. A curve passes through $(-2, -2)$ and its slope at the point (x, y) is given by $\frac{1}{x\sqrt{x^2-1}}$.

Which of the following points the curve also passes through?

(a) $\left(\frac{-2}{\sqrt{3}},\frac{-\pi}{6}-2\right)$ (b) $\left(\frac{-2}{\sqrt{3}},\frac{\pi}{6}-2\right)$ (c) $\left(-\sqrt{2},\frac{-\pi}{12}-2\right)$ (d) $\left(-\sqrt{2},\frac{\pi}{12}-2\right)$

303. Function $f(x)$ is such that $f(x)=a\ln x+\frac{x^2}{2}$ where $a>0$ is a parameter. If $\frac{f(x_1)-f(x_2)}{x_1-x_2}\geq 2\ \forall x_1,x_2\in(0,\infty)$ and $x_1\neq x_2$, then possible value of 'a' can be:

(a) 0 (b) 1/2 (c) 3/2 (d) 1

304. Let $f(x)$ be double differentiable function such that $|f''(x)|\leq 5\ \forall x\in[0,4]$ and f takes its largest value at an interior point of this interval. Then the value of $|f'(0)|+|f'(4)|$ can be:

(a) 18 (b) 19 (c) 20 (d) 21

305. If $\frac{x}{a}+\frac{y}{b}=1$ and $\frac{x}{c}+\frac{y}{d}=1$ where $a,b,c,d>0$ intersect the axes at four con-cyclic points and $a^2+c^2=b^2+d^2$, then the lines can intersect at which of the following given points?

(a) $(1, 1)$ (b) $(1,-1)$ (c) $(2,-2)$ (d) $(3, 3)$

306. The value of $\sum_{k=1}^{\infty}\frac{6^k}{(3^k-2^k)(3^{k+1}-2^{k+1})}$ can be equal to:

(a) $\lim_{x\to 0}\frac{1-\cos 2x}{x^2}$

(b) $\lim_{n\to\infty}\left(\sin\frac{\pi}{2n}\times\sin\frac{2\pi}{2n}\times\sin\frac{3\pi}{2n}\ldots\ldots\sin\frac{(n-1)\pi}{n}\right)^{1/n}$

(c) $\dfrac{\int_{\pi/2}^{0} \ln|\cos 2x|\,dx}{\pi \ln 2}$

(d) $\dfrac{2}{\pi \ln 2}\int_{0}^{\pi/4} \ln(1+\tan x)\,dx$

307. The equation of the normal to the curve $x^2 = y$ which form the shortest chord can be:

(a) $\sqrt{2}x-2y+2=0$ (b) $\sqrt{2}y+2x-2=0$

(c) $\sqrt{2}x+2y-2=0$ (d) $\sqrt{2}x+2y+2=0$

308. If $a_1, a_2, \ldots\ldots, a_n$ is a sequence of positive numbers which are in A.P. with common difference d and $a_1 + a_4 + a_7 + \ldots\ldots + a_{16} = 147$ then $a_1 + a_{16} = M$ and $a_1 + a_6 + a_{11} + a_{16} = N$.

Maximum value of $a_1 a_2 \ldots\ldots a_{16} = \left(\dfrac{S}{W}\right)^{16}$ (where S and W are coprime), then:

(a) $M=49$ (b) $N=98$ (c) $S=49$ (d) $W=2$

309. Let $\sum_{k=1}^{\infty} \sin^{-1}\left(\dfrac{\sqrt{k}-\sqrt{k-1}}{\sqrt{k(k+1)}}\right)=\theta$ Then:

(a) the value of $\sin\theta$ is equal to 1 (b) $\int_{0}^{\theta/2} \ln(1+\tan x)\,dx = \dfrac{-\pi}{8}\ln 2$

(c) $\lim_{x\to\theta}\left(1+\dfrac{x}{\tan x}\right)^{\frac{2}{x-\theta}} = e^{-\pi}$ (d) $\lim_{x\to\theta} \dfrac{(x-\cos x-\theta)}{x-\theta}=2$

310. The ends of the major axis of ellipse are (– 2, 4) and (2, 1). If the point (1, 3) lies on the ellipse. Then:

(a) The length of major axis is equal to 10.

(b) The length of minor axis is equal to $\dfrac{10}{\sqrt{24}}$.

(c) The length of latus rectum of ellipse is $\dfrac{5}{6}$.

(d) Square of the distance between the focii of ellipse is $\dfrac{125}{6}$.

311. Let $\int \dfrac{(x-1)e^x}{(x+1)^3}\,dx = f(x)+C$ where $f(x)=d+\sum_{i=0}^{n} \dfrac{a_i e^x}{(x+1)^i}$ with all $a_i=0$ for $i \ge n$ and $f(1)=\dfrac{e}{2}$. Then which of the following is/are correct?

(a) $a_0=0$ (b) $a_1=0$

(c) $a_2=1$ (d) $f(0)$ is irrational

312. A triangle with side lengths of a, b and c is a right triangle where $a<b<c$. Which of the following statements are possible?

(a) a, b and c form an A.P.
(b) a, b and c form an G.P.
(c) a, b and c form an H.P.
(d) None of these

313. If the equations $x^3 - 5x^2 + 7x \quad a = 0$ and $x^3 - 8x + b = 0$ have 2 common roots, then:

(a) $\log_4(a^3 + b^2 - 1)$ is equal to 2.

(b) $\int_0^{\pi} \ln(\sin ax)\, dx = \frac{-\pi}{2} \ln 2$

(c) $\lim_{x \to 0}\left(\left[\frac{a^2 \sin x}{x}\right] + \left[\frac{b^2 \tan x}{x}\right]\right)$ is equal to 12.

(d) $\tan^{-1}(\tan(a+b))$ is equal to $6 - 2\pi$.

[Note: $[\cdot]$ denotes the greatest integer function.]

314. Let $f : (0, \infty) \to R$ be a differentiable function satisfying the equation

$$2f(x) = f(x) + f\left(\frac{x}{y}\right) \forall x, y > 0.$$

If $f(1) = 0$ and $f'(1) = 1$, then:

(a) $f(x)$ has no local maxima and no local minima.

(b) $\lim_{x \to 0^+}\left[\frac{f(x+1)}{x}\right] = 0$

(c) $f(x) = ex$ has no roots.

(d) the equation $2e \cdot f(x) = x$ has one distinct solution.

[Note: Where $[k]$ denotes greatest integer function less than or equal to k]

315. Let $f : (0, \infty) \to R$ be a differentiable function satisfying the equation $f(xy) = e^{xy-x-y}(e^y f(x) + e^x f(y)) \forall x, y > 0$. If $f'(1) = e$, then:

(a) $\lim_{x \to e}\left[\frac{f(x) - e^x}{x - e}\right] = e^{(e-1)}$

(b) number of roots of the equation $f(x) = xe^x$ in $(0, \infty)$ is 2.

(c) $\int_1^e f(x)\, dx < e^e(e-1)$

(d) $f(x)$ is a strictly increasing function in $(0, \infty)$.

316. A polynomial function $f(x)$ with non-negative coefficient satisfy the equation $f(f(x)) = x\int_0^x f(t)\, dt$ and $f(0) = 0$, then:

(a) number of points where $|f(|x|)|$ is non derivable is 0.

(b) $\text{sgn}(f(x))$ is discontinuous at $x = 1$.

(c) derivative of $f(x)$ with respect to $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ at $x = \sqrt{3}$ is -4.

(d) $\lim_{x \to 0^+}\left(\frac{\sqrt{3} f(x)}{x}\right)^x = 1.$

317. Let $f(x)=1+x\ln(x+\sqrt{x^2+1})$ and $g(x)=\sqrt{1+x^2}$. Then:

(a) $f(x)>g(x)\ \forall\ x\in R^+$

(b) $f(x)<g(x)\ \forall\ x\in R^-$

(c) there exist $x=a>0$ for which $f(x)<g(x)$

(d) there exist $x=a<0$ for which $f(x)>g(x)$

318. If $f(x)=x^2+xg'(1)+g''(2)$ and $g(x)=f(1)x^2+xf'(x)+f''(x)$. Then:

(a) minimum value of $f(x)$ is equal to -2.25

(b) the value of $\int_2^3 \frac{dx}{f(x)-x+5}$ is equal to $\frac{\pi}{4}$.

(c) number of positive integral values in the domain of $\sqrt{\frac{f(x)}{g(x)}}$ is 4.

(d) number of points where $g(|x|)$ is non derivable is 1.

319. Let a be a positive integer such that the limit $\lim_{x\to 1}\left(\frac{1}{x-1}-\frac{1}{x^a-2x+1}\right)$ exists and is equal to b : (where $b\neq 0$),

(a) $\tan^{-1}(\tan a)$ is equal to $3-\pi$.

(b) $\tan^{-1}(\tan b)$ is equal to $3-\pi$.

(c) $\tan^{-1}(\tan(a+b))$ is equal to $5-2\pi$.

(d) $\tan^{-1}(\tan(a-b))$ is equal to 1.

320. If $y=\sqrt{x+\sqrt{x+\sqrt{x+\sqrt{x+\ldots\ldots}}}}$, where $x>0$, then $\frac{dy}{dx}$ can be:

(a) $\frac{1}{2y-1}$

(b) $\frac{x}{x+2y}$

(c) $\frac{1}{\sqrt{1+4x}}$

(d) $\frac{y}{2x+y}$

321. Given a function $f:R\to R$ defined as $f(x)=\begin{cases} x, & x<0 \\ \sin x, & 0\le x\le \pi/2. \\ 1, & x>\pi/2 \end{cases}$

If $f(x)=a\int_0^{\pi/2}|x-t|\sin t\,dt+bx+c$, then:

(a) $2a+1=0$

(b) $2b-1=0$

(c) $2c-1=0$

(d) $8abc-1=0$

322. The coefficients of the quadratic function $f(x)$ including the constant term, are all rational has local maximum at $x=0$. Let $g(x)=|f'(x)\ e^{f(x)}$ has maximum value $4\sqrt{e}$. If $g(x)=4\sqrt{e}$ has rational solutions then:

(a) $\int_{-1}^{0} g(x)\,dx=e-\frac{1}{e^7}$

(b) The value of $\text{sgn}(f(0))=-1$

(c) $g(x)$ is non derivable at one value of x.

(d) The value of $g\left(\tan\frac{\pi}{4}\right)=\frac{2}{e^7}$

[**Note:** Where $\text{sgn}(x)$ denotes signum function of x]

323. Let $f(x)$ be a continuous function defined for every real $x \in R$. For any real numbers 'a' and 'b' that satisfy $a < b$, $f(x)$ always satisfies $f(a) > f(b)$. Then which of the followings is/are correct?

(a) $\lim_{h \to 0} \frac{f(2+h)-f(2)}{h}$ exists and negative.

(b) There is always only one real root of $f(x) = 0$

(c) There is always only one real root of $f(x) = f(-x+1)$

(d) There is no real root of $f(x) = f(x+1)$

324. In ΔABC, $a = 11$ and $\sin A = \frac{3}{7}$ where 'a' is the side opposite to $\angle A$ and $0 < A < \frac{\pi}{2}$. If the side length of ΔABC are $11, b, c$ where 'b' is the largest possible side of ΔABC, then:

(a) circumradius R of ΔABC is equal to $\frac{77}{6}$

(b) inradius r of ΔABC is equal to $(11)(\sqrt{2})\left(\frac{\sqrt{5}-\sqrt{2}}{3}\right)$

(c) area of ΔABC is equal to $\frac{121\sqrt{10}}{3}$

(d) the value of $\sin 2A + \sin 2B + \sin 2C$ is equal to $\frac{24\sqrt{10}}{7}$

325. Which of the following statements are true?

(a) If $\lim_{x \to a} \frac{f(x)-f(a)}{x-a}$ exists, then f is differentiable at a.

(b) If f is continuous at a, then f is differentiable at a.

(c) If $\lim_{x \to a} f(x)$ exists, then f is differentiable at a.

(d) If f is differentiable at a, then $\lim_{x \to a} f(x) = f(a)$.

326. Let a function is defined as $f(x) = \begin{cases} \frac{a(1 - x\sin x) + b\cos x + 5}{x^2} & \text{if } x < 0 \\ 3 & \text{if } x = 0, \\ \left(1 + \left(\frac{cx + dx^3}{x^2}\right)\right)^{\frac{1}{x}} & \text{if } x > 0 \end{cases}$

where a, b, c and d be constants, if $f(x)$ is continuous at $x = 0$, then:

(a) $\lim_{x \to 0} \frac{e^{bx}-1}{\sin x} = -4$

(b) number of points of discontunity of $g(x) = [c - 3a\sin x]$ in $[0, \pi]$ is 5.

(c) the value of definite integral $\int_{1+a}^{b} \frac{dx}{x^2+16} = \frac{\pi}{4}$.

(d) the value of $2e^d + 7c - 3a - 5b$ is equal to 29.

[Note: $[k]$ denotes greatest integer function less than or equal to k.]

327. The smallest positive integral value of a for which the greater root of the equation $x^2 - (a^2 + a + 1)x + a(a^2 + 1) = 0$ lies between the roots of the equation $x^2 - a^2x - 2(a^2 - 2) = 0$, is less than:

(a) $\sqrt{\frac{27}{\sqrt{\frac{27}{\sqrt{\frac{27}{\sqrt{\dots}}}}}}}$

(b) $\sqrt{4\sqrt{4\sqrt{4\sqrt{\dots}}}}$

(c) $\sqrt{5}\sqrt{\sqrt{5}\sqrt{\sqrt{5}\sqrt{\sqrt{5}\sqrt{\dots}}}}$

(d) $\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{\dots}}}}$

328. Let $f(x)$ be a monic polynomial of degree 5. The graph of $|f(x)|$ and $f(|x|)$ are same. If $f(2) = 0$. Then:

(a) The value of $f(0) + f(1)$ equals 25

(b) $f(x) + f(-x) - 0 \ \forall x \in R$

(c) $\lim_{x\to 2} (1 + f(x))^{\frac{1}{1-\cos(x-2)}}$ equals e^{64}

(d) $\int \left(\frac{x}{f(x)}\right)^{\frac{1}{4}} dx = \ln(x + \sqrt{x^2 - 4}) + C$ where C is constant of integration

329. Let the equation $ax^2 - bx + c = 0$ has 2 distinct roots in the interval (0, 1) where $a, b, c \in N$. If $\lambda \le \log_5(abc)$ for all choices of natural numbers a, b, c then non-negative integral values of λ can be:

(a) 0 (b) 1 (c) 2 (d) 3

330. A quadratic equation $f(x) = ax^2 + bx + c = 0$ with $a \ne 0$, has positive distinct roots reciprocal of each other. Which of the following options is (are) **incorrect?**

(a) $af'(1) = 0$

(b) $af'(1) < 0$

(c) $af'(1) > 0$

(d) Nothing can be said about $af'(1)$

331. If $\int x \ln\left(1 + \frac{1}{x}\right) dx = f(x)\ln(x+1) + g(x)x^2 + kx + C$, where C is constant of integration, then:

(a) $\lim_{x\to 0} \frac{f(\cos x)}{x^2} = \frac{-1}{2}$

(b) $\lim_{x\to 0} \frac{g(1+x)}{x} = \frac{-1}{2}$

(c) $\lim_{x\to 0} (1 + f(x) + k)^{\frac{1}{x - \sin x}} = e^{-3}$

(d) $\int_1^e \frac{g(x)}{k} dx = 1$

332. Consider the piecewise defined function $f(x)=\begin{cases}\{x\}\sqrt{4x^2-12x+9}, & 1\le x\le 2\\ \cos\left(\frac{\pi}{2}(|x|-\{x\})\right), & -1\le x<1\end{cases}$ with $\{x\}$ denoting the fractional part of x. Which of the following is/are true?

(a) Range of $f(x)$ is equal to $[0, 1]$.

(b) The number of values of x for which function is continuous but not differentiable is 1.

(c) $f(x)=1$ has two solutions.

(d) Number of values of x for which $f(x)$ is discontinuous is 2.

333. Let $\int e^{x^2}\cdot e^x(2x^2+x+1)\,dx=e^{x^2}\cdot f(x)+C$ where $f(x)$ is some non-zero constant function and C is some arbitrary constant. If the local minimum value of $f(x)$ is equal to m, then:

(a) $f(x)$ is increasing in $(0,\infty)$

(b) the value of $\lim\limits_{x\to 0}(1+f(x))^{1/x}$ is equal to 1.

(c) the value of $\int_0^1 (f(x)+e^x)\,dx$ is equal to $2e$.

(d) the value of $\left[\frac{-1}{m}\right]$ is equal to 2.

[**Note:** $[\cdot]$ denotes greatest integer function.]

334. Let α and β are two roots of the equation $x^2+px+q=0$, where p and q are real numbers, and $q\neq 0$. Now suppose another quadratic equation $x^2+mx+n=0$ with roots $\alpha+\frac{1}{\alpha}$ and $\beta+\frac{1}{\beta}$ such that $m+n=0$. Then the possible integral values in the range of q can be:

(a) 1 (b) 2 (c) 3 (d) 4

335. Let $Y=\int_0^1 \frac{2x^2+3x+3}{(x+1)(x^2+2x+2)}\,dx$. Then which of the following option(s) are equal to Y?

(a) $\frac{\pi}{4}+2\ln 2-\text{arc}\tan 2$

(b) $\frac{\pi}{4}+2\ln 2-\text{arc}\tan\frac{1}{3}$

(c) $2\ln 2-\text{arc}\cot 3$

(d) $-\frac{\pi}{4}+2\ln 2+\text{arc}\cot 2$

336. Let $f(x)$ be a monic polynomial of degree 4 satisfying the following conditions:

(i) $f'(0)=0$

(ii) $f'(2)=16$

(iii) for some positive real k, $f'(x)<0$ in the intervals $(-\infty, 0)$ and $(0, k)$.

Then which of the followings is/are correct?

(a) Equation $f'(x)=0$ has one real root in the interval $(0, 2)$.
(b) Function $f(x)$ has a local maximum.
(c) If $f(0)=0$, then for all reals x, $f(x)\geq -\frac{1}{3}$.
(d) If $f(0)=0$, then the value of $\left|\int_{-1}^{1}\frac{f'(x)}{4}\,dx\right|=\frac{2}{3}$.

337. Let $f(x): R\to R$ and $g(x)\cdot R\to R$ be two differentiable functions, such that $f(x); g(x); x-g'(x)$ and $f'(x)+f(x)g'(x)$ are non-negative for all real x.
Then:
(a) $g(1)-g(0)\leq k\ \forall\ k\in(5,\infty)$
(b) $g(1)-g(0)\leq k\ \forall\ k\in(0,\infty)$
(c) Maximum value of $\frac{f(0)}{f(1)}$ is $e^{1/4}$
(d) Maximum value of $\frac{f(0)}{f(1)}$ is $e^{1/2}$

338. Let $f: D\to R$ be a function defined by $f(x)=\frac{x^2-x+c}{x^2+x+2c}$ where D is the domain of the function and R is the set of all real numbers. If $f(x)$ is surjective, then the possible integral values of 'c' can be:
(a) -6
(b) -4
(c) -2
(d) 0

339. Let $I=\int\frac{x^2}{(x\sin x+\cos x)^2}\,dx$. Which of the following options is equivalent to the given indefinite integral (ignoring arbitrary constant)?
(a) $I=\frac{\sin x+x\cos x}{x\sin x-\cos x}$
(b) $I=\frac{\sin x-x\cos x}{x\sin x+\cos x}$
(c) $I=\frac{x\sec x}{x\sin x+\cos x}-\int\frac{\sec x(1+x\tan x)}{x\sin x+\cos x}\,dx$
(d) $I=\frac{\sec x(1+x\tan x)}{x\sin x+\cos x}\,dx-\frac{x\sec x}{x\sin x+\cos x}$

340. Let $f'(x)$ be a continuous function which maps from $[0,1]\to[p(a), p(b)]$. If $p(x)$ is a differentiable function on $[a, b]$ such that $p(g(x))=x$, $g(0)=a$ and $g(1)=b$, then which of the following is/are true?
(a) $f(0)+2<f(1)$
(b) $f(1)\leq 1+f(0)$
(c) $\frac{\int_0^1 f'(x)\,dx}{\int_0^1 g'(x)\,dx}\leq p'(c)$ for some $c\in(a,b)$
(d) There exists $k\in[0,1]$ such that $f'(k)=k$

341. Let $I_n = \int_{-\pi}^{\pi} \frac{1}{1+2^{\sin\left(\frac{x}{2}\right)}} \left(\frac{\sin\left(\frac{nx}{2}\right)}{\sin\left(\frac{x}{2}\right)} \right)^2 dx$, for $n = 0, 1, 2, 3, \ldots\ldots$, then which of the following is/are always correct?

(a) $I_{n+1} - I_n = \pi \ \forall n = 0, 1, 2, 3, \ldots\ldots$ (b) $I_0, I_1, I_2, I_3, \ldots\ldots, I_n$ form an A.P.

(c) $\sum_{m=0}^{9} I_{2m} = 90\pi$ (d) $\sum_{m=0}^{10} I_m = 65\pi$

342. Let $P(x)$ and $Q(x)$ are two different polynomials with real coefficients satisfying the conditions:

(i) a and b are the roots of $P(x)$ and $Q(x)$ respectively.

(ii) $P(b) \cdot Q(a) > 0$.

Then:

(a) $P(c) - Q(c) = 0$ for some c.

(b) $P(c) - 3P^2(c) = Q(c) - 2Q^2(c)$ for some c.

(c) $P(c) - 2Q(c) = 0$ for some c.

(d) $P(c) - 2P^2(c) = Q(c) - 3Q^2(c)$ for some c.

343. The value of $3\sum_{n=1}^{\infty} \left(\frac{1}{\pi} \sum_{k=1}^{\infty} \cot^{-1}\left(1 + 2\sqrt{\sum_{r=1}^{k} r^3} \right) \right)^n$ is less than:

(a) 1 (b) 2 (c) 3 (d) 4

344. Given two function $F(x) = \left(1 + \frac{1}{x}\right)^x$, $G(x) = \left(1 + \frac{1}{x}\right)^{x+1}$ defined for all $x > 0$. Which of the following is decreasing function for $\forall x > 0$?

(a) $F(F(x) - G(x))$ (b) $G(F(x) - G(x))$

(c) $G(x) - F(x)$ (d) $G(F(x))$

345. If $\cos^2 x - a\sin x + b = 0$ has only one solution in $[0, \pi]$. Then:

(a) $a \in (-\infty, -2] \cup (-1, \infty)$ (b) $a \neq b$

(c) $a = b$ (d) $b \in (-\infty, -2] \cup (-1, \infty)$

346. Let P be any point on the line $x - y + 3 = 0$ and A be a fixed point $(3, 4)$. If the family of lines given by the equations $(3\sec\theta + 5\operatorname{cosec}\theta)x + (7\sec\theta - 3\operatorname{cosec}\theta)y + 11(\sec\theta - \operatorname{cosec}\theta) = 0$ are concurrent at a point B for all permissible value of θ, then:

(a) sum of the abscissa and ordinate of point B is equal to -1.

(b) product of the abscissa and ordinate of point B is equal to -2.

(c) maximum value of $|PA - PB|$ is $2\sqrt{10}$.

(d) minimum value of $PA + PB$ is $2\sqrt{34}$.

347. If $\sum_{m=1}^{6} \csc\left(\alpha+(m-1)\frac{\pi}{4}\right)\csc\left(\alpha+\frac{m\pi}{4}\right)=4\sqrt{2}$, where $\alpha \in (0, \pi)$ then α can be:

(a) $\frac{\pi}{12}$ (b) $\frac{\pi}{6}$ (c) $\frac{5\pi}{12}$ (d) $\frac{\pi}{3}$

348. If both $A-\frac{I}{2}$ and $A+\frac{I}{2}$ are orthogonal matrix, then which of the following statements are **incorrect**? (where I is an identity matrix order same as that of A.)

(a) A is skew-symmetric matrix of odd order.

(b) $A^2=\frac{3}{4}I$

(c) A is skew-symmetric matrix of even order.

(d) A is orthogonal

349. Consider a differentiable function $f: R \to R$ with $f(0)=0$ and $f'(0)=1$.

Which of the following statements are true for any such function f?

(a) $f(x)>0$ on $(0, q)$ for some positive q.

(b) $f(x)$ is increasing on (p, q) for some negative p and some positive q.

(c) There exists a differentiable function $g: R \to R$ such that $g''(x)-f(x)$ and

(d) $f'(x)$ is continuous.

350. If $(1+x)^n = C_0 + C_1x + C_2x^2 + \ldots + C_nx^n$, then the sum of the product of the coefficients taken two at a time can be represented by $\sum_{i=0}^{n}\sum_{j=i+1}^{n} C_iC_j = 2^a - \frac{b!}{c(d!)^2}$.

Then which of the following are correct?

(a) $a=2n-1$ (b) $b=2n$ (c) $c=2$ (d) $d-n$

351. Let S be the set of all 3×3 matrices having 3 entries equal to 1 and 6 entries equal to 0. A matrix M is picked uniformly at random from the set S. Then the correct statement(s) is(are):

(a) total number of matrices in the set S is 84

(b) probability that M is non-singular $=\frac{1}{14}$

(c) probability that M is identity matrix $=\frac{1}{14}$

(d) probability that M has trace equal to $0=\frac{5}{21}$

352. Let $f:(1,\infty)\rightarrow R$ be a differentiable function such that

$$\int_2^{150}(x-1)\ln(x-1)(2f(x)-(x-1)\ln(x-1))\,dx=\int_1^{150}f^2(x)\,dx.$$

Then:

(a) area bounded by the curve and x-axis is equal to 1/2.

(b) $f(x)$ is strictly decreasing in $\left(1, 1+\dfrac{1}{e}\right)$.

(c) number of solutions of the equation $f(x)=2$ is 2.

(d) $f(x)$ is monotonic in $\left(1+\dfrac{1}{e}, \infty\right)$.

353. If the equations, $x^2+ax+b=0$, $x^2+bx+a=0$ have a common root α then which of the following options might be true?

(a) $a+b=1$

(b) $\alpha+1=0$

(c) $a+b+1=0$

(d) $\alpha=1$

354. In ΔABC, where the opposite edges of $\angle A$, $\angle B$ and $\angle C$ are a, b and c respectively, c 2 and $\angle C=\dfrac{\pi}{3}$. If $2\sin 2A+\sin(2B+C)=\sin C$, then:

(a) the value of $\sin 2A+\sin 2B+\sin 2C$ is equal to $\sqrt{2}$

(b) inradius of ΔABC is $\dfrac{2\sqrt{3}}{3(\sqrt{3}+1)}$

(c) circumradius of ΔABC is $\dfrac{2\sqrt{3}}{3}$

(d) area of ΔABC is $\dfrac{2\sqrt{3}}{3}$

355. For any ΔABC, if the median through $\angle A$ is m_1, the median through $\angle B$ is m_2, the median through $\angle C$ is m_3 and $\begin{bmatrix}a^2\\b^2\\c^2\end{bmatrix}=M\begin{bmatrix}m_1^2\\m_2^2\\m_3^2\end{bmatrix}$ for a certain 3×3 matrix M. Then:

(a) trace of matrix M is equal to $\dfrac{-4}{3}$

(b) M is a symmetric matrix

(c) det. M is equal to $\dfrac{64}{81}$

(d) sum of all elements of matrix M is 4

356. Consider a cubic, $f(x)=ax^3+bx^2+cx+4$, $a,b,c\in R$ and $f''\left(\frac{-2}{3}\right)=0$, $f'(0)=3$, $f'\left(\frac{-2}{3}\right)=\frac{5}{3}$, and $g=f^{-1}$, then:

(a) $a+b=c$

(b) $abc=6$

(c) $(g(x)\cdot f(g(x)))'\big|_{x=4}=\frac{4}{3}$

(d) $(g(x)\cdot g(f(x)))'\big|_{x=4}=\frac{3}{4}$

357. Let f be a quadratic polynomial such that $f(-1-x)=f(-1+x)$, $\forall\, x\in R$. If $(f(1)-5)^2+(f(-1)-1)^2=f'(1)$, then which of the following is(are) equal to unity?

(a) $[\sin^{-1}f(x)]$, wherever defined

(b) $[\text{sgn}(f(x))]$

(c) $\left[\tan^{-1}\frac{1}{f(x)}\right]$

(d) $\left[\cot^{-1}\left[\frac{1}{2^{f(x)}}\right]\right]$

(where $[\cdot]$ denotes greatest integer function.)

358. Let $\int(x^2-1)e^x(x^2+4x+1)\,dx=e^x f(x)+C$ (where C is constant of integration).

If $g(x)=f(x)+f'(x)$, then:

(a) number of integral roots of $g(x)=0$ is 2.

(b) sum of square of integral roots of $g(x)=0$ is 2.

(c) if α is one non-integral root of $g(x)=0$, then $\alpha^4+4\alpha^3+2\alpha^2+4\alpha+2$ is equal to 1.

(d) $g'(0)=4$.

359. Which of the following functions are continuous $\forall\, x>1$?

(a) $f(x)=[x]+\{x\}^2$

(b) $f(x)=[x]^2+\{x\}+2[x]\{x\}$

(c) $f(x)=\left[\frac{\{x\}}{e^x}\right]$

(d) $f(x)=\left[\frac{\sin\pi x}{2}\right]\sin(\pi\{x\})$

[**Note:** Where $[k]$ denotes greatest integer function less than or equal to k and $\{k\}$ denotes fractional part function of k.]

360. If $a_1, a_2, \ldots\ldots, a_n$ is a sequence of positive numbers which are in A.P. with common difference d and $a_1+a_4+a_7+\ldots\ldots+a_{16}=147$

then $a_1+a_{16}=M$ and $a_1+a_6+a_{11}+a_{16}=N$

Maximum value of $a_1a_2\ldots\ldots a_{16}=\left(\frac{S}{W}\right)^{16}$ (where S and W are coprime), then:

(a) $M=49$

(b) $N=98$

(c) $S=49$

(d) $W=2$

361. Let $\sum_{k=1}^{\infty}\sin^{-1}\left(\dfrac{\sqrt{k}-\sqrt{k-1}}{\sqrt{k(k+1)}}\right)=\theta$. Then:

(a) the value of $\tan\dfrac{\theta}{2}$ is equal to $\sqrt{2}-1$

(b) $\lim\limits_{x\to\theta}\left(1+\dfrac{x}{\tan x}\right)^{\frac{2}{x-\theta}}=e^{-\pi}$

(c) the value of $\sin\theta$ is equal to 1

(d) $\lim\limits_{x\to\theta}\dfrac{(x-\cos x-\theta)}{x-\theta}=2$

362. If $\lim\limits_{x\to 0}\dfrac{\cos^2 x-\cos x-e^x\cos x+e^x-\dfrac{x^3}{2}}{x^n}=L$ (where L is non zero finite), then:

(a) $L=\dfrac{1}{2}$ (b) $n=3$ (c) $L=\dfrac{1}{4}$ (d) $n=4$

363. Let $f(x)=\begin{cases}x+1; & x>0\\ 2-x; & x\le 0\end{cases}$ and $g(x)=\begin{cases}3+x; & x<1\\ x^2-2x-2; & 1\le x<2\\ x-5; & x\ge 2\end{cases}$ then:

[**Note:** $[k]$ denotes greatest integer function less than or equal to k.]

(a) $\lim\limits_{x\to 0^+} g(f(x))=-3$

(b) $\lim\limits_{x\to 0^-} g(f(x))=-3$

(c) $\lim\limits_{x\to 0^+}[f(f(x))]=0$

(d) $\lim\limits_{x\to 0^-}[g(g(x))]=-1$

364. Let a and b be distinct real numbers such that b is a root of the equation $x^2+ax+10=0$ and a is the root of the equation $x^2+bx+10=0$, then which of the following is(are) **incorrect**?

(a) $a-b=0$

(b) $a+b=0$

(c) $a+b=2$

(d) No such a and b exists

365. Identify which of the following statement(s) is(are) **correct**?

(a) If $f(x)=\cos x$ and $g(x)=\ln x$, then range of $f(g(x))$ is $[-1, 1]$.

(b) If $f(x)=\dfrac{2}{\pi}(\sin^{-1}x+\cos^{-1}x)$ and $g(x)=\text{sgn}(x^2-x+1)$, then $f(g(x))$ and $g(x)$ both are identical functions.

(c) If $f:R\to[-2,2]$, $f(x)=\dfrac{2x}{1+x^2}$, then f is a bijective function.

(d) If $f(x)=\sin^{-1}x$ and $g(x)=\cos x$, then $f(g(x))$ is odd and $g(f(x))$ is even function.

366. If $\lim\limits_{x\to 0}\dfrac{ae^x+b\sin 2x+c\sqrt{1-x}}{x^2}$ exists finitely, then:

(a) $a+c=0$ (b) $2a+4b-c=0$ (c) $2a+3b=0$ (d) $3a+4b=0$

367. Which of the following statement(s) is(are) **incorrect**?

(a) The equation $\sin x - x = 0$ has a real root in $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$.

(b) The equation $\tan x - x = 0$ has a real root in $\left(\frac{\pi}{6}, \frac{\pi}{3}\right)$.

(c) If f is continuous function in $[a, b]$, then there exists atleast one $c \in [a, b]$ such that $f(c) = \frac{2f(a) + 3f(b)}{5}$.

(d) If $f(a)$ and $f(b)$ are of opposite signs then equation $f(x) = 0$ has necessarily atleast one root in (a, b).

368. If $f(x) = \begin{cases} \text{max. } (x^2, 1), & x \le 0 \\ \text{min. } (\{x\}, \ |1 - |x||), & x > 0 \end{cases}$, then:

[Note: Where $\{y\}$ denotes the fractional part of y.]

(a) $\lim_{x \to 0^+} f(x) = 1$

(b) $\lim_{x \to 3/4} f(x) = \frac{1}{4}$

(c) $f\left(f\left(\frac{-5}{2}\right)\right) = \frac{1}{4}$

(d) $f(f(-100)) = 0$

369. Let $f(x) = ax^2 + bx + c$, $a, b, x \in R$, $a \neq 0$

If $f(2018) - f(-2014) = \frac{\sin^2([\tan^{-1} x^2]\pi) + 3\tan^2([\cot^{-1} x^2]\pi)}{3\sin^2 x + \cos^2 x}$ $\forall x \in R$ and

$f(2018) + f(-2014) = 2(2016)^2 + 12$, then:

[Note: $[k]$ denotes greatest integer function less than or equal to k and $\text{sgn}(k)$ denotes signum function of k.]

(a) maximum value of $f(x)$ is 6.

(b) $f(1) + f(2) + f(3)$ is equal to 20.

(c) minimum value of $f(f(f(x)))$ is 490.

(d) number of solution(s) of the equation $f(x) = \text{sgn}(f(x))$ is 0.

370. Consider, $f(x) = 3(\tan^{-1}\sqrt{x-2})^2 - \text{cosec}^{-1}\sqrt{x}$. Identify which of the following statement(s) is(are) correct?

(a) Range of $f(x)$ is $\left[\frac{-\pi}{4}, \frac{3\pi^2}{4}\right)$.

(b) Range of $f(x)$ is $\left[\frac{-\pi}{4}, \frac{3\pi^2}{4} + \frac{\pi}{4}\right)$

(c) $\lim_{x \to 2^+} \frac{f(x) + (\pi/4)}{\sin(x-2)} = \frac{11}{4}$

(d) $\lim_{x \to 2^+} \frac{f(x) + (\pi/4)}{\sin(x-2)} = \frac{13}{4}$

371. Let λ be a real number satisfying

$$\tan^{-1}\left(\frac{\cos 2\alpha \sec 2\beta + \cos 2\beta \sec 2\alpha}{\lambda}\right) = \tan^{-1}(\tan^2(\alpha+\beta)\tan^2(\alpha-\beta)+1), \forall \alpha, \beta$$

wherever defined then:

(a) $\sin^{-1}(\sin\lambda) + \tan^{-1}(\tan\lambda) = 0$

(b) $\cos^{-1}(\cos\lambda) + \cot^{-1}(\cot\lambda) = 2\lambda$

(c) $\sec^{-1}(\sec\lambda) + \operatorname{cosec}^{-1}(\operatorname{cosec}\lambda) = \pi$

(d) Number of solutions of equation $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = \lambda$ is 3.

372. Let $P(x) = x^3 + ax^2 + bx$ be a polynomial whose roots are non-negative and are in arithmetic progression. If the sum of coefficients of $P(x)$ is 10, then:

(a) sum of the roots of $P(x)$ is equal to 9.

(b) sum of the roots of $P(x)$ is equal to 18

(c) the value of $(b-a)$ is equal to 9.

(d) the value of $(b-a)$ is equal to 27.

373. Let $f(x) = \frac{x(x-1)}{(2x-1)(x-2)}$ and range of $f(x)$ is (a, b). Then which of the following is(are) always correct?

(a) $\lim\limits_{y\to b} \frac{5^y}{2^y+3^y}$ does not exist

(b) $\lim\limits_{y\to b} \frac{5^y}{2^y+3^y+5^y} - 1$

(c) $\lim\limits_{y\to a+b} \frac{5^y}{2^y+3^y} = \frac{1}{2}$

(d) $\lim\limits_{y\to a+b} \frac{y^2}{e^y - y - 1} = 2$

374. Let $\vec{\mathbf{A}} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} + 5\hat{\mathbf{k}}$ and $\vec{\mathbf{B}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$.

If $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = 11$; $\vec{\mathbf{A}} \times \vec{\mathbf{B}} = -13\hat{\mathbf{i}} - 9\hat{\mathbf{j}} + 7\hat{\mathbf{k}}$, then:

(a) $x^2 + y^2 = 10$ (b) $y^2 + z^2 = 13$ (c) $[x+y+z] = 5$ (d) $\frac{x+y}{z} = 2$

[**Note:** $[k]$ denotes greatest integer function less than or equal to k.]

375. If $f(x)$ is a polynomial function such that $f(x) + f'(x) + f''(x) + f'''(x) = x^3$, $g(x) = \int \frac{f(x)}{x^3}\,dx$ and $g(1) = 1$, then:

(a) $g(x)$ is strictly increasing function in $(3, \infty)$

(b) the value of $\lim\limits_{x\to 1}(g(x))^{\frac{1}{x-1}}$ is equal to e^2

(c) number of solution of the equation $g(x) = 0$ is 2

(d) the value of $[g(e)]$ is equal to -1

[**Note:** Where $[k]$ denotes greatest integer function less than or equal to k.]

376. Let $g : R \to (-\infty, -1]$ be a function defined as:
$g(x) = (pq + 2p - q - 2)x^5 - (p^3 - 2p + 1)x^3 + (p^2 - 2p - 3)x^2 + (p^2 + 2q)x - 5$
where p, q are rational numbers. If $g(x)$ is surjective, then the possible value of $(p + q)$ is(are):

(a) $\frac{9}{2}$ (b) $\frac{7}{2}$ (c) $\frac{-7}{2}$ (d) $\frac{-9}{2}$

377. Which of the following limit tends to unity?

(a) $\lim\limits_{x\to 0} \frac{\sin(\tan x)}{\sin x}$ (b) $\lim\limits_{x\to \pi/2} \frac{\sin(\cos x)}{\cos x}$

(c) $\lim\limits_{x\to 0}\left(\frac{1}{x^2}\int_0^x \frac{t+t^2}{1+\sin t}\right)$ (d) $\lim\limits_{x\to 0} \frac{\int_0^x \sin^2 t\,dt}{\sqrt[3]{1+x^3} - 1}$

378. If $f(x)$ and $g(x)$ are differentiable functions for $0 \le x \le 1$ such that $f(0) = 2$, $g(0) = 0$, $f(1) = 6$, $g(1) = 2$, then which of the following are true for some $0 < c < 1$ (c in one options may be different from c in another)?

(a) $f'(c) - f(0) = g'(c)$ (b) $f'(c) - g(0) = 2g'(c)$

(c) $f'(c) + f(1) = 3g'(c)$ (d) $f'(c) + 2g(1) = 4g'(c)$

379. If the function $f(x) = \begin{cases} \dfrac{\sin[a(x+1)+\sin x]}{2x}, & x < 0 \\ c, & x = 0 \\ \dfrac{(x+bx^2)^{1/2} - x^{1/2}}{bx^{3/2}}, & x > 0 \end{cases}$ is continuous at $x = 0$, then which of the option(s) can be true (not necessary simultaneously)?

[**Note:** $[k]$ denotes greatest integer function less than or equal to k.]

(a) $a = 5/3$ (b) $b = 2$ (c) $c = 1/2$ (d) $f(1) = 1/3$

380. In triangle ABC, let a, b, c be the length of sides opposite to angles A, B, C respectively and $2s = a + b + c$. If $\frac{s-a}{4} = \frac{s-b}{3} = \frac{s-c}{2}$ and area of circle inscribed in triangle ABC is $\frac{8\pi}{3}$, then:

(a) the area of ΔABC is equal to $6\sqrt{6}$

(b) circumradius of ΔABC is equal to $\frac{35}{2\sqrt{6}}$

(c) angle A is equal to $\cos^{-1}\left(\frac{5}{7}\right)$

(d) the value of $\dfrac{8\sin^2\left(\frac{A+B}{2}\right)}{21\sin\left(\frac{A}{2}\right)\sin\left(\frac{B}{2}\right)\sin\left(\frac{C}{2}\right)}$ is equal to 2

381. If $S_n = \sum_{r=1}^{n} \cot^{-1}(r^2 + 3r + 3)$, then:

(a) $S_\infty = \cot^{-1}(2)$ (b) $S_5 = \cot^{-1}(3)$ (c) $S_6 = \cot^{-1}\left(\frac{17}{6}\right)$ (d) $S_8 = \cot^{-1}(5)$

382. Let $d(x, [a, b]) = \min. \{|x - y : a \le y \le b\}$.

A function $f : R \to [0, 1]$ is defined by $f(x) = \frac{d(x, [0, 1])}{d(x, [0, 1]) + d(x, [2, 3])}$, then which of the following is(are) **incorrect**?

(a) $f(x)$ is decreasing in $(-\infty, 0)$ and increasing in $(3, \infty)$.

(b) The function f is bijective

(c) Number of points where $f(x)$ is non-derivable is 4.

(d) Number of solution of the equation $f(x) - \frac{1}{2}$ is 2.

383. Let $f(x)$ be a polynomial function satisfying $0 < xf(y) < yf(x)\ \forall x, y$ such that $0 < x < y < 1$ and $f(0) = 0$ then:

(a) $f'(x) < f(1)$ (b) $f(1) < 2\int_0^1 f(x)\, dx$

(c) $3f\left(\frac{1}{3}\right) > 2f\left(\frac{1}{2}\right)$ (d) $6f\left(\frac{1}{6}\right) < 5f\left(\frac{1}{5}\right)$

384. An equilateral triangle ΔOAB has side length 1, P is a point on the plane of the triangle. If $\vec{\mathbf{OP}} = (2 - t)\vec{\mathbf{OA}} + t\vec{\mathbf{OB}}$, $t \in R$, then the possible value of $|\vec{\mathbf{AP}}|$ can be:

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{\sqrt{3}}{2}$ (d) 2

385. If $x, |x+1|, |x-1|$ are the first three terms of an arithmetic progression (in that order), then the sum of the first 20 terms of this arithmetic progression can be:

(a) 180 (b) 350 (c) 270 (d) 90

386. For $x, t \in R$, let $P_t(x) - (\sin t)x^2 - (2\cos t)x + \sin t - \frac{1}{3}$ be a family of quadratic polynomial in x, with variable coefficients. Also $A(t) = \int_0^1 (P_t(x))\, dx$.

Which of the following statement are true?

(a) $\lim_{t \to \pi/2} (A(t))^{\tan t}$ equals $e^{4/3}$ (b) $A(t)$ has infinitely many critical points.

(c) $A(t) = 0$ for infinitely many t. (d) $A'(t) > 0$ for all t.

387. Let $f: R \to R$ be a continuous function such that $\int_0^1 f(xt)\, dt = 0$ for all $x \in R$.

Which of the following statement(s) is(are) true?

(a) There exists 3 integral values of P such that the graph of $y = f(x)$ and $y = x^3 - 3x^2 + P$ intersects at 3 distinct points.

(b) $y = f(x)$ is a periodic function.

(c) If A denotes the minimum area bounded by the curves $y = f(x)$, $y = x^4 - 4x - a$ and the ordinates $x = 2$, $x = 4$ then $a = 69$.

(d) $f(x)$ is neither even function nor odd function.

388. Let function $f(x)$ satisfy $x^2 f'(x) + 2x f(x) = e^x$ and $f(2) = \frac{e^2}{4}$. Then:

(a) $f(x) = 1$ has exactly one real solution.

(b) $f(x) = 3$ has exactly three real solutions.

(c) $f(x)$ has local maxima but no local minima.

(d) $f(x)$ has local minima but no local maxima.

389. If (α, β) is a point on a circle whose centre is on x axis and has the coordinate $(\gamma, 0)$ which also touches the line $x + y = 0$ at $(2, -2)$, then.

(a) the radius of the circle is equal to $\sqrt{8}$.

(b) the greatest integral value of α is 7.

(c) γ is equal to 4

(d) length of tangent drawn from origin to the circle is $\sqrt{2}$.

390. In ΔABC with usual notation which of the following is(are) **correct**?

(a) $r_1 + r_2 + r_3 - r = 4R$

(b) $\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} = \frac{1}{r}$

(c) Length of angle bisector of ΔABC drawn through $\angle A$ is $\frac{2bc}{b+c} \sin \frac{A}{2}$.

(d) Length of median of ΔABC drawn through $\angle A$ is $\frac{\sqrt{2b^2 + 2c^2 - a^2}}{2}$.

391. Let d be the number of solutions of the equation $(\sec x - 1) = (\sqrt{2} - 1)\tan x$ in $[0, 2\pi]$. If d lies between the roots of the equation $x^2 + (k-1)x + k^2 + k - 11 = 0$, then k can be:

(a) -4 (b) -2 (c) 0 (d) 1

392. Cards are drawn one by one without replacement from a well shuffled pack of 52 playing cards until 2 aces are obtained for the first time. If N is the number of cards required to be drawn, then the probability

$P(N=n)=\frac{1}{k}(n-a)(n-b)(n-c)$, where $k, a, b, c \in N$ with $a>b>c$.

Then:

(a) the value of a is 52.
(b) the value of $b+c$ is 52.
(c) the value of $a+c$ is 52.
(d) 17 is a factor of k.

393. Let $f(x)$ is non-negative function defined for $x \geq 1$ such that $f'(x) \leq mf(x)$ holds everywhere in the domain for some positive real number m. If $f(1)=0$ then:

(a) $f(x)$ is neither odd nor even.

(b) $\lim\limits_{x\to 2}(5+f(x)+x^2-4x)^{\frac{e^2}{e^x-e^2(x-1)}}$ is equal to e^2.

(c) Number of solutions of the equation $f(x)=e^x-x^2$ is 1.

(d) $\int\limits_{f(e)-1}^{f(e^2)+1} \frac{dx}{2^x+1}$ is equal to $\frac{1}{2}$.

394. If a function $y=f(x)$ passes through the point $\left(\frac{1}{\sqrt{\ln 2}}, \frac{1}{2}\right)$ and satisfies the differential equation $x^2dy-2e^{\frac{-1}{x^2}}dx=0$, then : (Assume $f(0)=0$)

(a) $\int\limits_0^{1/\sqrt{2}} f(x)\,dx < \frac{1}{2e^2\sqrt{2}}$

(b) $\int\limits_0^{1/\sqrt{2}} f(x)\,dx > \frac{1}{2e^2\sqrt{2}}$

(c) $y=f(x)$ has exactly one point of inflection.

(d) $y=f(x)$ has exactly two points of inflection.

395. If $f(p)$ is the number of common tangent lines of two parabolas $x^2=2y$ and $\left(y+\frac{1}{2}\right)^2=4px$, then:

(a) $f(p)=1$ if $p \in \left(-\infty, \frac{-1}{3\sqrt{3}}\right)$
(b) $f(p)=2$ if $p \in \left(\frac{-1}{3\sqrt{3}}, \frac{1}{3\sqrt{3}}\right)$
(c) $f(p)=3$ if $p \in \left(\frac{-1}{3\sqrt{3}}, \frac{1}{3\sqrt{3}}\right)$
(d) $f(p)=4$ if $p \in \left(\frac{1}{3\sqrt{3}}, \infty\right)$

396. Event 'A' is independent of event $B, B \cup C$ and $B \cap C$. If $P(A)=\frac{1}{2}, P(B)=\frac{1}{3}$ and $P(C)=\frac{1}{4}$. Then:

(a) $P\left(\frac{A}{C}\right)=\frac{1}{2}$

(b) $P\left(\frac{\overline{B} \cup \overline{C}}{A}\right)=\frac{11}{12}$ (where B and C are independent events)

(c) $P\left(\frac{\overline{A}}{\overline{B \cap C}}\right)=\frac{1}{2}$

(d) A and C are not independent events

397. Let $S_n=\sum_{k=1}^{n} \tan^{-1}\left(\frac{1}{k(k+1)+1}\right)$ for positive integers $n \in N$, then:

(a) the value of S_{10} is equal to $\frac{\pi}{4}-\tan^{-1}\left(\frac{1}{11}\right)$.

(b) the value of $\lim_{n\to\infty} S_n$ is equal to $\frac{\pi}{2}$.

(c) the value of $5+\sum_{n=1}^{62} \frac{1+\tan S_n}{1-\tan S_n}$ is equal to 2020.

(d) the value of S_5 is equal to $\tan^{-1}(6)-\frac{\pi}{4}$.

398. Consider ΔABC, $A(5,-1)$, $B(\alpha,-7)$, $C(-2,\beta)$. Let $(-6,-4)$ is image of orthocentre of ΔABC in the point mirror M which is mid-point of the side BC. Also (p,q) is circumcentre of triangle ABC, then:

(a) the value of $\beta^2-\alpha^2+5\beta-\alpha$ is 12.

(b) the value of $2p+1$ is 0.

(c) the value of $2q+5$ is -6.

(d) the value of $q^2-\frac{p}{2}$ is $\frac{13}{2}$.

399. If $f(x)+g(x)+h(x)=2\ \forall\ x \in R$, then the value of the expression $\int_0^{3/4}(f^2(x)+g^2(x)+h^2(x))\,dx$, can be:

(a) $\frac{1}{2}$ (b) 1 (c) $\frac{3}{2}$ (d) 4

400. Let a, b, c denotes side lengths of ΔABC. If a, b, c are the roots of $8x^3 + (\lambda + 2)x^2 - (2k + \lambda)x \quad 27 = 0$ such that $\lambda^2 + 2\lambda(k+1) + 4k = 2^3 \cdot 3^5$, then which of the following is(are) **correct?**

(a) Circumradius of triangle ABC is $\frac{\sqrt{3}}{2}$.

(b) Distance between orthocentre and side AB is $\frac{\sqrt{3}}{4}$.

(c) Distance between orthocentre and circumcentre of ΔABC is $\frac{\sqrt{3}}{4}$.

(d) Distance between orthocentre and side BC is $\frac{\sqrt{3}}{2}$.

401. A and B shoot independently until each shoots their target. They have probabilities $\frac{3}{5}$ and $\frac{5}{7}$ respectively of hitting the target at each shot. Then:

(a) probability that B require more shots than A is $\frac{6}{31}$.

(b) probability that B require less shots than A is $\frac{10}{31}$.

(c) probability that A and B require same number of shots is $\frac{15}{31}$.

(d) probability that B require more shots than A is same as probability that A require more shots than B.

402. Which of the following is correct ?

(a) $\log_5\left(\sqrt{7\sqrt{7\sqrt{7....}}}\right) > 1$

(b) $\log_{(\sqrt{7}-\sqrt{6})}(\sqrt{3} - \sqrt{2}) < 1$

(c) $\log_3 10 > \log_{10} 70$

(d) $\log_3(3 + \sqrt{2}) > \log_2(2 - \sqrt{2})$

403. Which of the following is equal to integer?

(a) $7^{-\log_7 6} + 81^{(1-\log_9 2)}$

(b) $\log_6 3 \cdot \log_6 12 + (\log_6 2)^2$

(c) $\frac{1}{\log_5 3} + \frac{1}{\log_6 3} - \frac{1}{\log_{.0} 3}$

(d) $(\sqrt[3]{2} + \sqrt[3]{5})(\sqrt[3]{4} - \sqrt[3]{10} + \sqrt[3]{25})$

404. Let equation $x^{\log_2 x - 4} = 32$ has two real solutions x_1 and $x_2 (x_1 > x_2)$, then which of the following is correct?

(a) $x_1 \cdot x_2 = 32$

(b) $x_1 + x_2 = \frac{65}{2}$

(c) Characteristic of $\log_3(x_1)$ is 3

(d) Mantissa of $\log_2(x_2)$ is 0

405. Let $y=\dfrac{\sin x\cdot\sin 2x+\sin 3x\cdot\cos 6x+\sin 4x\cdot\cos 13x}{\sin x\cdot\cos 2x+\sin 3x\cdot\cos 6x+\sin 4x\cdot\cos 13x}$, then:

(a) if $x=\dfrac{\pi}{72}$, then $y=\sqrt{2}-1$ (b) if $x=\dfrac{\pi}{24}$, then $y=\sqrt{2}+1$

(c) if $x=\dfrac{\pi}{108}$, then $y=2+\sqrt{3}$ (d) if $x=\dfrac{5\pi}{108}$, then $y=2-\sqrt{3}$

406. Let $x=\alpha$ is a root of the equation $\log_3(9\cdot 2^x+9)\cdot\log_3(2^x+1)=\log_{\frac{1}{\sqrt{3}}}\left(\dfrac{1}{\sqrt{27}}\right)$, then α is less than:

(a) 1 (b) 2 (c) 3 (d) 4

407. Let x and y are positive real number such that $\log_9 x+\log_{27} y=\dfrac{7}{2}$ and $\log_{27} x+\log_9 y=\dfrac{2}{3}$, then:

(a) $xy=243$ (b) $xy=729$ (c) $\dfrac{x}{y}=3^{16}$ (d) $\dfrac{x}{y}=3^{17}$

408. If the expression $f(x)=x^4+2x^3+ax^2+bx+3$ has remainder $r(x)=4x+3$, when divided by $g(x)=x^2+x-2$, then:

(a) $a+b=1$ (b) $a-b=-3$ (c) $|b|=|2a|$ (d) $3b-2a=8$

409. Let $x=\sin\theta\cos^3\theta$ and $y=\sin^3\theta\cos\theta$, then:

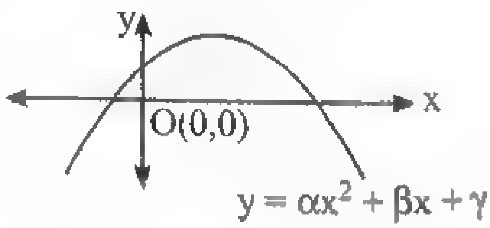
(a) if $0<\theta<\dfrac{\pi}{4}$, then $x+y>0$ (b) if $\dfrac{\pi}{4}<\theta<\dfrac{\pi}{2}$, then $x<y$

(c) if $\dfrac{\pi}{2}<\theta<\dfrac{3\pi}{4}$, then $x+y>0$ (d) if $\dfrac{3\pi}{4}<\theta<\pi$, then $x<y$

410. Let α,β,γ are positive real numbers such that $\log_\gamma(2\alpha)=\dfrac{1}{3}$, $\log_\gamma(5\beta)=\dfrac{1}{6}$ and $\log_\gamma(\alpha\beta)=\dfrac{3}{2}$, then:

(a) $\alpha^3=\dfrac{1}{16}$ (b) $\alpha^3=\dfrac{1}{80}$ (c) $\gamma=\dfrac{1}{10}$ (d) $\beta^{12}=\dfrac{5^{14}}{2}$

411. The following figure illustrate the graph of a quadratic trinomial $y=\alpha x^2+\beta x+\gamma$.



Then which of the following is(are) **correct**?

(a) $\alpha\beta<0$ (b) $\alpha^2+\beta\gamma>0$

(c) $\beta+\gamma-\alpha>0$ (d) $\alpha\beta\gamma>0$

412. Let $f(x) = (k-3)x^2 - 2kx + 3k - 6$ where $x \in R$. If the range of $f(x)$ is $[0, \infty)$, then the value of k can be:

(a) $\frac{3}{2}$ (b) 1 (c) 6 (d) 9

413. Let $f(\theta) = \left(1 + \frac{4\sin\theta}{\sin 6\theta}\right)\left(1 + \frac{4\sin 2\theta}{\sin 5\theta}\right)$, then:

(a) $f\left(\frac{\pi}{7}\right) = 25$ (b) $f\left(\frac{\pi}{7}\right) = -25$ (c) $f\left(\frac{2\pi}{7}\right) = 9$ (d) $f\left(\frac{2\pi}{7}\right) = -9$

414. Let $f(n) = \sum_{r=1}^{n} \log_{10}\left(\frac{9r+1}{9r-8}\right)$, then:

(a) $f(11) = 2$ (b) $f(11) = -2$ (c) $f(111) = 3$ (d) $f(1111) = 4$

415. If $2\sin^2\theta + 2\sqrt{2} = 3\,\text{cosec}^2\theta$, where $\theta \in (0, \pi)$, then:

(a) number of real solutions is 2. (b) number of real solution is 4.
(c) sum of all solutions is π. (d) sum of all solutions is 4π.

416. If the greatest value of $f(x) = -x^2 + 4x + \lambda - 4$, where $x \in [0, 5]$ is smaller than the least value of $g(x) = x^2 - 2\lambda x + 10 - 2\lambda$, where $x \in R$ then λ may be:

(a) $\frac{-3}{2}$ (b) $\frac{-17}{4}$ (c) $\frac{3}{11}$ (d) $\frac{-1}{8}$

417. Consider, $f(x) = \frac{(\sin x - 10)(x^2 - 4x + 3)(x^2 + x + 1)}{x^2 - 16}$.

Identify which of the following statement(s) is (are) correct.

(a) Number of integral values of x for which $f(x) \geq 0$ is 6.
(b) Sum of all the integral values of x for which $f(x) \geq 0$ is -2.
(c) Number of integral values of x for which $f(x) \leq 0$ is 10.
(d) Sum of all the integral values of x for which $f(x) \leq 0$ is 6.

418. If A and B are acute angles such that $A = \frac{1}{2}$ and $\tan B = \frac{1}{3}$, then:

(a) $\sin^2(A+B) = \frac{1}{2}$ (b) $\tan\left(\frac{A+B}{2}\right) = \sqrt{2} - 1$
(c) $\cot\left(\frac{A+B}{3}\right) = 2 - \sqrt{3}$ (d) $\cos(2A + 2B) = 0$

419. If sum of an infinite G.P. is $p\,(p \in R)$, then which of the following can be the common ratio of the G.P.?

(a) $\frac{1}{\sin^2\theta}\,(\theta \in R, \theta \neq n\pi, n \in I)$ (b) $e^{-t^2}\,(t \in R, t \neq 0)$
(c) $\frac{1}{2}\left(y^2 + \frac{1}{y^2}\right), (y \in R, y \neq 0)$ (d) $\frac{2}{x^2 - 4x + 7}, (x \in R)$

420. If equations $ax^2+(p+2)x+10q=0$ and $x^2+4x+5=0$ have a common root where $a, p, q \in N$ and $\lambda = (a+p+q)$ has the least value then:

(a) $\lambda = 8$
(b) $\lambda = 9$
(c) $a+q+\frac{3}{p}+\ldots\ldots\infty = 4$
(d) $a+q+\frac{3}{p}+\ldots\ldots\infty = \frac{9}{2}$

421. For the equation $3^{x} = 4^{x^2}$, the correct statement(s) is/are:

(a) number of solutions is 2.
(b) number of solution is 1.
(c) one of the solution is $\log_4\left(\sqrt{3\sqrt{3\sqrt{3\ldots\ldots\infty}}}\right)$
(d) the sum of all the solutions is $\log_4\left((1+\sqrt{2})^2 - 2^{\frac{3}{2}}\right)$

422. If $\sin x + 1 = \cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\ldots\ldots\infty}}}$ where $x \in (0, 2\pi)$, then:

(a) sum of all the solutions of the equation is 3π.
(b) sum of all the solutions of the equation is 6π.
(c) number of solutions of the equation is 2.
(d) number of solutions of the equation is 4.

423. If n^{th} term of the series $6+17+34+57+\ldots\ldots$ is $(\log_2 a)n^2 + (\log_3(b-a))n + \log_4 c$, then:

(a) $a+b=5$
(b) $b-c=13$
(c) $\log_5(2c+b) = \log_5(a+b)$
(d) $\log_2(2c+a) = 4$

424. Let $f(x) = px^2 - 3px + 14$. If $f(x) \geq |3\sin\theta - 4\cos\theta \ \forall\, x,\ \theta \in R$, then identify which of the following statement(s) is/are correct?

(a) Sum of all possible integral values of p is 10.
(b) Sum of all possible integral values of p is 15.
(c) If $f(x) \leq 14 + \sin^2\alpha\ \forall\, x,\ \alpha \in R$ then number of integral value of p is 1.
(d) If $f(x) \leq 14 + \sin^2\alpha\ \forall\, x,\ \alpha \in R$ then number of integral value of p is 0.

425. Let y_i ($i = 1$ to n) are the solution satisfying the equation $y^{\log_3(\sqrt{3y})} = y^{\log_3(3y)} - 6$ where $y > 0$. Then:

(a) $\sum_{i=1}^{n} y_i = \frac{28}{9}$
(b) $\prod_{i=1}^{n} y_i = \frac{1}{3}$
(c) Value of n is equal to 3.
(d) $\sum_{i=1}^{n} \frac{1}{y_i} = 9$

426. Let $P(x)=4\sin^3 x-\sin x+2\left(\sin\frac{x}{2}-\cos\frac{x}{2}\right)^2$, then which of the following is/are **correct**?

(a) Range of $P(x)$ is $[1, 3]$

(b) Range of $P(x)$ is $[0, 4]$

(c) Number of solution of $P(x)=1$ in $[0, \pi]$ is 2

(d) Number of solution of $P(x)=1$ in $[0, 2\pi]$ is 4

427. Four points A, B, C, D taken in order lie on the circumference of a circle to form a quadrilateral. Let $\alpha, \beta, \gamma, \delta$ denote four interior angles of the quadrilateral associated with A, B, C, D respectively. Which of the following is/are always true?

(a) $\cos\beta\cos\delta=1+\sin\beta\sin\delta$

(b) $\sin\alpha\cos\gamma+\cos\alpha\sin\gamma=0$

(c) $\sin^2\alpha+\cos^2\gamma=1$

(d) $\cos\beta+\cos\delta=0$

428. Let $a, b, c=5$ represents sides of triangle ABC. If $a, b(a<b)$ are the integral values of p for which graph of $f(x)=x^2-2(p+1)x+9(p-1)$ lies completely above x-axis, then:

(a) area of triangle is 6.

(b) circumradius of triangle is 5/2.

(c) value of $\cos A+\cos B+\cos C$ is 7/5.

(d) inradius of triangle is 2.

429. If the quadratic equation $(\log_2(\sin\theta))x^2+2x\quad 1=0$ has integral roots, then π/θ can be:

(a) 6/5 (b) 6/13 (c) 5/17 (d) 2

430. Consider, $f(n)=\begin{vmatrix} 2 & 1 & 0 \\ \frac{1}{(n+3)^2} & \frac{1}{(n+1)} & \frac{1}{(n+3)^2}-\frac{1}{n+1} \\ \frac{1}{(n+2)^2} & \frac{1}{(n+2)} & \frac{-(n+1)}{(n+2)^2} \end{vmatrix}$ where $n\in N$, then identify which of the following statement(s) is(are) **correct**?

(a) $\sum_{n=1}^{7} f(n)=\frac{49}{900}$

(b) $\sum_{n=1}^{7} f(n)=\frac{49}{450}$

(c) $\sum_{n=1}^{\infty} f(n)=\frac{1}{18}$

(d) $\sum_{n=1}^{\infty} f(n)=\frac{1}{9}$

431. Let P be a point on the line segment joining $A(5\cos\alpha, 5\sin\alpha)$ and $B(5\cos\beta, 5\sin\beta)$ such that $3PA = 2PB$ then the locus of P is:

(a) $x^2 + y^2 = 13$ if $|\alpha - \beta| = \pi/2$
(b) $x^2 + y^2 = 19$ if $|\alpha - \beta| = \pi/3$
(c) $x^2 + y^2 = 1$ if $|\alpha - \beta| = \pi$
(d) $x^2 + y^2 = 25$ if $|\alpha - \beta| = \pi/6$

432. For the equation $\sqrt{3}\sin 2x = \cos 2x + 2\tan\frac{x}{2}(1+\cos x)$ which of the following holds good?

(a) The number of solutions of the equation in $[0, 2\pi]$ is 4
(b) The number of solutions of the equation in $[0, 2\pi]$ is 3
(c) If α is the smallest positive root of the equation then $\dfrac{\tan 2\alpha + 2\cos 2\alpha}{\cot\alpha - \sin 3\alpha} = 2+\sqrt{3}$
(d) If α is the smallest positive root of the equation then $\dfrac{\tan 2\alpha + 2\cos 4\alpha}{\cot\alpha + \sin 3\alpha} - 2-\sqrt{3}$

433. If the straight line $3x - 4y + 7 - 0$ rotated through 90° about a point (3, 4) meets the coordinate axes at A and B and a ΔAOB is formed (O is the origin), then

(a) area of the triangle formed by orthocentre, circumcentre and centroid of the ΔAOB is 1 sq. units.
(b) area of the triangle formed by orthocentre, circumcentre and incentre of the ΔAOB is 1 sq. units.
(c) distance between orthocentre and incentre is 2 units.
(d) distance between orthocentre and circumcentre is 5 units.

434. In ΔABC if $AB - AC$ and lengths of the tangents to the incircle from the vertices A and C are 4 and 2 respectively, then identify which of the following statement(s) is(are) **correct**?

(a) $AI : BI : CI = \sqrt{3} : 1 : 1$
(b) Inradius of the triangle ABC is $\sqrt{2}$ sq. units
(c) $R = \frac{9}{4}\sqrt{2}$
(d) $\Delta R = a^2$

[**Note:** Symbols used have usual meaning in ΔABC]

435. If the point $(\alpha, 0)$ lies inside the quadrilateral formed by lines $2x + 5y = 15$, $5x - 4y = 21$, $3x + 5y + 17 = 0$ and $y = x + 3$, then which of the following is **true**?

(a) Number of prime value(s) of α is 4.
(b) Number of integral value(s) of α is 7.
(c) Minimum integral value of α is -3.
(d) Maximum integral value of α is 4.

436. Let (x^2+3x+2), (x^2-x-10) and $(x^2+x-4+y^2)$ are first three positive terms of an A.P., such that their squares will form a G.P., then which of the following is **true**?

(a) Sum of all possible integral value(s) of x is -3.

(b) If y, $y+3$ and z are in A.P., then z is equal to 6.

(c) Harmonic mean of given numbers is 2.

(d) Sum of first 20 terms of this A.P. is 40.

437. Two sides of a triangle have the joint equation $(x-3y+2)(x+y-2)=0$, the third side which is variable always passes through the point $(-5, -1)$, then the possible values of slope of third side such that origin is an interior point of the triangle is/are:

(a) $-\frac{4}{3}$ (b) $-\frac{2}{3}$ (c) $-\frac{1}{3}$ (d) $\frac{1}{6}$

438. Consider an obtuse angle triangle ABC of area $\frac{3\sqrt{3}}{4}$, where $\angle C$ is obtuse and sides a and b of the triangle satisfy the equation $(a-2b-1)^2+(2a-3b-3)^2=(a-2b-1)(2a-3b-3)$, then which of the following option(s) are **correct**?

(a) Inradius of ΔABC is $\frac{\sqrt{3}(4-\sqrt{13})}{2}$.

(b) Length of angle bisector drawn from vertex C is $\frac{3}{4}$.

(c) $\frac{a\sin 2B}{b}+\frac{b\sin 2A}{a}$ is equal to $\sqrt{3}$.

(d) Equation $a^2x^2+c^2x+b^2=0$ has no real roots (where a, b, c are length of sides of the triangle).

439. If $S: x^2+y^2+2gx+2fy+c=0$ intersects both the lines $xy-3x=0$ orthogonally and touches the circle $x^2+y^2-6x-12y+36=0$ externally, then:

(a) S neither intersects nor touches the x-axis

(b) radius of the circle S lies in $(1, 2)$

(c) radius of the circle S lies in $(2, 3)$

(d) equation of the transverse common tangent to both the circle is $x+y-3\sqrt{2}-9=0$

440. Let m, n be real numbers satisfying the relation:

$$\log_2(m^2+n^2+1)+\log_3\left(\frac{1+2\sin^2\left(\frac{2\pi}{7}\right)}{2-\cos\left(\frac{4\pi}{7}\right)}\right)=\log_2 n+\log_2(2m+2-n).$$

Identify which of the following statement(s) is(are) **correct**?

(a) $m+n=2$

(b) $||m|-|n||=4$

(c) Area of the figure enclosed by $|x|+|y|=|m|+|n|$ is 8 sq. units.

(d) Area of the figure enclosed by $|x|+|y|=|m|+|n|$ is 16 sq. units.

441. Consider, $f(x)=x^2+\lambda x+a^2+a+1$, where $a, \lambda \in R$. Identify correct statement(s) about $f(x)$.

(a) Least positive integral value of λ for which $f(x)=0$ has real roots for some real value of 'a' is 2

(b) If $\lambda=2$ then set of values of a for which $f(x)=0$ has real roots is $[-1, 0]$

(c) If both the roots of the equation $f(x)=0$ and $2x^2 \quad x+6=0$ are identical then sum of all possible values of 'a' is (-1)

(d) If $f(1+x)=f(1-x)\ \forall x \in R$, then $\lambda=2$

442. Let $a^3+b^3+c^3 \le 3abc$ where $a, b, c>0$. If the value of x is equal to a for which $y=\dfrac{(x-3)^2+3}{x-2}$ is least positive, then.

(a) $\log_2(a+b+c)=\log_2(abc)$

(b) $a+b<c$

(c) $\log_2 a+\log_2 b+\log_2 c=3\log_2 a$

(d) $\dfrac{b^2+3a}{4}=7$

443. If words are formed using all the letters of the word 'CHITRANJEEVI', then:

(a) number of words in which all vowels are separated is $7!\times {}^8C_5\times\dfrac{5!}{2!2!}$

(b) number of words in which vowels appear in alphabetical order is $\dfrac{12!}{5!}$

(c) number of words which contains the word 'CHITRA' is $\dfrac{7!}{2!}$

(d) number of words which contains the word 'IITJEE' is 7!

444. Let $<T_n>$ be a sequence such that $T_n^3+2T_n=T_{n+1}\ \forall n \in N$, and $T_1=1$, then:

(a) $\displaystyle\sum_{n=1}^{100}(T_n^3+T_n+1)=T_{101}+99$

(b) $\displaystyle\sum_{n=1}^{100}(T_n^3+T_n+1)=T_{101}+100$

(c) $\displaystyle\prod_{n=1}^{100}(T_n^2+2)=T_{100}$

(d) $\displaystyle\prod_{n=1}^{100}\left(\frac{T_{n+1}}{T_n}-T_n^2\right)=2^{100}$

445. Let straight line $y = mx + 4$ meets the curve $3x^2 - (1 \quad 3a)xy - ay^2 = 0$ at two points A and B such that $\angle AOB = 90^\circ \ \forall \ m \in R - \{m_1, m_2\}$ where $m_1 < m_2$ and 'O' is the origin. Identify which of the following statement(s) is/are correct?

(a) $m_1 + m_2 = \frac{10}{3}$

(b) $am_1 + m_2 = 2$

(c) If $m = 2$, then area of $\Delta AOB = \frac{80}{7}$ sq. units

(d) If $m = 2$, then area of $\Delta AOB = \frac{85}{7}$ sq. units

446. If P and Q are two points in ΔABC such that

$PA : PB : PC = \text{cosec}\left(\frac{A}{2}\right) : \text{cosec}\left(\frac{B}{2}\right) : \text{cosec}\left(\frac{C}{2}\right)$ and $AQ = BQ = CQ$ where $AB = 7$, $BC = 9$ and $CA = 8$, then:

(a) $PA^2 + PB^2 + PC^2 = 290$

(b) $\cos A + \cos B + \cos C = \frac{31}{21}$

(c) $PA^2 + PB^2 + PC^2 = 65$

(d) $AQ = \frac{21\sqrt{5}}{10}$

447. In the expansion of $\left(2^{\frac{1}{5}} + 7^{\frac{1}{7}}\right)^{105}$, which of the following holds good?

(a) Number of rational terms are 4.

(b) Number of irrational terms are 102.

(c) Exactly one middle term is irrational.

(d) Both middle terms are irrational.

448. If $f(x) = \frac{3}{1 + \tan^2 x} + \frac{9}{1 + \cot^2 x}$, then :

(a) number of integers in the range of $f(x)$ is 7.

(b) number of integers in the range of $f(x)$ is 5.

(c) sum of the integers in the range of $f(x)$ is 30.

(d) sum of the integers in the range of $f(x)$ is 42.

449. If $xyz = 2^3 \times 3^1 \times 5^2 \times 7^1$, then identify which of the following statement(s) is(are) correct?

(a) If $x, y, z \in N$, then number of ordered triplets (x, y, z) is 540.

(b) If $x, y, z \in I$, then number of ordered triplets (x, y, z) is 1620.

(c) If $P = xyz$, then number of divisors of P which are divisible by 12 is 12.

(d) If $P = xyz$, then product of divisors of P which are divisible by 12 is $(12P)^6$.

450. If α satisfies the equation $2\sqrt{2}\tan^3 x - 54\sqrt{2}\cot^3 x = 19$, then possible value of $(2\tan^2\alpha + \sqrt{2}\tan\alpha)$ can be equal to:

(a) 6 (b) 12 (c) 2 (d) $-4\sqrt{2}$

451. If -6α, β and $3\alpha^2 + 3\beta$ (in order) are the first three consecutive terms of an A.P. where α and β are natural numbers, then:

(a) the possible value of $(\alpha+\beta)$ is 4.
(b) the possible value of $(\alpha+\beta)$ is 2.
(c) the sum of the first 9 terms is 270.
(d) the sum of the first 9 terms is 540.

452. Consider an equation, $y(3^{|x|}-1) + 2|x|(2^y - 1) = 0$ where $y = \log_2(3x^2 - 1) - \log_2\sqrt{x^2+1}$. Identify which of the following statement(s) is(are) correct?

(a) Number of real solutions of the equation is 2.
(b) Number of real solutions of the equation is 5.
(c) Sum of squares of all the solutions is $\frac{7}{9}$.
(d) Sum of squares of all the solutions is $\frac{14}{9}$.

453. Which of the following function(s) is(are) surjective?

(a) $f: D_f \to R$, $f(x) = \ln(\tan(\pi[x]) + |x^2 + 2x - 3|)$

(b) $g: D_g \to R$, $g(x) = \dfrac{x^2+2x-3}{x-1}$

(c) $h: D_h \to R$, $h(x) = \ln\left(\dfrac{1-x}{1+x}\right)$

(d) $k: D_k \to R^+$, $k(x) = \sqrt{[x]+[-x]+1} + \sqrt{\{x\}+\{-x\}+1}$

[**Note:** $[m]$ and $\{m\}$ denotes greatest integer function less than or equal to m and fraction part function of m respectively, and D_l denotes the domain of the function $y = l(x)$.]

454. Let α and β be the roots of the equation $x^2 - 5x + 5 = 0$.

If $b = \dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha}$ and $t = x^2 - 4x + 3b - \dfrac{1}{5} + \dfrac{1}{x^2 - 4x + 9}$, $x \in R$ then:

(a) minimum value of $(b+t)$ is 8
(b) maximum value of $\log_{1/5}(t)$ is -1
(c) range of $y = \cot^{-1}(\log_5 t)$ is $\left(0, \dfrac{\pi}{4}\right]$
(d) range of $y = \cot^{-1}(\log_{1/5}(t))$ is $\left[\dfrac{\pi}{4}, \pi\right)$

455. If α, β and γ are the positive roots of the equation $x^3 - px^2 + qx - 7 = 0$ such that $\alpha\beta = 1$ and $p, q \in R$ and $p \le 9$ then:

(a) $|p+q| = 24$

(b) $p - q = -6$

(c) $\tan^{-1}\alpha + \tan^{-1}\gamma = \tan^{-1}\left(\frac{4}{3}\right)$

(d) $\tan^{-1}\alpha + \tan^{-1}\gamma = \tan^{-1}\left(\frac{4}{3}\right) = \pi$

456. Consider $f(x) = \tan^{-1}\left(\frac{2x}{\sqrt{9-4x^2}}\right) - \cos^{-1}\left(\frac{x}{3}\right)$. Identify which of the following statement(s) is(are) **correct**?

(a) Number of solutions of the equation $f(x) = \ln(-x)$ is 2.

(b) Number of solutions of the equation $f(x) = \ln(-x)$ is 1.

(c) If $f(x) - k = 0$ has a solution then number of integral values of k is 4.

(d) If $f(x) - k = 0$ has a solution then number of integral values of k is 3.

457. If $f(x) = x^2 - px + q$, $p, q \in R$ such that $f(x) = f(6-x)\ \forall\, x \in R$ and least value of $f(x)$ is $\frac{-81}{4}$ then:

(a) the least value of $\tan^{-1}(22 + [f(x)])$ is $\frac{\pi}{4}$

(b) the least value of $\tan^{-1}(22 + [f(x)])$ is $\frac{-\pi}{4}$

(c) largest integral value of k for which equation $\text{sgn}(f(x) + k) = 0$ has a solution is 20.

(d) largest integral value of k for which equation $\text{sgn}(f(x) + k) = 0$ has a solution is 21.

[**Note:** $[y]$ denotes greatest integer function less than or equal to y and sgn(y) denotes the signum function of y.]

458. If $5 \cdot 2^8 \cdot 3^{16}$ is one of the terms of a G.P. whose first term is 5 and all its terms are natural number then possible common ratio of the G.P. is:

(a) 6 (b) 12 (c) 18 (d) $(324)^2$

459. If $f(x)$ is a monic polynomial function of degree 4 satisfying $f(i) = \frac{1}{i}$ for $i = 1, 2, 3, 4$ then:

(a) number of zeroes at the end of $f(5)!$ is 4.

(b) number of divisors of $f(5)$ is 8.

(c) sum of even divisors of $f(5)$ is 56.

(d) sum of odd divisors of $f(5)$ is 18.

460. Consider $(1+x)^{2n}+(1+2x+x^2)^n=\sum_{r=0}^{2n} a_r x^r,\ n\in N$. If $\sum_{r=0}^{2n} a_r = f(n)$ then:

(a) $\sum_{n=1}^{\infty} \frac{1}{f(n)}=\frac{1}{6}$

(b) $\sum_{n=1}^{\infty} \frac{1}{f(n)}=\frac{3}{8}$

(c) largest value of p for which $f(5)$ is divisible by 2^p is 11.

(d) largest value of p for which $f(5)$ is divisible by 2^p is 9.

461. Let $y=f(x)$ be a cubic polynomial such that $\lim_{x\to 0}(1+f(x))^{\frac{1}{x}}=e^{-1}$; $\lim_{x\to 0}\left(x^3 f\left(\frac{1}{x}\right)\right)^{\frac{1}{x}}=e^2$,

then which of the following is/are **correct**?

(a) Sum of all real roots of $f(x)=0$ is -2

(b) Product of all real roots of $f(x)=0$ is 0.

(c) $\lim_{x\to\infty}\left(\frac{f(x)}{x^3}\right)=2$

(d) $\lim_{x\to\infty}\left(\frac{f(x)}{x^3}\right)=1$

462. Let $f(x)=\frac{\cos^{-1}(1-\{x\})\sin^{-1}(1-\{x\})}{\sqrt{2\{x\}}\,(1-\{x\})}$, then which of the following is/are **correct**?

[**Note:** $\{k\}$ denotes fractional part function of k.]

(a) $\lim_{x\to 0^+} f(x)=\sqrt{2}\lim_{x\to 0^-} f(x)$

(b) $\lim_{x\to 0^-} f(x)=\sqrt{2}\lim_{x\to 0^+} f(x)$

(c) $\lim_{x\to 0^-} f(x)=\frac{\pi}{2\sqrt{2}}$

(d) $\lim_{x\to 0^-} f(x)=\sqrt{2}\pi$

463. Let $f(x)=\begin{cases}\cos^{-1}x, & -1\le x<0\\ \sin^{-1}x, & 1\le x\le 0\end{cases}$ and $g(x)=\begin{cases}\sin^{-1}x, & -1\le x<0\\ \cos^{-1}x, & 1\ge x\ge 0\end{cases}$. If $h(x)=\min\{f(x), g(x)\}$, then:

(a) $h(x)$ is continuous $\forall\, x\in[-1,1]$

(b) $h(x)$ is non derivable at exactly one point in $x\in(-1,1)$

(c) minimum value of $h(x)$ is equal to $\frac{-\pi}{4}$

(d) maximum value of $h(x)$ is equal to $\frac{\pi}{4}$

464. Let S be a circle with centre O and radius 2. A and B be two points on the circle. $\angle AOB = x$, tangents at A and B intersect at D and OA and BD intersect at C. Then which of the following must be **correct**?

(a) $\lim_{x\to 0}\frac{\text{Area }(\Delta OBC)}{\text{Area }(\Delta OAB)}=1$

(b) $\lim_{x\to 0}\frac{\text{Area }(\Delta OBC)}{\text{Area }(\Delta OAB)}=2$

(c) $\lim_{x\to 0}\frac{\text{Area }(\Delta ADB)}{(\text{Area }(\Delta OAB))^3}=\frac{1}{16}$

(d) $\lim_{x\to 0}\frac{\text{Area }(\Delta ADB)}{(\text{Area }(\Delta OAB))^3}=\frac{1}{4}$

465. If $x^4+3x^3+2(1-a)x^2-3ax+a^2=0$ has only real roots then which of the following may be the value of a?

(a) -1 (b) 0 (c) 1 (d) 2

466. A shopkeeper places before you 41 different toys out which 20 toys are to be purchased. Suppose m number of ways in which 20 toys can be purchased without any restriction and n – number of ways in which a particular toy is to be always included in each selection of 20 toys, then $(m-n)$ can be expressed as:

(a) $\frac{2^{10}}{20!}(1\cdot 3\cdot 5\ldots\ldots 39)$

(b) $\frac{2^{20}(1\cdot 3\cdot 5\ldots\ldots 19)}{10!}$

(c) $\prod_{r=0}^{19}\left(\frac{4r+2}{20-r}\right)$

(d) $\left(\frac{21}{1}\right)\left(\frac{22}{2}\right)\left(\frac{23}{3}\right)\ldots\ldots\left(\frac{40}{20}\right)$

467. Let $y=f(x)$ be a differentiable function such that $f(3-x)=f(3+x)\ \forall x\in R$ and the equation $f(x)=0$ has exactly 5 distinct real roots x_1, x_2, x_3, x_4 and x_5. If $x_1<x_2<x_3<x_4<x_5$, then which of the following is/are must be **correct**?

(a) $x_1+x_2+x_3+x_4+x_5=15$

(b) $f'(x_3)=0$

(c) $y=|f(x)|$ is not differentiable at $x=x_1, x_2, x_4$ and x_5.

(d) $y=|f(x)|$ is a differentiable function.

468. In the binomial expansion of $\left(\sqrt{y}+\frac{1}{2\sqrt[4]{y}}\right)^n$ the first three coefficients form an arithmetic progression. Then:

(a) the value of n is 7

(b) the value of n is 8

(c) number of terms in the expansion where the power of y is natural is 2

(d) number of terms in the expansion where the power of y is natural is 3

469. Let $f(x)=\begin{cases}\left[\tan^{-1}x+2\operatorname{sgn}\left(\dfrac{x}{1+x^2}\right)\right], & x\geq 0\\ \{[x]\}, & x<0\end{cases}$ and $g(x)=|x+2|-\tan 1\ \forall\ x\in R$

If $h(x)=\min.\{f(x), g(x)\}$, then:

(a) minimum value of $h(x)$ is $-\tan 1$.

(b) maximum value of $h(x)$ is 3.

(c) number of points where $h(x)$ is discontinuous is 2.

(d) number of points where $h(x)$ is non-derivable is 5.

[Note: $[y]$, $\{y\}$ and $\operatorname{sgn}(y)$ denote greatest integer, fractional part and signum function of y respectively.]

470. Let $f(x)=\lim\limits_{n\to\infty}\left(a^{\frac{1}{n}}+\ln b+\cos\dfrac{x}{\sqrt{n}}\right)^n$ where $a, b>0$ be a non-constant function and $L=\lim\limits_{x\to 0}\dfrac{f(x)-a}{1-\cos x}$. Identify which of the following statement(s) is(are) **correct**?

(a) The number of solution(s) of the equation $f(x)=|x|$ are 3.

(b) The number of solution(s) of the equation $f(x)=|x|$ are 2.

(c) $a+L=0$

(d) $a+L+3be=2$

471. Let $f(x)=(\sqrt{\pi^2\ \ 1}\cos x+\sin x)\cos(x\ \ \operatorname{cosec}^{-1}\pi)$. If m and M are respectively minimum and maximum values of $f(x)$, then which of the following is(are) **correct**?

(a) $m+M=p$

(b) $m+M=0$

(c) $\cos(m+M)=\cos m+\cos M$

(d) $\sin(m+M)=\sin m+\sin M$

472. If $f(x)=\begin{cases}\tan^{-1}(|x|-1), & |x|\leq 1\\ a|x|^3+bx^2+c, & |x|>1\end{cases}$ is non-derivable at exactly one point in R and $f'(2)=0$, then:

(a) $a+b=\dfrac{2}{3}$

(b) $c-2a=0$

(c) $\lim\limits_{x\to 3}\dfrac{f(x)-f(3)}{x-3}=-3$

(d) $\lim\limits_{x\to -3}\dfrac{f(x)-f(-3)}{x+3}=-3$

473. 50 identical marbles are to be distributed among four boys, A_1, A_2, A_3 and A_4. The number of marbles receiving by them in the distribution are as follows:

A_1 : 1, 3, 5, 7, ...

A_2 : 4, 6, 8, 10, ...

A_3 : 5, 7, 9, 11, ...

A_4 : 2, 4, 6, 8, ...

Identify which of the following statement(s) is(are) **correct**?

(a) The total number of ways of distribution is $^{22}C_3$

(b) The total number of ways of distribution is $^{20}C_3$

(c) If A_4 is receiving not more than 14 marbles, then number of ways of distribution is 960.

(d) If A_4 is receiving not more than 14 marbles, then number of ways of distribution is 1085.

474. Let a, b and $b-2$ are the first three terms (in order) of a G.P. where $a, b \in N$. Identify which of the following statement(s) is(are) correct?

(a) If $a \in [1, 8]$, then $r = \frac{1}{2}$

(b) If $a \in [1, 8]$, then $S_\infty = 16$

(c) If $a \in (8, 11]$, then S_∞ can be equal to 27

(d) If $a \in (1, 8]$, then $S_\infty = \frac{27}{2}$

475. If $\tan\alpha$ and $\tan\beta$ are the roots of the equation $x^2 - 3x - 2 = 0$ where $\alpha, \beta \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ and $\beta > \alpha$, then:

(a) $\beta - \alpha \in \left(0, \frac{\pi}{2}\right)$

(b) $\beta - \alpha \in \left(\frac{\pi}{2}, \pi\right)$

(c) $\tan 2\alpha = \frac{1+\sqrt{17}}{1-\sqrt{17}}$

(d) $\tan 2\alpha = \frac{1-\sqrt{17}}{1+\sqrt{17}}$

476. Let $f(x) = \left[\frac{4^x + 2^x + 1}{2^x - 2^{x/2} + 1}\right]$ and $g(x) = \left[\frac{9}{x^2 + 5}\right]$. Identify which of the following statement(s) is(are) **correct**?

[**Note:** where $[y]$ denotes greatest integer function less than or equal to y.]

(a) Number of points of discontinuities of $f(x)$ in $(-\infty, 0]$ is 2.

(b) Number of points of discontinuities of $g(x)$ in $(-\infty, \infty)$ is 2.

(c) Number of points of discontinuities of $f(x) \cdot g(x)$ in $(-\infty, \infty)$ is 7.

(d) Number of points of discontinuities of $f(x) \cdot g(x)$ in $(-\infty, \infty)$ is 6.

477. Let $f : R \rightarrow (0, \infty)$ be a real valued function satisfying $\int_0^x t f(x-t)\, dt = e^{2x} - 2x - 1$, then which of the following is(are) **correct**?

(a) The value of $(f^{-1})'(4)$ equals $\frac{1}{8}$

(b) Derivative of $f(x)$ with respect to e^x at $x = 0$ is equal to 8

(c) The value of $\lim_{x \to 0} \frac{f(x) - 4}{x}$ equals 4

(d) The value of $f(0)$ is equal to 4

478. Let $f(x) = \begin{cases} 1 + \ln(c^2 + c + 1) \tan^2(x-1)^{\frac{1}{(\ln x)^2}}, & x \neq 1 \\ 3c, & x = 1 \end{cases}$, where $c \in R$.

If $\lim_{x \to 1} f(x)$ exists but $f(x)$ is discontinuous at $x = 1$, then c can take the value:

(a) 1 (b) 2 (c) 3 (d) 4

479. Let $f(x) = x^2 - px + q$, $p, q \in R$. If x_1, x_2, x_3, x_4, x_5 (where $x_i \in I$) are the 5 points where $g(x) = |f(|x|)|$ is non-derivable and $\sum_{i=1}^{5} |x_i| = 10$, then $p + q$ can be:

(a) 7 (b) 9 (c) 11 (d) 13

480. Which of the following definite integral vanishes?

(a) $\int_{-\pi}^{\pi} (\cos 2x \cdot \cos 2^2 x \cdot \cos 2^3 x \cdot \cos 2^4 x \cdot \cos 2^5 x)\, dx$

(b) $\int_{-1}^{1} \ln(x + \sqrt{x^2 + 1})\, dx$

(c) $\int_0^1 \tan^{-1}\left(\frac{2x - 1}{1 + x + x^2}\right) dx$

(d) $\int_0^{\pi/2} \ln(\tan x)\, dx$

481. If $a^2 + b^2 + c^2 + ab + bc + ca \leq 0$, where $a, b, c \in R$ and $f(x) = a[x] + b|x| + c\,\text{sgn}(x)$, then in $(-2, 2)$, which of the following is not true?

[**Note:** $[y]$ denotes greatest integer function less than or equal to y.]

(a) $f(x)$ is discontinuous at exactly two points

(b) $f(x)$ is discontinuous at exactly three points

(c) $f(x)$ is continuous and derivable for every x

(d) $f(x)$ is non-derivable at exactly one point

482. Let ABC be a triangle such that $AB = AC$. If equation of the side $AB : 7x + y = 0$, $AC : x + y = 0$ and line BC is passing through (2, 3), then which of the following may be **correct**?

(a) $BC : 2x + y = 7$

(b) $BC : x - 2y + 4 = 0$

(c) Area of ΔABC is $\frac{147}{5}$

(d) Area of ΔABC is $\frac{16}{5}$

483. If $\int \frac{3x\sin^2 x\cos x - 3\sin^3 x}{x^4} dx = f(x) + C$, where $\lim\limits_{x \to 0} f(x) = 1$ and C is the constant of integration, then:

(a) the value of $\lim\limits_{x \to 0} \frac{\int_0^x t f(t)\, dt - 2x^2}{1 - \cos x} = -3$

(b) the value of $\lim\limits_{x \to 0} \frac{(f(x))^{\frac{1}{3}} - x^2}{x^2} = \frac{-1}{6}$

(c) if $h(x) = x \cdot \sqrt[3]{f(x)}$, then $\int_0^{\blacksquare} h^4(x)\, dx = \frac{3\pi}{8}$

(d) if $h(x) = x \cdot \sqrt[3]{f(x)}$, then $\int_0^{\frac{\pi}{2}} e^{h(x)} (\cos^2 x - \sin x)\, dx = -1$

484. Let $f : (0, \infty) \to [-2, \infty)$, $f(x) = ax^2 \quad bx + c$ (where $a, b, c \in R$) be a surjective function such that $\lim\limits_{x \to 0} f(x) = 3$. If $g : [1, \infty) \to [-2, \infty)$, $g(x) = f(x)$ is an invertible function, then identify which of the statement(s) is(are) **correct**?

(a) The value of $40 \cdot g'(1)$ is equal to 0.

(b) If domain of $g(g(x))$ is $\left[1 + \sqrt{\frac{p}{q}}, \infty\right)$, then $(q - p)$ equal to 2.

(c) The number of solution(s) of the equation $g(x) = g^{-1}(x)$ is 2.

(d) The value of $\frac{d}{dx}(90g^{-1}(x))$ at $x = 43$ is 3.

485. Consider $f(x) = \{x^2 - 1\}[|x|]$, then:

[**Note:** Where $\{y\}$ denotes fractional part function and $[y]$ denotes greatest integer function less than or equal to y.]

(a) number of points where f is discontinuous in $[-2, 2]$ is 4

(b) number of points where f is discontinuous in $[-2, 2]$ is 6

(c) number of solution(s) of the equation $2f(x) = |x|$ is 5

(d) number of solution(s) of the equation $2f(x) = |x|$ is more than 5

486. Let $f(x)=\lim_{n\to\infty}(-n)\left(\left|2\tan^{-1}x-\frac{1}{n}\right|\tan^{-1}x\right), x\in R$. Identify the correct statement(s).

(a) The number of points where $f(x)$ is discontinuous is 1

(b) The number of points where $g(x)=|f(x)|$ is discontinuous is 1

(c) $f(1)+f(2)=2$

(d) The least positive integral value of λ for which the equation $f(x)=\left|x+\frac{5}{\lambda}\right|$ has a solution is 6

487. Let $S=\{14, 15, 16, \ldots\ldots, 22\}$. If N is the number of subsets of S containing two or more elements such that sum of the least and greatest elements has odd number of divisors, then:

(a) the number of prime factors of N is 3

(b) the number of divisors of N is 8

(c) sum of the digits of N is 8

(d) the number of divisors of N is 16

488. Consider, $f(f(x-2))=(x^2+3)^2+1\ \forall\, x\in R$.

Identify which of the following statement(s) is(are) **correct**?

(a) Least value of $f(x)$ is 1

(b) Least value of $f(x)$ is 10

(c) $\left.\frac{d(f(f(x)))}{dx}\right|_{x=0}=56$

(d) If $\int\frac{dx}{f(x)}=g(x)+C$ where C is a constant and $g(-2)=0$, then $g(0)+g(1)=\frac{3\pi}{4}$

489. Let $P(x)$ be a polynomial satisfying $\lim_{x\to\infty}\frac{x^2P(x)}{x^5+5x+6}=2$ and $P(1)=2, P(2)=16$, $P(3)=54$, then:

(a) $P(4)=64$

(b) $P(4)=128$

(c) area bounded by $y=f(x)$, $x=0$, $x=2$ and x-axis is 4 sq. units.

(d) area bounded by $y=f(x)$, $x=0$, $x=2$ and x-axis is 8 sq. units.

490. Let a, b, c be three distinct non-zero real numbers satisfying equation $\frac{1}{a}+\frac{1}{a-1}+\frac{1}{a-2}=1$, $\frac{1}{b}+\frac{1}{b-1}+\frac{1}{b-2}=1$ and $\frac{1}{c}+\frac{1}{c-1}+\frac{1}{c-2}=1$, then:

(a) $(1-a)(1-b)(1-c)=1$

(b) $abc=2$

(c) $abc=1$

(d) $(1-a)(1-b)(1-c)=2$

491. If $A=[a_{ij}]_{n\times n}$ and $a_{ij}=(i^2+j^2-ij)(j-i)$, where n is odd, then the value of $tr.(A)$ is equal to:

(a) 0 (b) $|A|$ (c) $|adj.A|$ (d) $2|A|$

492. If $f:A\to B, f(x)=\sin^{-1}\left(\frac{[x]}{\{x\}}\right)$ and $g:C\to D, g(x)=\cos^{-1}\left(\frac{[x]}{\{x\}}\right)$, then which of the following is always **correct**?

[**Note:** $[\cdot]$ and $\{\cdot\}$ denotes greatest integer and fractional part function respectively.]

(a) $A=C$

(b) $f(x)$ and $g(x)$ both are injective

(c) B and D both are singleton sets

(d) Number of integral solution of the equation $f(x)+g(x)=\frac{\pi}{2}$ is zero.

493. If $I_n=\int_{-1}^{1}|x|\left(1+x+\frac{x^2}{2}+\frac{x^3}{3}+......+\frac{x^{2n}}{2n}\right)dx$, then:

(a) $I_2=\frac{4}{3}$ (b) $I_2=\frac{7}{6}$ (c) $\lim_{n\to\infty} I_n=\frac{3}{2}$ (d) $\lim_{n\to\infty} I_n=\frac{5}{4}$

494. Given that $g(x)=2f\left(\frac{x^2}{2}\right)+f(6-x^2)$, $\forall x\in R$ and $f''(x)>0\ \forall x\in R$, then:

(a) $g(x)$ increases for $x\in(-\infty,-2)\cup(0,2)$

(b) $g(x)$ increases for $x\in(-2,0)\cup(2,\infty)$

(c) $g(x)$ decreases for $x\in(-\infty,-2)\cup(0,2)$

(d) $g(x)$ decreases for $x\in(-2,0)\cup(2,\infty)$

495. Let $f:R\to(0,1)$ be a continuous function, then which of the following pair of vectors are linearly dependent for some $x\in(0,1)$?

(a) $\vec{a}=f(x)\hat{i}+2\hat{j};\ \vec{b}=x^2\hat{i}+3\hat{j}$ (b) $\vec{a}=f(x)\hat{i}+3\hat{j};\ \vec{b}=x^2\hat{i}+2\hat{j}$

(c) $\vec{a}=\left(\int_0^{1-x}f(t)\,dt\right)\hat{i}+3\hat{j};\ \vec{b}=x\hat{i}+2\hat{j}$ (d) $\vec{a}=\left(\int_0^{1-x}f(t)\,dt\right)\hat{i}+2\hat{j};\ \vec{b}=x\hat{i}+3\hat{j}$

496. Matrices of order 2×2 are formed by using the elements of the set $A=\{-2,-1,0,1,2\}$, then probability that matrix is either symmetric or skew-symmetric, is greater than:

(a) $\frac{1}{10}$ (b) $\frac{2}{10}$ (c) $\frac{3}{10}$ (d) $\frac{4}{10}$

497. If $L = \dfrac{\int_0^{n\pi} e^{-x}(\sin^4 ax + \cos^2 ax)\,dx}{\int_0^{\pi} e^{-x}(\sin^4 ax + \cos^2 ax)\,dx}$, where $a \in R$ then:

(a) If $a = 1$, then $\lim\limits_{n\to\infty} L < 1$

(b) If $a = 2$, then $\lim\limits_{n\to\infty} L > 1$

(c) If $a = 3$, then $\lim\limits_{n\to\infty} L < 1$

(d) If $a = 4$, then $\lim\limits_{n\to\infty} L > 1$

498. Let L be a straight line passing through origin. Suppose that all the points on L are at constant distance from the two planes $P_1: x + 3y - z + 1 = 0$ and $P_2: 3x - y + z - 1 = 0$ then which of the following points lie(s) on the line L:

(a) $(1, -2, -5)$ (b) $(1, -2, 5)$ (c) $(-1, -2, 5)$ (d) $(-1, 2, 5)$

499. Let $f(x) = \dfrac{2 + \ln x}{x^2}$, $x > 0$. Identify which of the following is(are) **correct** about $f(x)$?

(a) $f'(x) = 0$ for some $x \in \left(0, e^{\frac{-7}{6}}\right)$

(b) $\lim\limits_{x\to 0} f'(x) = \infty$

(c) $\lim\limits_{x\to 0} f(x) = 0$

(d) Rolle's Theorem is applicable for $f'(x)$ in some interval of $(0, \infty)$

500. Let $E - ABCD$ be a pyramid on square base $ABCD$ where A is the origin and B and D are lying on positive x-axis and y-axis respectively. If E is $(0, 2, 3)$ and $\vec{DE} \cdot (\hat{i} + \hat{j}) = \vec{0}$, then:

(a) image of the point D in the plane ABE is $\left(0, \dfrac{-10}{13}, \dfrac{24}{13}\right)$

(b) image of the point D in the plane ABE is $\left(0, \dfrac{-6}{13}, \dfrac{30}{13}\right)$

(c) volume of the tetrahedron $ABDE$ is 2 cubic units

(d) perpendicular distance of the point D from the plane ABE is $\dfrac{9}{\sqrt{13}}$

501. Let α, β and γ be the roots of the equation $x^3 - 4x + 1 = 0$. If $T_n = \alpha^n + \beta^n + \gamma^n$, $n \geq 1$ then which of the following is(are) **true**?

(a) $T_6 - 4T_4 = 3$

(b) $T_{96} + 2T_{99} + T_{102} = 16T_{98}$

(c) $T_{96} + 2T_{99} + T_{102} = 16T_{100}$

(d) $[\alpha] + [\beta] + [\gamma] = -2$

[**Note:** $[k]$ denotes greatest integer function less than or equal to k.]

502. If $y=f(x)$ satisfies the differential equation $(x^2 y''+y)dx=xdy+(x^2\sin x+x^3\cos x)\,dx$ such that $f'(\pi)=1$ and $f(\pi)=\pi$, then : $\left(\text{where } y''=\dfrac{d^2y}{dx^2}\right)$

(a) $\lim\limits_{x\to 0}\dfrac{f'(x)+1}{x^2}=\dfrac{1}{2}$

(b) $\lim\limits_{x\to 0}\dfrac{f'(x)+1}{x^2}=\dfrac{3}{2}$

(c) maximum value of $\dfrac{f(x)}{x}+\sin^2 x$ is $\dfrac{5}{4}$

(d) minimum value of $\dfrac{f(x)}{x}+\sin^2 x$ is -1

503. Identify which of the following statement(s) is(are) **correct**?

(a) If $0\le \arg z\le\dfrac{\pi}{3}$, then minimum value of $|2\sqrt{3}z-6i|$ is equal to 6

(b) If $\arg\left(\dfrac{z-2}{z+2i}\right)=\dfrac{\pi}{2}$, then minimum value of $|z+3-3i|$ is equal to $3\sqrt{2}$

(c) If z_1 and z_2 are lying on $|z|=5$ such that $|z_1-z_2|=2$, then the value of $|z_1+z_2|$ is equal to $4\sqrt{6}$

(d) If $\dfrac{\pi}{4}\le\arg(z-1)\le\dfrac{3\pi}{4}$ and $\operatorname{Im}(z)\le 2$, then area of the region in which z lies is 4 sq. units

504. Consider a conic $\dfrac{1}{x+y-2}+\dfrac{1}{x-y+2}+\dfrac{1}{y-x+2}=0$ in two dimensional co-ordinate plane. Identify which of the following statement(s) is(are) **correct**?

(a) Length of latus rectum of the conic is $4\sqrt{2}$

(b) Length of latus rectum of the conic is $2\sqrt{2}$

(c) Focus of the conic is $\left(\dfrac{3}{2},\dfrac{3}{2}\right)$

(d) Vertex of the conic is $(0, 0)$

505. In ΔABC, if $AB=AC$ and internal bisector of angle B meet AC at D such that $BD+AD=BC=4$, then identify the **correct** statement(s).

(a) $R=2\sec 10^\circ$

(b) $R=4\sec 10^\circ$

(c) $\Delta=16\sin 10^\circ\sin 40^\circ\sin 70^\circ$

(d) $\angle A+\angle B=\dfrac{7\pi}{9}$

506. The equations of the sides of a triangle having $(4, -1)$ as a vertex, if the lines $x - 1 = 0$ and $x - y - 1 = 0$ are the equations of two internal bisectors of its angles, are:

(a) $2x - y + 3 = 0$ (b) $x + 2y - 6 = 0$ (c) $2x + y - 7 = 0$ (d) $x - 2y - 6 = 0$

507. If $a^2 + 8b^2 + 2c^2 + 2d^2 - 4ab - 4bc - 4bd = 0$ (where $a, b, c, d \in R$), then the value of $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ is:

(a) $\frac{a^2}{4}$ (b) b^2 (c) c^2 (d) d^2

508. An ellipse with eccentricity $\frac{1}{2}$ passes through $P(3,4)$ whose nearer focus is $S(0,0)$ and equation of tangent at P on ellipse is $3x + 4y - 25 = 0$. If a chord through S parallel to tangent at P intersects the ellipse at A and B then:

(a) length of AB is 15
(b) length of latus rectum of ellipse is 15
(c) focal length of ellipse is 10
(d) centre of ellipse is $(-3, -4)$

509. A contest consisting of ranking 10 songs of which 6 are Indian classic and 4 are western songs. Number of ways of ranking so that:

(a) there are exactly 3 indian classic songs in top 5 is $(5!)^3$
(b) top rank goes to indian classic song is 6.9!
(c) the ranks of all western songs are consecutive is 4! 7!
(d) the 6 indian classic songs are in a specified order is ${}^{10}P_4$

510. Let $f(x)$ be a derivable function and $f(\alpha) = f(\beta) = 0 (a < \beta)$, then in the interval (α, β):

(a) $f(x) + f'(x) = 0$ has at least one real root
(b) $f(x) - f'(x) = 0$ has at least one real root
(c) $f(x) f'(x) = 0$ has at least one real root
(d) $(f'(x))^2 + f(x) f''(x) = 0$ has at least two real roots

511. *D-ABC* is a tetrahedron with $A = (2, 0, 0)$, $B - (0, 4, 0)$ and $CD = \sqrt{14}$. Edge CD lies on the line $\frac{x-1}{1} = \frac{y-1}{2} = \frac{z-3}{3}$. If locus of centroid of tetrahedron is $\frac{x - \frac{3}{2}}{1} = \frac{y - y_1}{a} = \frac{z - z_1}{b}$, then which of the following is/are true:

(a) $a + b = 5$ (b) $y_1 + z_1 = 6$ (c) $y_1 - z_1 = 1$ (d) $a + b + y_1 = 8$

512. Let $f(x) = x^3 - x^2 + x + 1$ and $g(x) = \begin{cases} \max. f(t); 0 \le t \le x, & \text{for } 0 \le x \le 1 \\ 3 - x, & \text{for } 1 < x \le 2 \end{cases}$, then $g(x)$ is:

(a) continuous for $x \in [0, 2] - \{1\}$
(b) continuous for $x \in [0, 2]$
(c) derivable for all $x \in [0, 2]$
(d) derivable for all $x \in [0, 2] - \{1\}$

513. Let $f(x)=\sin^{-1}\left(\frac{2x}{1+x^2}\right)$, $g(x)=\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ and $h(x)=\tan^{-1}\left(\frac{2x}{1-x^2}\right)$, then:

(a) $f(x)+2\tan^{-1}x=\pi\ \forall\, x\geq 1$
(b) $\frac{f(x)}{g(x)}=1\ \forall\, x\in[0,1]$
(c) $g(x)+h(x)=\forall\, x\in(-1,0)$
(d) $\frac{\lim\limits_{x\to 1^+}(f(x)+g(x)+h(x))}{\lim\limits_{x\to 1^-}(f(x)+g(x)+h(x))}=3$

514. Let : $f:[0,\infty]\to A; f(x)=\sqrt{\tan^{-1}x}+\sqrt{\pi-\tan^{-1}x}$ is an onto function, then:

(a) $f(x)$ is injective
(b) $f(x)$ is many-one
(c) set A is $[\sqrt{\pi},\sqrt{2\pi})$
(d) set A is $[\sqrt{\pi},2\sqrt{\pi})$

515. Let $S_n=\tan^{-1}\left(\sin 1\cdot\sum_{r=1}^{n}\sec(r-1)\sec r\right)$, then:

(a) $S_5=5-\pi$ (b) $S_5=5-2\pi$ (c) $S_5=10-3\pi$ (d) $S_{10}=3\pi-10$

516. Let $f(x)=x^2-2x-3$, then $\lambda=|f(|x|)|$ has:

(a) exactly one solution, if $\lambda<0$
(b) exactly two solutions, if $\lambda=\{0\}\cup(4,\infty)$
(c) exactly three solutions, if $\lambda=3$
(d) exactly four solutions, if $\lambda=\{4\}\cup(0,3)$

517. Let $S_n=\frac{1}{1^3}+\frac{1+2}{1^3+2^3}+\frac{1+2+3}{1^3+2^3+3^3}+\ldots\ldots$ (n terms), where $n=1,2,3,4,\ldots\ldots$, then S_n is always less than:

(a) 1 (b) 2 (c) 3 (d) 4

518. Let $f(x)=ax^2+2bx-3c$ has no real root and $\frac{3c}{4}<a+b$, then:

(a) $a>0$ (b) $c<0$ (c) $a+|b|>\frac{3c}{4}$ (d) $b<0$

519. Let $f(x)=\sin^2 x-\sin x+k$, $x\in R$. Then:

(a) $f(x)\geq 0$ if $k\geq\frac{1}{4}$
(b) $f(x)\geq 0$ if $k\leq\frac{1}{4}$
(c) $f(x)\leq 0$ if $k\geq -2$
(d) $f(x)\leq 0$ if $k\geq -2$

520. Let $f(x)=\cot^{-1}\left(\frac{x^{2018}+5}{(x-5)(x-10)}\right)$, then:

(a) $\lim\limits_{x\to 5^-}f(x)=0$
(b) $\lim\limits_{x\to 5^+}f(x)=\pi$
(c) $\lim\limits_{x\to 10^-}f(x)=\pi$
(d) $\lim\limits_{x\to 10^+}f(x)=0$

521. If $f(x) = x^3 + 3x^2 + (4-k)x + b$ is an injective function $\forall\, x \in R$, then:

(a) maximum positive integral value of k is 1.

(b) minimum positive integral value of k is 1.

(c) number of positive integral value of k is 1.

(d) number of non-negative integral value of k is 1.

522. Consider the function $f(x) = \begin{cases} \dfrac{\max.\left\{x, \left[\dfrac{1}{x}\right]\right\}}{\max.\left\{x, \left[\dfrac{1}{x}\right]\right\}}, & \text{when } x \neq 0 \\ 1, & \text{when } x = 0 \end{cases}$, then:

(a) $\lim\limits_{x\to 0^+} f(x) \neq 0$

(b) $\lim\limits_{x\to 0^-} f(x) = 0$

(c) $f(x)$ is continuous for all $x \neq 0$

(d) $f(x)$ is derivable for all $x \neq 0$

523. Let $h(x) = \int\left(\int\left(\int g'''(x)\,dx\right)dx\right)dx$ with $h(3) = g(3)$, $h(1) = g(1)$ and $h(0) - g(0) = 6$.

If $f(x) = h(x) - g(x)$, then:

(a) $f(x)$ decreases in the interval $(1, 3)$

(b) $f(x)$ decreases in the interval $(-\infty, 2)$

(c) $f(4) = 6$

(d) $f(2) = 6$

524. Let $P(x)$ be a polynomial function on R such that $P(x) + P(2x) = 5x^2 - 18\ \forall\, x \in R$.

(a) number of solutions of $P(x) = e^x$ is 1

(b) number of solutions of $P(x) = e^x$ is 2

(c) $\displaystyle\int_0^\infty \frac{dx}{P(x)+25} = \frac{\pi}{4}$

(d) $\displaystyle\int_0^\infty \frac{dx}{P(x)+25} = \frac{\pi}{8}$

525. If $f(x) = 2 + \displaystyle\int_{-1}^{1}\left(\frac{tx^2}{2} + \frac{9x}{14}\right)f(t)\,dt$, then:

(a) Rolle's Theorem is applicable for $y = f(x)$ in $[-2, -1]$

(b) $\lim\limits_{x\to 0} f(x) = 0$

(c) f is continuous and derivable on R

(d) maximum value of $f(x)$ does not exist

526. Let f be real-valued function such that $e^{-2x} f(x) = x + 3 + \displaystyle\int_0^x \frac{dt}{\sqrt{t^6+1}}$ for all $x \in (-1, 1)$ and let $y = g(x)$ be a function whose graph is reflection of the graph of $y = f(x)$ w.r.t. line $y = x$, then $g'(3)$ is not equal to:

(a) 1

(b) $\dfrac{1}{2}$

(c) $\dfrac{1}{4}$

(d) $\dfrac{1}{8}$

527. Let $f(x)=\begin{cases}-x^3+a, & 0\le x<1\\ x, & 1\le x\le 3\end{cases}$:

(a) if $f(x)$ has a absolute minimum at $x=1$, then minimum positive integral value of a is 2.

(b) if $f(x)$ has a absolute minimum at $x-1$, then minimum positive integral value of a is 3.

(c) if $f(x)$ has a absolute maximum at $x-3$, then maximum positive integral value of a is 3.

(d) if $f(x)$ has a absolute maximum at $x=0$, then minimum positive integral value of a is 3.

528. Let $y=P(x)$ be a differentiable function $\forall\, x\in[0,\infty)$ such that $\frac{d}{dx}(P(x))+(x\ \ 1)^3\ge P(x)+1\,\forall x\in[0,\infty)$. If $P(x)\le x^3+3x+1\,\forall\, x\in[0,\infty)$ and $P(0)=1$, then which of the following is/are **correct**?

(a) $y=P(x)$ is a monotonic function

(b) Area bounded by $y=P(x)$; x-axis; $x=0$ and $x=1$ is $\frac{11}{4}$

(c) $\int_{-1}^{1}P(x)\,dx=2$

(d) $y=P(x)$ is a bijective function

529. Let $f:R\to[-3,3]$ be a twice differentiable function such that $f'(0)=f(1)=f(3)=2$, then which of the following must be **correct**?

(a) $y=f(x)$ is monotonic for some set of values of x

(b) There must be at least one $c\in[-3,0)$ such that $f'(x)\le 2$

(c) $f''(x)\ge\frac{-1}{3}$ for some $c\in(-3,3)$

(d) For some values of $c\in(-3,3)$, $f''(c)\ge-2$

530. Let $f:(0,\pi)\to R$ be a differentiable function defined as $f(x)=\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{2^{2r}}\sec^2\frac{x}{2^r}$.

Then which of the following must be **correct**?

(a) $f\left(\frac{\pi}{2}\right)=1-\frac{4}{\pi^2}$

(b) $f'\left(\frac{\pi}{2}\right)=\frac{16}{\pi^3}$

(c) $\lim_{x\to 0^+}f(x)=\frac{1}{3}$

(d) $f(x)=0$ has at least one real root

531. For $n \geq 1$, Let G_n be the geometric mean of $\left\{\sin\frac{k\pi}{2n} : 1 \leq k \leq n\right\}$ then $\lim_{n\to\infty} G_n$ equals:

(a) $\lim_{x\to 0}\frac{1-\cos x}{x}$

(b) $\lim_{x\to 0}\frac{1-\cos x}{x^2}$

(c) $\frac{2}{\pi}\int_0^{\pi/2}\sin^2 x\,dx$

(d) $\lim_{x\to 0^-}\left[\frac{e^x-1}{x}\right]$

[**Note:** $[k]$ denotes greatest integer function less than or equal to k.]

532. Let $y = f(x)$ be function defined as $x = y^3 + y^2 + y + 1$, then which of the following is/are **correct**?

(a) $2f'(0) = 1$ (b) $f''(0) = \frac{1}{2}$ (c) $\int_0^4 f(x)\,dx = \frac{4}{3}$ (d) $\int_0^4 f(x)\,dx = 0$

533. Let $P = \begin{bmatrix}1 & 1 & 0\\0 & 1 & 0\\0 & 1 & 1\end{bmatrix}$ and Q be an orthogonal matrix of order 3×3. Let $A = P^{2018}$ and $B = QPQ^T$, then which of the following is/are **correct**?

(a) Trace of matrix A is 3

(b) $Q^T B^{2018} Q = A$

(c) $\det(B^5) = 1$

(d) $\det(adj\,(A)) = \det(adj\,(B))$

534. Let $y = f(x)$ be a quadratic polynomial such that $[f(2)\ \ f(1)\ \ f(0)]\begin{bmatrix}x\\y\\1\end{bmatrix} = [2x + y + 2]\ \forall x, y \in R$, then which of the following is/are **correct**?

(a) Range of $f(x)$ is $[1, \infty)$

(b) Range of $f(x)$ is $[2, \infty)$

(c) Area bounded by $y = f(x)$ and $y = 2 - x$ is $\frac{1}{2}$

(d) Area bounded by $y = f(x)$ and $y = 2 - x$ is $\frac{1}{6}$

535. Let ΔABC be an isosceles triangle with $AB = AC$. If $AB: 4x + y = 7$, $AC: x + 4y = 7$ and BC is passing through $(1, 1)$, then possible equation of BC is:

(a) $3x + 2y = 5$ (b) $x + y = 2$ (c) $2x + 3y = 5$ (d) $x - y = 0$

536. Let A be a square matrix of order 3 such that $adj(adj(adj(A))) = \begin{bmatrix}16 & 0 & 4\\5 & 4 & 0\\1 & 4 & 3\end{bmatrix}$ and $\det(A)$ is positive, then which of the following must be **correct**?

(a) $8\cdot\text{trace}\,(A^{-1}) = 23$

(b) $8\cdot\text{trace}\,(A^{-1}) = 35$

(c) $\det(adj\,A) = 4$

(d) $\det(adj\,A) = 2$

537. Let two parabolas be $S_1 : y^2 = 4ax$ and $S_2 : y^2 = -4ax$. From any point P on S_1, tangents are drawn to S_2 touching it at Q and R, then:
(a) line QR is tangent to S_1
(b) line QR neither touches nor intersect S_1
(c) if normal at any point $A(t)$ on S_1 is tangent to S_2 then $t^2 = \sqrt{2} - 1$
(d) if normal at any point $A(t)$ on S_1 is tangent to S_2 then $t^2 = \sqrt{2} + 1$

538. Consider a 3×3 non-singular matrix $A = [a_{ij}]$. A matrix $B = [b_{ij}]_{3\times 3}$ is formed such that b_{ij} is the sum of all the elements of i^{th} row in A except a_{ij}. If there exists a matrix C such that $AC = B$, then:
(a) C is symmetric matrix
(b) C is a diagonal matrix
(c) $|B| = \frac{|A|}{2}$
(d) $|B| = 2|A|$

539. If points of intersection of three non-concurrent lines $x + 2y = 3$, $ax - y = 1$ and $x + 3y = 5$ lies on a circle and one of the line is diameter of that circle, then:
(a) sum of possible values of a is 5
(b) there will be unique value of a
(c) $\left(\frac{-1}{7}, \frac{11}{7}\right)$ may be centre of the circle
(d) $\left(\frac{1}{14}, \frac{23}{14}\right)$ may be centre of the circle

540. Consider a differentiable function $f : R \to R$ for which $f'(0) = \ln 2$ and $f(x+y) = 2^x f(y) + 4^y f(x)$, $\forall\, x, y \in R$. Which of the following is(are) **correct**?
(a) $f(4) = 240$
(b) $f'(2) = 24 \ln 2$
(c) The minimum value of $y = f(x)$ is $\frac{-1}{4}$
(d) The number of solution of $f(x) = 2$ is 1

541. Let f be a real valued function defined on R (the set of all real numbers) as $f(x) = \pi\left\{\frac{x}{\pi}\right\}$, then which of the following is(are) **correct**?
[Note: where $\{\cdot\}$ denotes fractional part function.]
(a) Range of $f(x)$ is $[0, \pi)$
(b) $\lim_{x \to \frac{\pi}{2}} f(x) = \frac{\pi}{2}$
(c) $\int_0^{2\pi} f(x)\,dx = \pi^2$
(d) $f'\left(\frac{5\pi}{2}\right) = 1$

542. Let $f(x)$ and $g(x)$ be two derivable function on R (the set of all real numbers) satisfying

$$f(x)=\frac{x^3}{2}+1-x\int_0^x g(t)\,dt \text{ and } g(x)=x-\int_0^1 f(t)\,dt, \text{ then:}$$

(a) $\int_0^1 f(t)\,dt=\frac{3}{2}$

(b) $\int_0^1 f(t)\,dt=\frac{5}{4}$

(c) number of points of non-derivability of $f(|x|)$ is zero

(d) number of points of non-derivability of $f(|x|)$ is one

543. If the coordinates of the point where the line $x-2y+z-1=0=x+2y-2z-5$ intersects the plane $x+y-2z=7$ is (α, β, γ), then:

(a) $\alpha+\beta+\gamma=7$

(b) $\alpha-\beta+\gamma+1=0$

(c) $\alpha^2+\beta^2+\gamma^2=21$

(d) $\alpha\beta+\beta\gamma+\gamma\alpha=2$

544. If the complex number z satisfies the condition $\left|z-\frac{25}{z}\right|=24$, then which of the following is(are) **correct**?

(a) Maximum distance of z from origin is 5

(b) Maximum distance of z from origin is 25

(c) Minimum distance of z from origin is 1

(d) Minimum distance of z from origin is 4

545. Let $A=[a_{ij}]$ be a matrix of order 3 where $a_{ij}=\begin{cases}0, & i=j\\ (i+2j\ \ 3)x, & i>j\\ i, & i<j\end{cases}$.

If $f(x)=\det.(A)$, then which of the following is(are) correct statement(s)?

(a) $\int_{-1}^{1} f(x)\,dx=\frac{8}{3}$

(b) $|f(|x|)|$ is non-differentiable at 2 points

(c) $f(|x|)=k$ has four distinct solution for $k\in\left(0,\frac{1}{4}\right)$

(d) $\int_0^1 \frac{dx}{f(x)+2x+1}=\frac{1}{3}$

546. In ΔABC, $AB = c$, $BC = a$ and $CA = b$ and b^2, a^2 and c^2 are in A.P. such that $a = 2$ and point A is variable point. $\angle CAB = \theta$, length of median drawn from A to BC is 'L'. Then which of following is/are must be **correct** ?

(a) $L = \sqrt{3}$
(b) locus of A is circle
(c) $\cos\theta$ must be positive
(d) $\cot A$, $\cot B$ and $\cot C$ in A.P.

547. Let $A = \begin{bmatrix} a & b & 1 \\ 2 & 1 & 3 \\ 1 & c & 2 \end{bmatrix}$ and $A^{-1} = (5A - A^2)$, then:

(a) $|A| = 3$
(b) $|A| = -3$
(c) $Tr(A) = 5$
(d) $Tr(A) = a + b + c$

548. Let $f : [0, 8] \to R$ be differentiable function such that $f(0) = 0, f(4) = 2, f(8) = 2$, then which of the following holds good?

(a) There exist some $C_1 \in (0, 8)$ where $f'(C_1) = \frac{1}{2}$

(b) There exist some $C_1 \in (0, 8)$ where $f'(C_1) - \frac{1}{10}$

(c) There exist some C_1 and $C_2 \in (0, 8)$ where $8f'(C_1) \cdot f(C_2) = 1$

(d) There exist some $C_1 \in (0, 1)$ and $C_2 \in (1, 2)$ such that

$$\int_0^{8} f(t)\, dt = 3(C_1^2 f(C_1^3) + C_2^2 f(C_2^3))$$

549. If the system of equation $2x \quad y + z - 0, x \quad 2y + z = 0$ and $ax - y + 2z = 0$ has infinitely many solutions and $f(x)$ be a continuous function such that $f(5 + x) + f(x) = 2 \ \forall\, x \in R$, then $\int_0^{-2a} f(x)\, dx$ is equal to:

(a) -10
(b) $-2a$
(c) $\int_0^a [x] dx$
(d) $2a$

[**Note:** $[k]$ denotes greatest integer less than or equal to k.]

550. Suppose $g'(x) < 0 x \ \forall\, x \geq 0$ and $\int_0^x t g'(t)\, dt \ \forall x \geq 0$. Which of the following statement(s) are **correct**?

(a) f is not increasing
(b) f is continuous $\forall\, x > 0$
(c) $f(x) = xg(x) - \int_0^x g(t)\, dt$
(d) $f'(x)$ exists $\forall\, x > 0$

551. Let $A=\begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$ and $B=\begin{bmatrix} bc-a^2 & ca-b^2 & ab-c^2 \\ ca-b^2 & ab-c^2 & bc-a^2 \\ ab-c^2 & bc-a^2 & ac-b^2 \end{bmatrix}$ be two non-singular matrices such that $(A^2-2I)B=O$ where $a>b>c>0$, then which of the following statement(s) is(are) **correct**?

(a) $Tr.(AB)=6\sqrt{2}$
(b) $Tr.(AB)=-6\sqrt{2}$
(c) $\det.(A-\sqrt{2}B)=54\sqrt{2}$
(d) $\det.(A-\sqrt{2}B)=-54\sqrt{2}$

[**Note:** I is an identity matrix of order 3 and $Tr.(P)$ and $\det.(P)$ denote trace and value of the determinant of square matrix P respectively.]

552. Let $f:R\to R$ and $g:(-2,2)\to R$ be two functions defined by $f(x)=\max.(|1-|x||, x^3+1)$ and $g(x)=[f(x)]$. Identify which of the following statement(s) is(are) **correct**?

[**Note:** $[y]$ denotes greatest integer function less than or equal to y.]

(a) Number of points where $f(x)$ is discontinuous is 0.
(b) Number of points where $f(x)$ is non-derivable is 2.
(c) Number of points where $g(x)$ is discontinuous is 9.
(d) Number of points where $g(x)$ is non-derivable is 8.

553. Let $f:[1,2]\to R$ be a differentiable function with $f'(x)$ as a non-decreasing function such that $f(1)=2$ and $f'(2)\le 1$, then identify the correct statement(s):

(a) $f(x)\le x+1\ \forall\ x\in[1,\ 2]$
(b) $f(x)\ge x+1\ \forall\ x\in[1,\ 2]$
(c) $f'(2)-f(2)\ge -2$
(d) $\int_1^2 e^{f(x)}\,dx\le\int_1^2 e^{x^2+1}\,dx$

554. If $f(x)\int\frac{\tan^3 x}{2+\tan^2 x}\,dx-\ln\left|\frac{2-g(x)}{\cos x}\right|+C$, where $f(0)=\ln 2$ and C is the constant of integration, then:

(a) $\lim_{x\to 0}\frac{g(x)}{\sqrt{x^2-x^2\cos x}}=2$
(b) $\int_0^{\frac{\pi}{2}} g(x)\,dx=\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{3}\right)$
(c) $\lim_{x\to 0^+}[x^2-g(x)]=0$
(d) $\int_0^{\frac{14\pi}{3}}\sqrt{g(x)}\,dx=\frac{19}{2}$

[**Note:** $[k]$ denotes greatest integer function less than or equal to k.]

555. If $P(t)=\lim_{n\to\infty}\sum_{r=2}^{n}\frac{\sqrt{t^{2r-3}}(1-t)}{(\sqrt{t^{2r-1}}+1)(\sqrt{t^{2r-3}}+1)}$, then $P\left(\frac{1}{2}\right)$ lies in the interval:

(a) $\left(0, \sin\frac{\pi}{6}\right)$ (b) $\left(0, \tan\frac{\pi}{4}\right)$ (c) $\left(\tan\frac{\pi}{4}, \tan\frac{\pi}{3}\right)$ (d) $\left(\cos\frac{\pi}{6}, \cos\frac{\pi}{3}\right)$

556. Let a, b, c (in order) be the first three terms of a sequence and satisfying $\log\left(\frac{8b^3-a^3-c^3}{6abc}\right)=0$ and $\log b=\log(a^2-4)=\log(c\quad 2)$. If T_n and S_n denote n^{th} term and sum of first n terms of the sequence, then:

(a) $T_{10}=31$
(b) $S_{10}=120$
(c) $T_{21}^2-2T_{21}T_1+2T_1^2=1609$
(d) $S_{11}-S_{10}=23$

557. Let $I_1=\int_0^1\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)dx; I_2=\int_0^1\cos^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)dx$ and

$I_3=\lim_{n\to\infty}\left(\frac{1}{n+1}+\frac{1}{n+2}+\ldots\ldots+\frac{1}{2n}\right)$, then:

(a) $I_1+I_3=I_2$
(b) $I_1+I_2+I_3=\ln 2$
(c) $I_2+I_3=I_1$
(d) $I_1+I_2+I_3=\frac{\pi}{2}+\ln 2$

558. If two tangents can be drawn to the different branches of the hyperbola $x^2-\frac{y^2}{4}=1$ from the point (α, α^2), then:

(a) $\alpha\in(-\infty,-3)$
(b) $\alpha\in(3,\infty)$
(c) $\alpha\in(-2,0)\cup(0,2)$
(d) $a\in(2,\infty)$

559. There are six students S_1, S_2, S_3, S_4, S_5 and S_6 and for them there are six seats R_1, R_2, R_3, R_4, R_5 and R_6 arranged in a row, where the initially seat R_i is allotted to the students S_i, $i=1, 2, 3, 4, 5, 6$. But on a day all are allotted seats randomly.

(a) The probability that S_4 gets allotted, seat R_4 and none of the remaining students gets the seat previously allotted to them, is $\frac{11}{180}$.

(b) The probability that S_5 gets allotted seat R_5 and none of the remaining gets the seat previously allotted to them is $\frac{1}{6}$.

(c) The probability that even numbered of students are seating at even numbered seat and none of the students is seating on the seat previously allotted is $\frac{1}{180}$.

(d) The probability that even numbered of students are seating at even numbered seat and none of the students is seating on the seat previously allotted is $\frac{1}{30}$.

560. If $f(x)$ be such that $f(x) = \max(|3-x|, 3-x^3)$ then:

(a) $f(x)$ is continuous $\forall x \in R$

(b) $f(x)$ is derivable $\forall x \in R$

(c) $f(x)$ is non-derivable at three points only

(d) $f(x)$ is non-derivable at four points only

561. Let a function $f: R \to R$ be defined as $f(x) = x + \sin x$ and $I = \int_0^{\pi} f^{-1}(x)\,dx$ then:

(a) $I > \int_0^1 \frac{1}{1+x^3}\,dx$

(b) $I < \int_0^1 e^{x^2}\,dx$

(c) $2 < I < 3$

(d) $\frac{\pi}{4} < I < \frac{\pi}{2}$

562. If graph of $xy=1$ is reflected in $y=2x$ to give the graph $12x^2 + rxy + sy^2 + t = 0$ then:

(a) $r=1, s=12, t=25$

(b) $r=-1, s=12, t=1$

(c) $r=-7, s=-12, t=25$

(d) $r+s=-19$

563. A parallelopiped is formed using three non-collinear vectors, $\vec{a}$, $\vec{b}$ and $\vec{c}$ with fixed magnitudes. Angles between any of the vector with normal of the plane determined by the other two is α and volume of the parallelopiped is T and its surface area is Y. If

$$\frac{Y}{T} = 4\left(\frac{1}{|\vec{a}|} + \frac{1}{|\vec{b}|} + \frac{1}{|\vec{c}|}\right) \text{then:}$$

(a) $\cos^2\alpha + \cos\alpha = \frac{3}{4}$

(b) $\sin^2\alpha + \sin^4\alpha = \frac{21}{16}$

(c) $\cos^2\alpha + \cos\alpha = \frac{3+2\sqrt{3}}{4}$

(d) $\sin^2\alpha + \sin^4\alpha = \frac{5}{16}$

564. Let a and b be two real numbers such that $a^2 - 3b^2 + 4a + 1 = 0$. If the line $ax + by + 1 = 0$ touches a fixed circle $\forall$ A and b, then which of the following is/are correct?

(a) Centre of the circle is (2, 0)

(b) Radius of the circle is $\sqrt{3}$

(c) Circle is passing through (2, 3)

(d) Radius of the circle is 3

565. Let $A_1, A_2; G_1, G_2$ and H_1, H_2 be two AM's, GM's and HM's respectively between two positive real numbers a and b, then:

(a) $A_1H_2 = ab$

(b) $A_1H_2 = a^2b^2$

(c) $G_1G_2 = ab$

(d) $A_2H_1 = ab$

566. For $0 < x < \pi/2$ if $\sin x$, $(\sin x + 1)$ and $6(\sin x + 1)$ are in G.P., then:

(a) common ratio is $3\sqrt{2}$

(b) common ratio is 1/2

(c) fifth term = 162

(d) $S_n = 1 - (1/2)^n$

567. If x is real and $\Delta(x)=\begin{vmatrix} x^2+x & 2x-1 & x+3 \\ 3x+1 & x^2+2 & x^3-3 \\ x-3 & x^2+4 & 2x \end{vmatrix}=a_0x^7+a_1x^6+a_2x^5+......+a_6x+a_7$,

then:

(a) $a_7=21$

(b) $\sum_{k=0}^{6} a_k=111$

(c) $\Delta(-1)=32$

(d) $\Delta(1)=121$

568. In ΔABC, if $\cos A+\cos B=4\sin^2\frac{C}{2}$, then which of the following are true?

(a) $a+b=2c$

(b) a, b, c are in H.P.

(c) $\tan\frac{A}{2}, \tan\frac{C}{2}, \tan\frac{B}{2}$ are in A.P.

(d) $\tan\frac{A}{2}, \tan\frac{C}{2}, \tan\frac{B}{2}$ are in H.P.

569. If $P(\alpha, \beta)$, the point of intersection of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{a^2(1-e^2)}=1$ and the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{a^2(E^2-1)}=\frac{1}{4}$, is equidistant from the foci of the two curves (all lying in the right of y-axis) then:

(a) $2\alpha=a(2e+E)$

(b) $a-e\alpha=E\alpha-\frac{a}{2}$

(c) $E=\frac{\sqrt{e^2+24}-3e}{2}$

(d) $E=\frac{\sqrt{e^2+12}-3e}{2}$

570. In an experimental performance of a single throw of a pair of unbiased normal dice, let three events E_1, E_2 and E_3 are defined as follows:

E_1 : getting prime numbered face on each dice.

E_2 : getting the same number on each dice.

E_3 : getting total on two dice equal to 4.

Which of the following is/are **true**?

(a) The probabilities $P(E_1), P(E_2), P(E_3)$ are in A.P.

(b) The events E_1 and E_2 are independent.

(c) $P(E_3/E_1)=\frac{2}{9}$

(d) $P(E_1+E_2)+P(E_2-E_3)=\frac{17}{36}$

571. If three planes $P_1 \equiv 2x + y + z - 1 = 0$, $P_2 \equiv x - y + z - 2 = 0$ and $P_3 \equiv \alpha x - y + 3z - 5 = 0$ intersects each other at point P on XOY plane and at point Q on YOZ plane, where O is the origin then identify the correct statement(s)?

(a) The value of α is 4.

(b) Straight line perpendicular to plane P_3 and passing through P is $\frac{x-1}{4} = \frac{y+1}{-1} = \frac{z}{3}$.

(c) The length of projection of $\overrightarrow{\mathbf{PQ}}$ on x-axis is 1.

(d) Centroid of the triangle OPQ is $\left(\frac{1}{3}, \frac{-1}{2}, \frac{1}{2}\right)$

572. Let a, b, c be distinct complex numbers with $|a| = |b| = |c| = 1$ and z_1, z_2 be the roots of the equation $az^2 + bz + c = 0$ with $|z_1| = 1$. Also P and Q are the points representing the complex numbers z_1 and z_2 respectively in the complex plane with $\angle POQ = \theta$ (where O being the origin) then which of the following is/are **correct**?

(a) $b^2 = ac$ (b) $\theta = \frac{2\pi}{3}$

(c) $PQ = \sqrt{3}$ (d) $|z_1 + z_2| = 1$

573. Which of the following is(are) correct?

(a) If A and B are two square matrices of order 3 and A is a non-singular matrix such that $AB = O$, then B must be a null matrix.

(b) If A, B, C are three square matrices of order 2 and det. $(A) = 2$, det. $(B) = 3$, det. $(C) = 4$, then the value of det. $(3ABC)$ is 216.

(c) If A is a square matrix of order 3 and det $(A) = \frac{1}{2}$, then det. $(adj.\, A^{-1})$ is 8.

(d) Every skew symmetric matrix is singular.

574. Let z satisfies $z\bar{z} + (-4+5i)\bar{z} + (-4-5i)z - 40 = 0$. If $a = \max.|z+2-3i|$ and $b = \min.|z+2-3i|$, then:

(a) $a + b = 20$ (b) $a^2 + b^2 = 362$

(c) $a - b = 18$ (d) $ab = 19$

575. An ellipse is orthogonal to the hyperbola $x^2 - y^2 = 2$. The eccentricity of the ellipse is reciprocal of that of the hyperbola. Then:

(a) equation of the ellipse is $x^2 + 2y^2 = 8$

(b) focus of the ellipse is at $(-4\sqrt{2}, 0)$

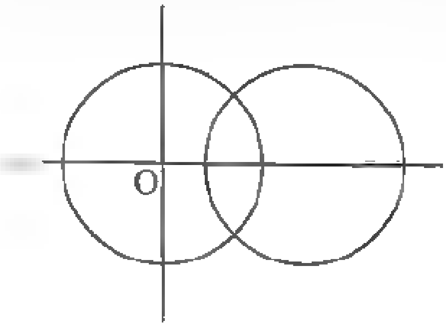
(c) equation of directrix of ellipse is $x + 4\sqrt{2} = 0$

(d) equation of director circle of ellipse is $x^2 + y^2 = 12$

Paragraph Type Questions

Paragraph for Question nos. 576 and 577

Let $C_1 : x^2 + y^2 - r^2$ and $C_2 : (x - p)^2 + (y - q)^2 = r^2$ be 2 circles with radius $r (r > 0)$ and have n points of intersection, (x_i, y_i) for $i \in \{1, 2, \ldots\ldots, n\}$. If C_1 and C_2 are orthogonal at all points of intersection, then:



576. As we move the centre of C_2 along $p + bq = 0$ for some constant $b \neq 0$, then $\frac{dr}{dq}$ is equal to:

(a) $\frac{b^2q + q}{r}$ (b) $\frac{b^2q + q}{2r}$

(c) $\frac{b^2q + q}{3r}$ (d) $\frac{b^2q + q}{4r}$

577. As we move the centre of C_2 along the curve $q - a$ for some constant $a \neq 0$, then $\frac{dr}{dp}$ is equal to:

(a) $\frac{p}{4r}$ (b) $\frac{p}{3r}$ (c) $\frac{p}{2r}$ (d) $\frac{p}{r}$

Paragraph for Question nos. 578 and 579

Let $f(x)$ be a function such that $f(x) = e^x (2x - 1) - ax + a$ where a is a parameter and $a < 1$. If there exist one and only one $x_0 \in I$ such that $f(x_0) < 0$. Then the range of a is $\left[\frac{p}{qe}, r\right)$ where p, q are co-prime.

578. The value of $(p + q + r)$ is:

(a) 4 (b) 5 (c) 6 (d) 7

579. The value of $\tan^{-1}(\tan p) + \cos^{-1}(\cos q)$ is:

(a) $4 - \pi$ (b) $5 - \pi$ (c) $6 - \pi$ (d) $7 - \pi$

Paragraph for Question nos. 580 to 582

Let $f(x)$ and $g(x)$ are two continuous function defined for $0 \le x \le 1$ $f(x) = \int_0^1 e^{x+t} f(t)\,dt$;

$g(x) = x + \int_0^1 e^{x+t} g(t)\,dt$:

580. The value of $f(1)$ is:

(a) 0 (b) 1 (c) $\frac{1}{e}$ (d) e

581. The value of $g(0)$ is:

(a) $\frac{2}{3-e^2}$ (b) $\frac{2}{e^2-2}$ (c) $\frac{2}{e^2-1}$ (d) 0

582. The value of $\frac{g(0)}{g(2)}$ is:

(a) 0 (b) $\frac{1}{3}$ (c) $\frac{1}{e^2}$ (d) $\frac{2}{e^2}$

Paragraph for Question nos. 583 to 584

Let $y = f(x)$ be a differentiable function passing through $(1, 0)$.
Let slope of the tangent at the point $(x, f(x))$ be m_1 and slope of line joining the point and origin be m_2. Also $\left|\frac{\log(m_1 + m_2)}{\log x}\right| = \frac{2}{1}$.

If $f_1(x)$ and $f_2(x)$ are 2 function satisfying the above property where $f_1(x)$ is an algebraic function and $f_2(x)$ is a transcendental function.

583. The value of $\int_{-1}^{1}\left(f_1(x) + \frac{1}{4x}\right)dx$ is equal to:

(a) 0 (b) 4 (c) 8 (d) 12

584. Which one of the following statement is correct?

(a) $f_2(x)$ is an increasing function $\forall\, x > 0$.
(b) $f_2(x)$ is a decreasing function $\forall\, x > e$.
(c) $f_1(x)$ is a decreasing function $\forall\, x > 0$.
(d) $f_1(x)$ is an increasing function $\forall\, x \in R$.

Paragraph for Question nos. 585 and 586

Let $f(x) = \left(\cos^{-1}\frac{x}{2}\right)^2 + \pi\sin^{-1}\frac{x}{2} - \left(\sin^{-1}\frac{x}{2}\right)^2 + \frac{\pi^2}{12}(x^2 + 6x + 8)$. Also M is the maximum value and m is the minimum value of $f(x)$.

585. The value of M is:

(a) $\frac{7\pi^2}{4}$ (b) $\frac{9\pi^2}{4}$ (c) $\frac{5\pi^2}{4}$ (d) $\frac{11\pi^2}{4}$

586. The value of 'm' is equal to:

(a) $\frac{\pi^2}{3}$ (b) $\frac{\pi^2}{4}$ (c) $\frac{\pi^2}{6}$ (d) $\frac{\pi^2}{12}$

Paragraph for Question nos. 587 and 588

Let $f(x)=(c-1)x^2+2cx+c+4$ and $g(x)=cx^2+2(c+1)x+(c+1)$, where $c\in R$.

587. If $g(x)$ is always negative $\forall\, x\in(0,1)$, then the number of integral values of c in the interval $[-5, 5]$ is:

(a) 4 (b) 5 (c) 6 (d) 7

588. If the system of equation $f(x)\le 0$ and $g(x)\ge 0$ has a unique solution then the sum of all real values of c is equal to:

(a) $\frac{7}{12}$ (b) $\frac{1}{3}$ (c) $\frac{4}{3}$ (d) $\frac{3}{4}$

Paragraph for Question nos. 589 and 590

Let $f(x)=\lim_{n\to\infty}\left(\cos\left(\frac{x}{\sqrt{n}}\right)\right)^n$ and $g(x)=\frac{-1}{2\ln(f(x))}$

589. The value of $\sum_{n=1}^{\infty}\tan^{-1}\left(\frac{g(n)}{2}\right)$ equals:

(a) $\tan^{-1}(2)$ (b) $\tan^{-1}(1)$ (c) $\tan^{-1}(\sqrt{3})$ (d) $\frac{\pi}{2}$

590. Number of roots of the equation $f(x)=g(x)$ is:

(a) 0 (b) 1 (c) 2 (d) 4

Paragraph for Question nos. 591 and 592

Let $f: R\to R$ be a continuous function satisfying $f(x)=f(2x)\ \forall\, x\in R$. Also $f(1)=3$.

591. The value of $\tan^{-1}(\tan f(x))+\sin^{-1}(\sin f(x))+\cos^{-1}(\cos f(x))$ at $x=10$, is equal to:

(a) π (b) $6-\pi$ (c) $3-\pi$ (d) 3

592. The value of $\lim_{x\to 0}(x^3-\tan^3 x-2+f(x))^{\frac{1}{x^5}}$ is equal to:

(a) e (b) $\frac{1}{e}$ (c) e^2 (d) $\frac{1}{e^2}$

Answer the following by appropriately matching the lists based on the information given in the paragraph. List-I contains function and List-II contains range.

List - I	List-II
(I) $f(x)=\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$	(P) $\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$
(II) $g(x)=\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$	(Q) $[0,\pi]$
(III) $h(x)=\pi\left(\frac{\sqrt{x+7}-4}{x-9}\right)$	(R) $[-\pi,\pi]$
(IV) $k(x)=\frac{\pi}{\sqrt{2}}(\sin\sqrt{x^2}+\cos\sqrt{x^2})$	(S) $\left(0,\frac{\pi}{4}\right]-\left\{\frac{\pi}{8}\right\}$

593. Which of the following option is the only **correct** combination?

(a) (I) (Q) (b) (III) (P) (c) (IV) (R) (d) (II) (S)

594. Which of the following option is the only **incorrect** combination?

(a) (II) (P) (b) (I) (Q) (c) (III) (S) (d) (IV) (R)

Paragraph for Question nos. 595 and 596

A quadratic polynomial $f(x)$ with positive leading coefficient such that $g(x)=f(\ln x)\ \forall\ x>0$. Also the curve $y=g(x)$ satisfy the following condition.

(a) There is exactly one value for a positive number p such that $(p, g(p))$ is its extremum point and $(p^2, g(p^2))$ is its inflection point.

(b) Exactly one tangent line can be drawn from the point (0, 0) to the curve $y=g(x)$.

595. The value of $\frac{f(10)}{f(2)}$ equals:

(a) 25 (b) 36 (c) 41 (d) 64

596. The value of definite integral of $\int_1^2 \frac{f(0)}{f(x)}dx$ equals:

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

Paragraph for Question nos. 597 and 598

On the ellipse $E=\frac{x^2}{64}+\frac{y^2}{9}=1$, tangents drawn at the point $P_1, P_2, P_3, \ldots\ldots P_n$ on the ellipse intersecting the major axis at $T_1, T_2, T_3 \ldots\ldots, T_n$ respectively.

597. If the value of $\sum_{i=1}^{n}\frac{\text{Area}(\Delta P_iT_iS)\cdot\text{Area}(\Delta P_iT_iS')}{(P_iT_i)^2}=18$. where S and S' represents the foci of the ellipse, then 'n' equal to:

(a) 6 (b) 8 (c) 10 (d) 12

598. If the area of the quadrilateral formed by the tangents at the ends of the latus rectum of the ellipse E, is $\frac{16\lambda}{\sqrt{55}}$, then λ equals:

(a) 8 (b) 16 (c) 32 (d) 64

Paragraph for Question nos. 599 and 600

Let $f(x)$ be a polynomial of degree 3 such that $f(0)=1$, $f(1)=2$ and zero is a critical point of $f(x)$ having no local extreme.

599. The value of $\lim_{x\to 0}(f(x))^{\frac{1}{\tan x\ x}}$ is equal to:

(a) e^2 (b) e^{-2} (c) e^3 (d) e^{-3}

600. If the value of definite integral $\int_{-1}^{1}\frac{f(x)}{\sqrt{x^2+7}}dx$ is equal to $2\ln\left(\frac{\sqrt{a}+1}{\sqrt{b}+c}\right)$ then the value of $(a+b+c)$, is:

(a) 8 (b) 15 (c) 16 (d) 17

Answer the following by appropriately matching the lists based on the information given in the paragraph.

Two AP's having same number of terms equal to k. The ratio of the last term of the first progression to the first term of the second progression equals the ratio of the last term of the second progression to the first term of the first progression both of which are numerically equal to 4. The ratio of the sum of k terms of the first progression to the sum of k terms of second progression is equal to 2. Let α be the ratio of the common difference of the first and the second progressions. Let λ be the ratio of their k^{th} terms. Then:

List-I	List-II
(I) The ratio of the first term of first A.P. to second A.P. is	(P) 26
(II) The value of α is equal to	(Q) 33
(III) The value of λ is equal to	(R) 7
(IV) The value of $\alpha+2\lambda$ is equal to	(S) 2/7
	(T) 7/2

601. Which of the following options has the **correct** combination considering List-I and List-II?

(a) (III) (P) (b) (IV) (R) (c) (I) (S) (d) (I) (T)

602. Which of the following options has the **incorrect** combination considering List-I and List-II?

(a) (II) P (b) (III) (T) (c) (III) (S) (d) (IV) (Q)

Answer the following by appropriately matching the lists based on the information given in the paragraph.

Let $f(x)$ is defined as $f(x)=\begin{vmatrix}\sin(x+\alpha) & \sin(x+\beta) & \sin(x+\gamma)\\ \cos(x+\alpha) & \cos(x+\beta) & \cos(x+\gamma)\\ \cos(\beta-\gamma) & \cos(\gamma-\alpha) & \cos(\alpha-\beta)\end{vmatrix}$ and $f(9)=\lambda\neq 0$

and $P=\begin{bmatrix}\cos(\pi/9) & \sin(\pi/9)\\ -\sin(\pi/9) & \cos(\pi/9)\end{bmatrix}$, where α, β and γ be non-zero numbers such that $(\alpha P^6+\beta P^3+\gamma I)$ is the zero matrix and where I is an identity matrix of order 2.

List-I	List-II
(I) The value of $\dfrac{\sum_{k=1}^{9} f(k)}{f(9)}$ equals	(P) 1
(II) The absolute value of $\dfrac{\alpha}{\beta}$ is equal to	(Q) 2
(III) The value of $(\alpha^2+\beta^2+\gamma^2)^{(\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)}$, is	(R) 5
(IV) The absolute value of $\dfrac{2\beta}{\gamma}$ is equal to	(S) 7
	(T) 9

603. Which of the following options has the **correct** combination considering List-I and List-II?

(a) (IV) (P) (b) (III) (Q) (c) (II) (R) (d) (I) (T)

604. Which of the following options has the **incorrect** combination considering List-I and List-II?

(a) (II) P (b) (I) R (c) (III) P (d) (IV) Q

Paragraph for Question nos. 605 to 607

Let a denotes number of digits in 2^{50}, b denotes number of zero's after decimal and before first significant digit in 3^{-50} and c denotes number of natural numbers have characteristic 3 with base 5, then

(Given: $\log_{10}2=0.301$ and $\log_{10}3=0.4771$)

605. The value of $c-a\times b$ is:

(a) 132 (b) 140 (c) 7632 (d) 11132

606. Sum of the digits of the number c is:

(a) 4 (b) 5 (c) 6 (d) 22

607. Number of factors of the number $a+b$ are:

(a) 2 (b) 3 (c) 4 (d) 6

Paragraph for Question nos. 608 and 609

Consider $a = \log_2 3$, $b = \log_3 5$ and $c = \log_5 7$.

608. $\log_{14} 63$ equals:

(a) $\frac{ab+c}{abc+1}$ (b) $\frac{abc+2a}{abc+1}$ (c) $\frac{bc+2}{abc+1}$ (d) none of these

609. Which of the following is **incorrect**?

(a) $abc<3$ (b) $abc>2$ (c) $abc<1$ (d) $abc<4$

Paragraph for Question nos. 610 and 611

Let A, B and C be three angles such that $A+B+C=\pi$.

610. If $\sin^2 A+\sin^2 B+\sin^2 C=\sin A\sin B+\sin B\sin C+\sin C\sin A$, then the value of $\frac{\cos A\cdot\tan B}{\sin C}$ can be:

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) $\frac{\sqrt{3}}{2}$

611. Which of the following must be **correct**?

(a) $\tan A+\tan B+\tan C=\tan A\tan B\tan C$

(b) $\cot A\cot B+\cot B\cot C+\cot C\cot A=1$

(c) $\sin 2A+\sin 2B+\sin 2C=4\sin A\sin B\sin C$

(d) All of the above

Paragraph for Question nos. 612 and 613

Let $f(x)=x^2-ax-b$ and $g(x)=2x^2+3x+a^2$ where $a, b, x\in R$.

612. If $g(x)\ge f(x)\ \forall\ x,\ a\in R$ then sum of integral values of b in $[0, 6]$ is:

(a) 9 (b) 11 (c) 15 (d) 18

613. If $|a-1|+|b-2|=0$ and the range of $y=\frac{f(x)}{g(x)}$ is $R-\{\alpha,\beta\}$, then $(\alpha+\beta)$ is equal to:

(a) $\frac{5}{2}$ (b) $\frac{7}{2}$ (c) $\frac{-5}{2}$ (d) $\frac{9}{2}$

Paragraph for Question nos. 614 and 615

Let $f(x)=x^3-6x^2+14x-12$ and $g(x)=x^2-ax+b$ where $a, b\in N$ have two common roots.

614. If α,β are the roots of $g(x)=0$, then $\left(\frac{1}{4-\alpha}\right)^2+\left(\frac{1}{4-\beta}\right)^2=$

(a) 9 (b) $\frac{1}{9}$ (c) -9 (d) $\frac{-1}{9}$

615. Number of integral values of x, such that $8 < g(x) \le 18$ is:

(a) 4 (b) 5 (c) 8 (d) 9

Paragraph for Question nos. 616 and 617

Consider a circle $S : x^2 + y^2 - 6x - 4y - 3 = 0$ with centre C and P be the point $(-1, -1)$. Also PA and PB are tangent drawn to the circle S.

616. The area of the quadrilateral $PACB$ is equal to:

(a) 12 (b) 24 (c) $3\sqrt{15}$ (d) $4\sqrt{15}$

617. Radius of the circle circumscribing the triangle PAB is:

(a) 1 (b) $\frac{3}{2}$ (c) $\frac{4}{3}$ (d) $\frac{5}{2}$

Paragraph for Question nos. 618 and 619

Let $P = \int_0^1 \sqrt{\frac{x}{1-x}} \ln\left(\frac{x}{1-x}\right) dx$, $Q = \pi \ln\left(\frac{\sqrt{\alpha+1}+1}{2}\right)$ and $R = \int_0^8 e^{Q/P}\, d\alpha$.

618. The value of P is equal to:

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) π

619. The value of $3R$ is equal to:

(a) 36 (b) 37 (c) 38 (d) 39

Paragraph for Question nos. 620 and 621

Equation of an altitude of an equilateral triangle is $\sqrt{3}x + y = 2\sqrt{3}$ and one of its vertex is $(3, \sqrt{3})$. Then:

620. Which of the following can't be the vertex of the triangle?

(a) $(0, 0)$ (b) $(0, 2\sqrt{3})$ (c) $(2, 0)$ (d) $(3, -\sqrt{3})$

621. If orthocentre $H(a, b)$ of the triangle lies in the first quadrant, then $a^2 + b^2$ is equal to:

(a) 1 (b) 2 (c) 3 (d) 4

Paragraph for Question nos. 622 and 623

Consider the following set of points in the x-y plane $A = \{(a, b) \mid a, b \in I$ and $|a| + |b| \le 2\}$.

622. The number of straight line which pass through at least 2 points in A, is:

(a) 20 (b) 22 (c) 32 (d) 40

623. The number of triangles whose vertices are points in A, is:

(a) 256 (b) 276 (c) 286 (d) 289

Paragraph for Question nos. 624 and 625

Let a line L_1 passing through a point $A(2, 0)$ and making an angle θ with positive x-axis in anticlockwise direction, where $\tan\theta = \frac{1}{2}$. Now, L_1 is rotated about the point A in anticlockwise direction through an angle of $(\pi - 4\theta)$. If the line in new position is L_2, then.

624. The positive slope of the angle bisector between lines L_1 and L_2 is:

(a) 2 (b) $\frac{3}{2}$ (c) $\frac{3}{4}$ (d) $\frac{4}{3}$

625. The radius of the largest circle which touches L_1, L_2 and the y-axis is:

(a) $4(\sqrt{3}-1)$ (b) $4(\sqrt{3}+1)$ (c) $4(\sqrt{5}-2)$ (d) $4(\sqrt{5}+2)$

Paragraph for Question no. 626 and 627

Consider, $f(x) = \lim\limits_{n\to\infty} \dfrac{\text{sgn}(\sqrt{ac}-b)e^{nx}+x^2+f}{2e^{nx+x}+x+d}$ where $a > b > c > 0$ and $d, f \in R$

[Note: $\text{sgn}(y)$ denotes the signum function of y.]

626. If a, b and c are in A.P. and $f(x)$ is continuous for all $x \in R$, then the value of $(2f + d + 1)$ is equal to:

(a) 0 (b) 1 (c) –1 (d) $\frac{-1}{2}$

627. If a, b and c are in G.P. and $f(x)$ is continuous for all $x \in R (d < 0)$, then number of solution(s) of the equation $f(x) = ||x-4|-2|-1$ is(are):

(a) 0 (b) 2 (c) 3 (d) 4

Paragraph for Question nos. 628 and 629

Let f be a differentiable function satisfying

$\sqrt[3]{f(x+y)} = \sqrt[3]{f(x)} + \sqrt[3]{f(y)} + 1 \;\forall\, x, y \in R$ and $f'(0) = 3$

628. If $h(x) = f(x) - x^3$, then number of point(s) where $y = h(|x|)$ is non-derivable is(are):

(a) 0 (b) 1 (c) 2 (d) 3

629. If x_0 is solution of the equation $f(x) = f^{-1}(x)$, then $\cos^{-1}(\cos 2x_0) + 4\tan^{-1}\left(\tan\frac{x_0}{2}\right)$ is equal to:

(a) 0 (b) $4\pi - 2x_0$ (c) 2π (d) $3\pi - x_0$

Paragraph for Question nos. 630 and 631

Let $f: R \to R$ is a function defined by $f(x) = \begin{cases} 1, & \text{if } x = 1 \\ e^{(x^{10}-1)} + (x-1)^2 \sin\left(\frac{1}{x-1}\right), & \text{if } x \neq 1 \end{cases}$

630. The value of $f'(1)$ is:

(a) 0 (b) 1 (c) 10 (d) 100

631. If $\lim_{x \to \infty}\left(x\left(\sum_{k=1}^{100} f\left(1+\frac{k}{x}\right) - 100\right)\right) = \lambda$, then the value of $\frac{\lambda}{100}$ is:

(a) 50 (b) 505 (c) 5050 (d) 50500

Paragraph for Question nos. 632 and 633

Suppose that f is defined on R by the rule $f(x) = (1-x)(1+x^2)$. The function is invertible and its inverse is denoted by f^{-1}.

632. If $h = f^{-1}(\ln(f(x)))$, $x < 1$, then the value of $\left(3 + \frac{1}{h'(0)}\right)$ is:

(a) 2 (b) 3 (c) 5 (d) 6

633. The value of $\int_0^1 \frac{(1-x)\ln(1+x)}{f(x)}\,dx$ is equal to:

(a) $\frac{\pi \ln 2}{2}$ (b) $\pi \ln 2$ (c) $\frac{\pi \ln 2}{4}$ (d) $\frac{\pi \ln 2}{8}$

Paragraph for Question nos. 634 and 635

Let PAB be a triangle where $A(1,1)$, $B(3,3)$ and P be a variable point such that $PA^2 + PB^2 = 6$. The locus of point P is $S = 0$. From point $Q(3, 7)$, pair of tangents are drawn to the curve $S = 0$ which touches the curve $S = 0$ at C and D. Let $S_1 = 0$ be the circumcircle of ΔQCD. Then:

634. Equation of common chord of $S = 0$ and $S_1 = 0$ is:

(a) $2x - 3y = 1$ (b) $y = x$ (c) $3x - 5y = 1$ (d) $x + 5y = 13$

635. If θ is the acute angle between $S = 0$ and $S_1 = 0$, then $\tan\theta$ equals:

(a) 3 (b) 5 (c) 7 (d) 9

Paragraph for Question nos. 636 and 637

Let $f(x)$ and $g(x)$ be two differentiable function such that:

$$f(x) + \int_0^x g(t)\,dt = 2\sin x - 3$$

$$f'(x)g(x) = \cos^2 x$$

636. The value of $\lim_{x\to 0}(f(x)+g(x)+3)^{1/x}$ equal to:

(a) $\frac{1}{e}$ (b) e (c) $\frac{1}{e^2}$ (d) e^2

637. The value of definite integral $\int_{-\pi/4}^{\pi/4}\frac{x^2(f(x)+3)+1}{2g^2(x)+1}dx$ is:

(a) $\frac{\sqrt{3}\pi}{9}$ (b) $\frac{\sqrt{3}\pi}{3}$ (c) 0 (d) $\frac{5}{3}$

Paragraph for Question nos. 638 and 639

Let $f(x)=x^2-5x+6$, $g(x)=f(|x|)$, $h(x)=|g(x)|$.

638. If $h(x)=k$, where $k\in I$ has more than two solutions, then the probability that $h(x)=k$ will have exactly 8 real and distinct solutions, is equal to:

(a) 0 (b) $\frac{1}{7}$ (c) $\frac{2}{7}$ (d) 1

639. The number of integral values of x satisfying the equation $g(x)+|g(x)|=0$ is equal to:

(a) 0 (b) 1 (c) 2 (d) 4

Paragraph for Question nos. 640 and 641

Consider two lines $L_1: 2\hat{i}+\hat{j}-\hat{k}+\lambda(\hat{i}+2\hat{k})$ and $L_2: 3\hat{i}+\hat{j}-\hat{k}+\mu(\hat{i}+\hat{j}-\hat{k})$. Let Π be the plane which contains the line L_1 and parallel to L_2 and intersecting coordinate axes at A, B and C respectively.

640. The shortest distance between L_1 and L_2 is:

(a) $\frac{1}{\sqrt{5}}$ (b) $\frac{1}{\sqrt{6}}$ (c) $\frac{1}{\sqrt{8}}$ (d) $\frac{1}{\sqrt{14}}$

641. Volume of tetrahedron $OABC$, (where O is origin) is:

(a) $\frac{2}{3}$ (b) $\frac{4}{9}$ (c) $\frac{2}{9}$ (d) $\frac{4}{3}$

Paragraph for Question nos. 642 and 643

Let f be a continuous function such that $g(x)=\int_{-1}^{1}f(t)|x-t|dt$ where $x\in(-1,1)$.

642. The value of $\int_0^1\frac{g''(x)}{f(x)}dx$ is equal to:

(a) 0 (b) 1 (c) 2 (d) 3

643. If $f(x)=x^2$, then the value of $g'(1)$ is equal to:

(a) 1 (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{4}{3}$

Paragraph for Question nos. 644 and 645

If $A = [a_{ij}]_{n \times n}$, where $a_{ij} = i^2 + j^2, \forall\, i$ and j, then:

644. $\lim\limits_{n \to \infty} \dfrac{tr.(A)}{n^3}$ is equal to:

(a) $\dfrac{1}{6}$ (b) $\dfrac{1}{3}$ (c) $\dfrac{2}{3}$ (d) $\dfrac{4}{3}$

645. If $\lim\limits_{n \to \infty} \sum\limits_{i=1}^{n} \tan^{-1}\left(\dfrac{1}{a_{ii}}\right) = \cot^{-1} \lambda$, then λ is equal to:

(a) -1 (b) 1 (c) 2 (d) 3

Question nos. 646 to 648

Let $X = \{1, 2, 3, \ldots\ldots, 10\}$. A, B, C are three sets such that $A \subseteq X$, $B \subseteq X$ and $C \subseteq X$.

Column-1: Contains types of three subsets of X.

Column-2: Contains number of ways of selecting three subsets of X according to column-1

Column-3: Contains conditional probabilities $P\left(\dfrac{E}{E_1}\right)$ or $P\left(\dfrac{E}{E_2}\right)$ where

E: Selecting three subsets of X according to column-1

E_1: Selecting three subsets of X such that $n(A \cap B) = 5$

E_2: Selecting three subsets of X such that $n(A \cup B) = 5$.

Column-1	Column-2	Column-3
(I) $A \cap B \cap C \supseteq \{2, 3, 4, 5, 6\}$ and $A = B = C$	(i) 32	(P) $P\left(\dfrac{E}{E_1}\right) = 0$
(II) $A \cup B \cup C = \{3, 4, 5\}$	(ii) 242	(Q) $P\left(\dfrac{E}{E_1}\right) = \dfrac{1}{{}^{10}C_5 \cdot 12^5}$
(III) $A \cap B \cap C = \{3, 4, 5, 6, 7\}$ and $A = B \neq C$	(iii) 243	(R) $P\left(\dfrac{E}{E_2}\right) = \dfrac{31}{{}^{10}C_5 \cdot 12^5}$
(IV) $A \cup B \cup C = \{6, 7, 8, 9, 10\}$ and $A = B \neq C$	(iv) 343	(S) $P\left(\dfrac{E}{E_2}\right) = 0$

[**Note:** $S \supseteq T$ denotes S is a superset of T, means S contains atleast all elements of T.]

646. Which of the following options is the only **correct** combination?

(a) (I) (i) (P) (b) (II) (ii) (S) (c) (III) (ii) (R) (d) (IV) (iv) (P)

647. Which of the following options is the only **correct** combination?

(a) (II) (iv) (P) (b) (III) (iii) (P) (c) (III) (i) (R) (d) (IV) (ii) (R)

648. Which of the following options is the only **incorrect** combination?

(a) (I) (i) (Q) (b) (II) (iv) (P) (c) (II) (iv) (S) (d) (IV) (ii) (S)

Question nos. 649 to 651

Consider, $E: \dfrac{(x-1)^2}{16}+\dfrac{(y-2)^2}{9}=1$ and $H:(x-1)^2-(y-2)^2=\dfrac{7}{2}$.

Column-1 contains equation of tangent to either E or H.

Column-2 contains image of foci (whose abscissa is greater than 1) of the conic in its tangent.

Column-3 contains area (in sq. units) of the triangle formed by joining foci of the conic (according to column-2), its image in the tangent and centre of the conic.

Column-1	Column-2	Column-3
(I) $y=x+6$	(i) $(1, \sqrt{7}+2)$	(P) $\dfrac{7}{2}$
(II) $y=x+1$	(ii) $(-4, \sqrt{7}+7)$	(Q) $\dfrac{(5\sqrt{7}+7)}{2}$
(III) $x+y=3$	(iii) $(6, \sqrt{7}-3)$	(R) $\dfrac{7}{4}$
(IV) $x-y-4=0$	(iv) $(1, 2-\sqrt{7})$	(S) $\dfrac{(5\sqrt{7}-7)}{2}$

649. Which of the following options is the only **correct** combination?

(a) (I) (ii) (Q) (b) (II) (i) (R) (c) (III) (iii) (S) (d) (IV) (iii) (R)

650. Which of the following options is the only **correct** combination?

(a) (I) (i) (P) (b) (II) (iv) (P) (c) (III) (iv) (P) (d) (III) (iii) (Q)

651. Which of the following options is the only **incorrect** combination?

(a) (III) (iv) (P) (b) (IV) (ii) (S) (c) (II) (i) (P) (d) (I) (ii) (Q)

Paragraph for Question nos. 652 and 653

Let A be a variable point on locus of feet of perpendicular drawn from focus upon any tangent to the curve $|z-2|+|z+2|=6$ and B be a variable point on $(1-i)z+(1+i)\bar{z}=10\sqrt{2}$, then

652. Minimum value of AB is:

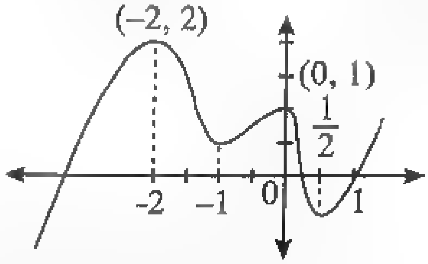
(a) 1 (b) 2 (c) 3 (d) 4

653. If a variable circle touches both the loci on which A and B externally lie then latus rectum of locus of centre of variable circle is:

(a) 2 (b) 4 (c) 8 (d) 16

Paragraph for Question nos. 654 and 655

Graph of $y = P(x) = ax^5 + bx^4 + cx^3 + dx^2 + ex + f$, is given



654. If $P''(x) = 0$ has real roots α, β, γ then $[\alpha] + [\beta] + [\gamma]$ is equal to:

[**Note:** Where [ ] denotes greatest integer function]

(a) -2 (b) -3 (c) -1 (d) 0

655. The minimum number of real roots of the equation $(P''(x))^2 + P'(x) \cdot P'''(x) = 0$ is:

(a) 5 (b) 7 (c) 6 (d) 4

Paragraph for Question nos. 656 and 657

Let $f(x) = \frac{\pi}{4} + \cos^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right) - \tan^{-1} x$ and a_i $(a_i < a_{i+1}\ \forall\ i = 1, 2, 3, \ldots\ldots, n)$ be the positive integral values of x for which $\operatorname{sgn}(f(x)) = 1$ where $\operatorname{sgn}(\cdot)$ denotes signum function.

656. The value of $\sum_{i=1}^{n} a_i$ is equal to:

(a) 1 (b) 2 (c) 3 (d) 4

657. If $P(x) - x^2 - 4kx + 3k^2$ is negative for all values of x lying in the interval (a_1, a_2) then set of real values of k is:

(a) $\left(\frac{1}{3}, 1\right)$ (b) $\left[\frac{2}{3}, 2\right]$ (c) $\left(\frac{1}{3}, 2\right)$ (d) $\left[\frac{2}{3}, 1\right]$

Paragraph for Question nos. 658 and 659

Let $f(x) = x^{2010} + x^{1010} - x^{510} + x^{210} + x^2$. If $f(x)$ is divided by $x^2(x^2 - 1)$, then we get remainder as $g(x)$, function of x.

658. If $g(x)$ is defined from R to R, then:

(a) $g(x)$ is injective but not surjective (b) $g(x)$ is surjective but not injective
(c) $g(x)$ is neither injective nor surjective (d) $g(x)$ is injective and surjective

659. If roots of $g(x) = 0$ lies between the roots of the equation $x^2 - 2(a+1)x + a(a-1) = 0$ then number of integral values of a will be:

(a) 0 (b) 1 (c) 2 (d) 3

Question nos. 660 to 662

If equation of column-I is given along with condition of column-II then match it with appropriate value given in column-III.

Column-I	Column-II	Column-III
(I) $\int_0^1 f(x)\,dx=1$	(i) $f(2x)=2f(x)$	(P) $\int_1^2 f(x)\,dx=5$
(II) $\int_0^2 f(x)\,dx=2$	(ii) $f(2x)=3f(x)$	(Q) $\int_2^4 f(x)\,dx=6$
(III) $\int_0^3 f(x)\,dx=3$	(iii) $f(3x)=3f(x)$	(R) $\int_1^3 f(x)\,dx=8$
(IV) $\int_0^4 f(x)\,dx=4$	(iv) $f(3x)=4f(x)$	(S) $\int_3^9 f(x)\,dx=24$

660. Which of the following is **correct** combination?
(a) (I) (i) (P) (b) (I) (ii) (P) (c) (II) (ii) (Q) (d) (II) (iii) (Q)

661. Which of the following is **incorrect** combination?
(a) (II) (i) (Q) (b) (I) (iii) (R) (c) (II) (ii) (Q) (d) (I) (ii) (P)

662. Which of the following is **correct** combination?
(a) (III) (iv) (R) (b) (II) (iii) (R) (c) (III) (iii) (R) (d) (III) (iii) (S)

Question nos. 663 to 665

Match the condition of column-I with corresponding number of real roots of $f(x)=0$ is column-II and number of points of non-derivability of $y=f(|x|)$ in column-III, where $f(x)=ax^2+bx+c$.

Column-I	Column-II		Column-III	
(I) $a^2+b^2+c^2-ab-bc-ca\le 0$	(i)	0	(P)	0
(II) $a^2+b^2+c^2+ab+bc+ca\le 0$	(ii)	1	(Q)	1
(III) $3(a^2+b^2+c^2+1)\le 2(a+b+c+ab+bc+ca)$	(iii)	2	(R)	3
(IV) $a^2+b^2+c^2\le 2a+6b+4c+14$	(iv)	∞	(S)	5

663. Which of the following is **correct** combination?
(a) (I) (i) (P) (b) (I) (iii) (Q) (c) (II) (iv) (P) (d) (II) (ii) (Q)

664. Which of the following is **incorrect** combination?
(a) (III) (i) (Q) (b) (II) (i) (P) (c) (I) (i) (Q) (d) (II) (iv) (P)

665. Which of the following is **correct** combination?
(a) (III) (i) (P) (b) (II) (i) (R) (c) (IV) (iii) (R) (d) (IV) (iii) (Q)

Paragraph for Question nos. 666 and 667

Let $f(x)=(ax^2+bx+c)\operatorname{sgn}(2\sin x-1)$ be a continuous function $\forall x \in (0,6)$ where $a, b, c \in R$.

[**Note:** sgn $(\cdot)$ represents signum function.]

666. If $a \neq 0$, then which of the following must be **incorrect**?

(a) $y=f(x)$ is not differentiable at exactly two points

(b) $y=f(x)$ is differentiable

(c) if $a=1$, then $\cos b=-1$

(d) $b^2-4ac>0$

667. If $c=0$, then $(a+b)$ equals:

(a) π (b) $\frac{\pi}{6}$ (c) $\frac{5\pi}{6}$ (d) 0

Paragraph for Question nos. 668 to 669

Let $\alpha<\beta<\gamma$ be three numbers in G.P. Let $f(x)=x^3-ax^2+bx-8$ be a polynomial such that $f(x)=0$ has three roots α, β and γ, where α, γ are integers.

668. Let $g(x)-f(x-\beta)$, then the roots of the equation $g(x)=0$ are in:

(a) A.P. (b) G.P. (c) H.P. (d) none of these

669. Let $y=g(x)$ be a twice differentiable function such that $g(\alpha)=0$, $g(\beta)=2$, $g(\gamma)=-3$, $g(a)=5$, $g(b)=0$, then number of minimum distinct real roots of the equation $(g'(x))^2+g(x)\cdot g''(x)=0$ in $[1, 14]$ is:

(a) 2 (b) 6 (c) 3 (d) 5

Paragraph for Question nos. 670 to 671

Let $f(a,b)=\sqrt{49+a^2-7\sqrt{2}a}+\sqrt{b^2+50-10b}+\sqrt{a^2+b^2-\sqrt{2}ab}\ (a, b \in R^+)$

$g(a,b)=\sqrt{a^2+b^2}+\sqrt{a^2+b^2\;\;2a+1}+\sqrt{a^2+b^2-2a+1}+\sqrt{a^2+b^2-6a-8b+25}$

$(a, b \in R)$ and $h(a)=\left|\sqrt{a^2+4a+5}-\sqrt{a^2+2a+5}\right|\ (a \in R)$

670. Least value of (a, b):

(a) is equal to 12 (b) is equal to 13

(c) is equal to 15 (d) does not exist

671. The least value of $g(a, b)$ is equal to m and the greatest value of $h(a)$ is n at $a=\alpha$ then $m+n+\alpha$ is equal to:

(a) $5+2\sqrt{2}$ (b) $8+2\sqrt{2}$ (c) $2+2\sqrt{2}$ (d) none of these

Paragraph for Question nos. 672 to 673

Consider, $f(x) = \left|\sin\left((2r_1 - 1)\frac{\pi}{6}\right)x\right| + \left|\cos\left(\frac{r_2\pi}{6}\right)x\right|$.

Let a fair dice is thrown twice and r_1, r_2 are the numbers obtained on the dice its first and second throw respectively.

672. The probability that $f(1)$ is an integer, is:

(a) $\frac{5}{9}$ (b) $\frac{4}{9}$ (c) $\frac{1}{3}$ (d) $\frac{2}{3}$

673. The probability that $f(2)$ is irrational, is:

(a) $\frac{7}{18}$ (b) $\frac{5}{9}$ (c) $\frac{2}{3}$ (d) $\frac{8}{9}$

Paragraph for Question nos. 674 to 675

Let ω be the complex number representing the point $M\left(\frac{-1}{2}, \frac{\sqrt{3}}{2}\right)$ and $a, b, c, \alpha, \beta, \gamma$ be non-zero complex numbers such that

$$a + b + c = \alpha$$

$$a + b\omega + c\omega^2 = \beta$$

$$a + b\omega^2 + c\omega = \gamma.$$

674. If $|\alpha|^2 + |\beta|^2 + |\gamma|^2 = \lambda(|a|^2 + |b|^2 + |c|^2)$ then λ is equal to:

(a) 1 (b) 2 (c) 3 (d) 4

675. Number of distinct complex numbers z satisfying the equation:

$$(z+1)\begin{vmatrix} z+\omega^2 & 1 \\ 1 & z+\omega \end{vmatrix} + \omega\begin{vmatrix} 1 & \omega \\ z+\omega & \omega^2 \end{vmatrix} + \omega^2\begin{vmatrix} \omega & z+\omega^2 \\ \omega^2 & 1 \end{vmatrix} = 0$$

is equal to:

(a) 0 (b) 1 (c) 2 (d) 3

Match the Column Type Questions

676.

Column-I		Column-II	
(A)	Sum of the solutions of the equation $\|x-1\|^{\log_2 x^2 - 2\log_x 4} = (x-1)^7$ is	(P)	Divisible by 2
(B)	The value of $\dfrac{\log_5 9 \cdot \log_7 5 \cdot \log_3 7}{\log_3 \sqrt{6}} + \dfrac{1}{\log_4 \sqrt{6}}$ is	(Q)	Divisible by 3
(C)	The value of $3^{\log_3 5} + 3^{\sqrt{\log_3 5}} - 5^{\log_5 3} - 5^{\sqrt{\log_5 3}}$ is	(R)	Divisible by 4
(D)	Let n is the number of natural numbers N such that $\log_2 N = 5 + m_1$ and $\log_5 N = 2 + m_2$ where $m_1, m_2 \in [0, 1)$, then n is	(S)	Divisible by 6
		(T)	Divisible by 8

677.

Column-I		Column-II	
(A)	If $P = 2^{\left(\frac{\log_3(\log_2 6)}{\log_3 2}\right)}$ and $A^P = 4$, then the value of A^{P^2} is equal to	(P)	16
(B)	If $a + ar + ar^2 + \ldots.. \infty = 7$ and $a^2 + a^2r^2 + a^2r^4 + \ldots \infty = \dfrac{147}{11}$ where $0 < r < 1$, then $7(a+r)$ is equal to	(Q)	21
(C)	If $(101)!$ is divisible by 7^p where $p \in N$, then largest value of p is	(R)	25
		(S)	36

678.

Column-I		Column-II	
(A)	If z_1 and z_2 satisfy $z + \bar{z} = 2\|z-1\|$ and $\arg(z_1 - z_2) = \dfrac{\pi}{4}$, then $\operatorname{Im}(z_1 + z_2)$ is not less than	(P)	0
(B)	A tangent to $x^2 = 8y$ cuts the hyperbola $xy = c^2$ in two points P and Q then length of latus rectum of locus of mid-point of PQ, is	(Q)	1
(C)	If the equation $x^2 - 4x + \log_{\left(\frac{1}{2}\right)} a = 0$ does not have distinct real roots, then number of integral value of a is equal to	(R)	2
		(S)	4

679.

	List-I		List-II
(P)	If $\lim\limits_{x\to 0}\dfrac{f(x)}{x}=1$, then $\lim\limits_{x\to 0}\dfrac{f(x)}{x^2}$ is equal to	(1)	3
(Q)	If $\lim\limits_{x\to 0}\dfrac{f(x)-5}{x}=3$, then $\lim\limits_{x\to 0}\dfrac{f^2(x)-25}{\sin x}$ is equal to	(2)	8
(R)	If $\lim\limits_{x\to 2}\dfrac{\sqrt{f(x)-2}-4}{x-2}=1$ and $l=\lim\limits_{x\to 2}\dfrac{\sqrt{f(x)}-3\sqrt{2}}{x-2}$, then $9l^2$ is equal to	(3)	30
(S)	If $\lim\limits_{x\to 1}(f(x))^{\frac{1}{x^2-1}}=e^2$, then $\lim\limits_{x\to 1}\dfrac{4(x^3-1)}{f(x)-1}$ is equal to	(4)	Does not exist

Code:

	P	Q	R	S		P	Q	R	S
(a)	4	1	2	3	(b)	1	2	3	4
(c)	3	2	1	4	(d)	4	3	2	1

680. If $t_1, t_2, t_3, \ldots\ldots, t_9$ are positive numbers such that $t_1\cdot t_2\cdot\ldots\ldots\cdot t_9=3^7$ and $m=t_1+t_2+\ldots\ldots+t_7+3t_8+3t_9$ and $n=\dfrac{1}{t_1}+\dfrac{1}{t_2}+\ldots\ldots+\dfrac{1}{t_7}+\dfrac{1}{3t_8}+\dfrac{1}{3t_9}$, then:

	List-I		List-II
(P)	Least value of m is equal to	(1)	3
(Q)	Least value of n is equal to	(2)	13/3
(R)	If m is least, then $\sum\limits_{i=1}^{9} t_i$ is equal to	(3)	23
(S)	If n is least, then $\sum\limits_{i=1}^{9} t_i$ is equal to	(4)	27

Code:

	P	Q	R	S		P	Q	R	S
(a)	2	1	4	3	(b)	4	1	3	3
(c)	4	3	2	1	(d)	3	2	1	3

681. Let $f(x)=\begin{cases}(15-3b)\{x\}-(b^2-4b-5)\operatorname{sgn}(x+1), & \frac{-\pi}{2}<x<0\\ k([x]+[-x]), & 0\le x\le \pi\\ \dfrac{(a+2\cos x)(1+\tan x)}{\ln(1+\pi^2-2\pi x+x^2)}, & \pi<x<\frac{3\pi}{2}\end{cases}$

where $[y]$, $\{y\}$ and sgn(y) denote greatest integer function, fractional part function and signum function of y respectively.

	List-I		List-II
(P)	If f is continuous in $\left(\frac{-\pi}{2},0\right)$, then the value of b is	(1)	0
(Q)	If f is continuous at $x=\pi$, then value of $(a+k)$ is	(2)	1
(R)	If f is continuous in $\left(\frac{-\pi}{2},\pi\right)$, then value of $(b+k)$ is	(3)	5
(S)	If f has exactly four points of discontinuity in $\left(\frac{-\pi}{2},\frac{3\pi}{2}\right)$, then $(a+b+k)$ is equal to	(4)	6

Code:

	P	Q	R	S
(a)	3	2	3	4
(b)	3	1	3	3
(c)	4	1	4	4
(d)	4	2	4	4

682. Consider $f(x)=k^2(2-x)+k(x-1)^2+2x^2-6$.

	List-I		List-II
(P)	Number of integral value(s) of k for which equation $f(x)=0$ has exactly one real solution (identical real roots) is(are)	(1)	1
(Q)	Number of integral value(s) of k for which equation $f(x)=0$ has more than 2 solutions is(are)	(2)	2
(R)	Least positive integral value of k for which equation $f(x)=0$ has two solutions (two distinct real roots) is	(3)	5
(S)	Largest integral value of k for which equation $f(x)=0$ has both imaginary roots, is	(4)	7

Code:

	P	Q	R	S
(a)	3	1	1	2
(b)	2	4	4	3
(c)	2	1	1	3
(d)	3	4	4	2

683. If $\alpha+\beta, \beta+\gamma$ and $\gamma+\alpha$ are the roots of the equation $x^3-3x-1=0$.

	List-I		List-II
(P)	The value of $\alpha^2+\beta^2+\gamma^2$ is equal to	(1)	-8
(Q)	The value of $\alpha^3+\beta^3+\gamma^3$ is equal to	(2)	0
(R)	The value of $(\alpha+\beta-\gamma)(\beta+\gamma-\alpha)(\gamma+\alpha-\beta)$ is equal to	(3)	-3
(S)	The value of $(\alpha^3-3\alpha+1)(\beta^3-3\beta+1)(\gamma^3-3\gamma+1)$ is equal to	(4)	6
		(5)	8

(a) $P\to3; Q\to4; R\to5; S\to2$
(b) $P\to4; Q\to2; R\to1; S\to3$
(c) $P\to3; Q\to3; R\to2; S\to1$
(d) $P\to4; Q\to3; R\to5; S\to5$

684. Let x, y be positive real numbers such that $xy^3=81$, then:

	List-I		List-II
(P)	The least value of $(x+y)^4$ is	(1)	$2(12)^4$
(Q)	The least value of $(x+3y)^4$ is	(2)	$9(2)^8$
(R)	The least value of $(3x+y)^4$ is	(3)	$27(2)^8$
(S)	The least value of $(2x+3y)^4$ is	(4)	$3(2)^8$
		(5)	$(12)^4$

(a) $P\to4; Q\to5; R\to2; S\to1$
(b) $P\to4; Q\to3; R\to1; S\to2$
(c) $P\to3; Q\to2,5; R\to3; S\to1$
(d) $P\to1; Q\to4; R\to2; S\to3$

685. Let $\cos(\theta+70°)=\dfrac{-1}{3}$ where $\theta\in(0°,110°)$

	List-I		List-II
(P)	$\tan(\theta+70°)=$	(1)	$2\sqrt{2}$
(Q)	$\cos(160°+\theta)=$	(2)	$\dfrac{9+4\sqrt{2}}{7}$
(R)	$\sin(20°-\theta)=$	(3)	$-2\sqrt{2}$
(S)	$\tan(25°+\theta)=$	(4)	$\dfrac{-2\sqrt{2}}{3}$
		(5)	$\dfrac{-1}{3}$

(a) $P\to5; Q\to3; R\to4; S\to1$
(b) $P\to3; Q\to4; R\to5; S\to2$
(c) $P\to3; Q\to5; R\to2; S\to4$
(d) $P\to1; Q\to2; R\to4; S\to3$

686. If a, b, c and d are the positive roots of the equation $x^4 - px^3 + qx^2 - rx + \frac{15}{32} = 0$ such that $\frac{a}{2} + \frac{b}{3} + \frac{c}{4} + \frac{d}{5} = 1$, then:

List-I		List-II	
(P)	$a =$	(1)	$\frac{3}{4}$
(Q)	$b =$	(2)	$\frac{7}{2}$
(R)	$c =$	(3)	$\frac{1}{2}$
(S)	$d =$	(4)	$\frac{9}{2}$
		(5)	1

(a) $P \to 5; Q \to 3; R \to 1; S \to 4$
(b) $P \to 3; Q \to 5; R \to 4; S \to 1$
(c) $P \to 3; Q \to 1; R \to 5; S \to 2$
(d) $P \to 4, Q \to 2; R \to 3; S \to 5$

Question nos. 687 to 689

Column-1 represents a condition to form trigonometric equation. Column-2 represents the value of $\sin\theta + \cos\theta$ and column-3 represents the general value of θ satisfying the trigonometric equation.

Column-1		Column-2		Column-3	
(I)	If $2^{\sin\theta}, \sqrt{2}$ and $2^{\cos\theta}$ are three terms of a decreasing G.P.	(i)	$\frac{\sqrt{3}+1}{2}$	(P)	$\theta = 2n\pi - \frac{\pi}{2}$
(II)	If $\cos\theta, \sec\theta$ and $\cot\theta$ are three positive numbers in H.P.	(ii)	$\sqrt{2}$	(Q)	$\theta = 2n\pi + \frac{\pi}{6}$
(III)	If $2\log\sec\theta$, $\log 2$ and $2\log \text{cosec}\,\theta$ are in A.P.	(iii)	-1	(R)	$\theta = 2n\pi + \frac{\pi}{2}$
(IV)	If G.M. of $(2+\sin\theta), (3+\sin\theta)$ and $(4+\sin\theta)$ is equal to cube root of 6.	(iv)	1	(S)	$\theta = 2n\pi + \frac{\pi}{4}$

687. Which of the following options is the only **correct** combination?
(a) (I) (iii) (R) (b) (II) (i) (Q) (c) (III) (ii) (P) (d) (IV) (ii) (S)

688. Which of the following options is the only **correct** combination?
(a) (I) (ii) (P) (b) (II) (iii) (R) (c) (III) (ii) (S) (d) (IV) (i) (P)

689. Which of the following options is the only **correct** combination?
(a) (I) (i) (Q) (b) (II) (iv) (R) (c) (III) (ii) (R) (d) (IV) (iii) (P)

Question nos. 690 to 692

Column-1 represents a quadratic equation with some given conditions. Column-2 represents number of non-positive integral values of 'k' and column-3 represents number of prime values of 'k'. Then match the following.

Column-1		Column-2		Column-3	
(I)	Let α and β are real roots of $x^2 - 8x + k^2 - 6k = 0$ such that $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = 2$.	(i)	0	(P)	0
(II)	If one root of the equation $(k-2)x^2 - (8-2k)x + (3k+8) = 0$ is negative and other is positive.	(ii)	1	(Q)	1
(III)	If difference between the real roots of equation $4x^2 - 2kx + 1 = 0$ is less than $\sqrt{3}$.	(iii)	2	(R)	2
(IV)	If quadratic expression $2kx^2 - (4k-5)x - 10$ is negative for exactly three distinct integral values of x.	(iv)	3	(S)	3

690. Which of the following options is the only **correct** combination?

(a) (I) (iii) (S) (b) (II) (ii) (R) (c) (III) (i) (P) (d) (IV) (i) (Q)

691. Which of the following options is the only **correct** combination?

(a) (I) (iv) (Q) (b) (II) (iv) (P)
(c) (III) (ii) (S) (d) (IV) (iii) (R)

692. Which of the following options is the only **correct** combination?

(a) (I) (ii) (P) (b) (II) (i) (Q)
(c) (III) (iv) (S) (d) (IV) (ii) (P)

693. In ΔABC, if $\frac{a+b}{c} + \frac{c(a+b)}{ab} \le 4$ and $r = \sqrt{3}$, then:

List-I		List-II	
(P)	$2(\cos A + 2\cos B)$ is equal to	(1)	$2\sqrt{3}$
(Q)	R is equal to	(2)	$3\sqrt{3}$
(R)	$\tan A + Ar.(\Delta ABC)$ is equal to	(3)	3
(S)	$(r_1 + r_2 + r_3)$ is equal to	(4)	$9\sqrt{3}$
		(5)	$10\sqrt{3}$

[Note: Symbols used have usual meaning in ΔABC and $Ar.(\Delta PQR)$ denotes area of triangle PQR.]

(a) P → 2; Q → 1; R → 5; S → 3
(b) P → 3; Q → 2; R → 4; S → 3
(c) P → 3; Q → 1; R → 5; S → 4
(d) P → 2; Q → 2; R → 4; S → 4

694.

	List-I		List-II
(P)	The number of solution(s) of the equation $\sin^{-1}\left(\frac{1}{1+x^2}\right)=\text{sgn}(x^2+1)$, is (are)	(1)	0
(Q)	If the range of the function $f(x)-\cos^{-1}\left(\frac{x^2}{1+x^2}\right)$ is $(a, b]$, then $[a+b]$ is equal to	(2)	1
(R)	If α and β are the roots of the $2x^2-3x-2=0$, then $\frac{12}{17}\left(\tan^2\left(\sin^{-1}\frac{\alpha}{\sqrt{1+\alpha^2}}\right)+\tan^2\left(\cos^{-1}\frac{1}{\sqrt{1+\beta^2}}\right)\right)$ is	(3)	2
(S)	If $f(x)=\sin(\cos^{-1}(\sin(\cos^{-1}x))+\sin^{-1}(\cos(\sin^{-1}x)))$, then the value of $\sum_{x=1}^{4} f\left(\frac{3x}{16}\right)$ is equal to	(4)	3
		(5)	4

[Note: sgn z and $[z]$ denotes signum function and greatest integer less than or equal to z respectively.]

(a) P → 1; Q → 1; R → 5; S → 5
(b) P → 3; Q → 1; R → 4; S → 4
(c) P → 1; Q → 2; R → 5; S → 4
(d) P → 3; Q → 2; R → 4; S → 5

695. If a variable line $L: 3x\quad 2y-4+\lambda(x-2y+4)=0$ where λ is a parameter is passing through a fixed point $P(a, b)$ and $S: x^2+y^2=8$ is a circle, then:

	List-I		List-II
(P)	$(a+b)$ is equal to	(1)	$2\sqrt{2}$
(Q)	Length of the tangent from 'P' to the circle 'S' is	(2)	$4\sqrt{2}$
(R)	Least distance of 'P' to the circle 'S' is	(3)	$6\sqrt{2}$
(S)	Least radius of the circle whose centre is 'P' and it contains the circle 'S', is	(4)	$2\sqrt{6}$
		(5)	8

(a) P → 5; Q → 2; R → 2; S → 4
(b) P → 5; Q → 4; R → 1; S → 3
(c) P → 5; Q → 4; R → 3; S → 3
(d) P → 5; Q → 2; R → 2; S → 4

696. If $\int (x^4+2x^2+1)\sin x\, dx = f(x)\sin x + g(x)\cdot\cos x + C$, then:

[**Note:** Where $f(x)$ and $g(x)$ are polynomial functions defined from R to R and 'C' is the constant of integration.]

List-I		List-II	
(P)	$f(x)$ is	(1)	One-one function
(Q)	$g(x)$ is	(2)	Many-one function
(R)	Number of points where $\|f(x)\|$ is non-derivable is	(3)	Onto function
(S)	Number of points where $\|g(x)\|$ is non-derivable is	(4)	Into function
		(5)	3
		(6)	4

(a) P → 2, 3; Q → 2, 4; R → 5; S → 6
(b) P → 2; Q → 2; R → 5; S → 5
(c) P → 2, 4; Q → 2, 3; R → 6; S → 5
(d) P → 2; Q → 2, 3; R → 6; S → 6

697.

List-I		List-II	
(P)	If $g:[1,3]\to[1,3]$ is a continuous decreasing function, then $\int_1^3 (g(x)-g^{-1}(x))dx$ is equal to	(1)	0
(Q)	If $f(x)=\dfrac{5-\cos 3x}{3+\cos 5x}$, then maximum value of $f(x)$ is	(2)	1
(R)	Let $f:R\to R$ be a function defined by $f(x)=x^3+px^2+qx-3$. If f is monotonic decreasing in the interval (1, 3) only, then $(p+q)$ is equal to	(3)	2
(S)	If $\lim_{n\to\infty}\sum_{r=1}^{n}\dfrac{n}{(n+r)(2n+r)}=\ln\left(\dfrac{a}{b}\right)$ where a and b are co-prime, then $\|a-b\|$ is equal to	(4)	3
		(5)	8

(a) P → 5; Q → 3; R → 5; S → 3
(b) P → 1; Q → 4; R → 1; S → 5
(c) P → 5; Q → 3; R → 5; S → 2
(d) P → 1; Q → 4; R → 4; S → 2

698. Consider a system of linear equations $3x + y - z = 0$, $x - \frac{py}{4} + z = 2$ and $2x - y + 2z = q$ where $p, q \in I$ and $p, q \in [1, 10]$, then identify the correct statement(s).

	List-I		List-II
(P)	Number of ordered pairs (p, q) for which system of equation has unique solution is	(1)	1
(Q)	Number of ordered pairs (p, q) for which system of equation has no solution is	(2)	9
(R)	Number of ordered pairs (p, q) for which system of equation has infinite solutions is	(3)	10
(S)	Number of ordered pairs (p, q) for which system of equation has atleast one solution is	(4)	90
		(5)	91

(a) P → 4; Q → 2; R → 1; S → 5
(b) P → 4; Q → 3; R → 1; S → 4
(c) P → 4; Q → 3; R → 2; S → 5
(d) P → 4; Q → 2; R → 2; S → 4

699. Let $f(x) = \sin^{-1}(2x-1) + \cos^{-1}(2\sqrt{x-x^2}) + \tan^{-1}\left(\frac{1}{1+[x^2]}\right)$ where $[k]$ denotes greatest integer less than or equal to k.

	List-I		List-II
(P)	$f\left(\frac{1}{6}\right)$ is equal to	(1)	$\frac{\pi}{6}$
(Q)	$f\left(\frac{3}{4}\right)$ is equal to	(2)	$\frac{\pi}{4}$
(R)	$\sin^{-1}(\tan(f(1)))$ is equal to	(3)	$\frac{\pi}{3}$
(S)	$\sum_{r=1}^{10} f\left(\frac{r}{20}\right)$ is equal to	(4)	$\frac{7\pi}{12}$
		(5)	$\frac{5\pi}{2}$

(a) P → 2; Q → 4; R → 1; S → 3
(b) P → 2; Q → 4; R → 1; S → 5
(c) P → 5; Q → 4; R → 1; S → 3
(d) P → 5; Q → 4; R → 1; S → 5

700. Let N be the number of words which can be formed using all the letters of the word **'DARJEELING'** so that there are atleast two consonants between any two vowels.

	List-I		List-II
(P)	If N is divisible by 2^n $(n \in N)$, then n can be	(1)	1
(Q)	If N is divisible by 6^p $(p \in N)$, then p must be less than	(2)	2
(R)	Number of odd divisors of N is greater than	(3)	3
(S)	Number of zeroes at the end of N is less than	(4)	4
		(5)	5
		(6)	6

(a) $P \to 3, 4, 5, 6$; $Q \to 5, 6$; $R \to 3, 4, 5, 6$; $S \to 4, 5, 6$

(b) $P \to 4, 5, 6$; $Q \to 5, 6$; $R \to 4, 5, 6$; $S \to 4, 5, 6$

(c) $P \to 1, 2, 3, 4, 5, 6$; $Q \to 4, 5, 6$; $R \to 1, 2, 3, 4, 5, 6$; $S \to 2, 3, 4, 5, 6$

(d) $P \to 1, 2, 3, 4, 5, 6$; $Q \to 4, 5, 6$; $R \to 2, 3, 4, 5, 6$; $S \to 2, 3, 4, 5, 6$

INTEGER TYPE QUESTIONS

701. Let F be the set of all continuous real valued functions which are solutions to $f^2(x)=100+\int_0^x (f(t)f'(t)-f(t)-f'(t)-1)\,dt$. Find the value of $\frac{1}{|F|}\sum_{f(x)\in F}|f(100)|$.

702. Let S_n where $n\in N$ be the sum of infinite geometric progression whose first term is n and common ratio is $\frac{1}{n+1}$. If $\lim_{n\to\infty}\frac{S_1S_n+S_2S_{n-1}+S_3S_{n-2}+\ldots\ldots+S_nS_1}{S_1^2+S_2^2+S_3^2+\ldots\ldots+S_n^2}=\frac{p}{q}$ where p and q are co-prime, then find the value of $(p+q)$.

703. Let $f(x)=\cos^{-1}\left(\sqrt{\sin^{-1}\left(\sec\left(\ln\left(\frac{2x^2+3x-2}{x^2-3x+2}\right)\right)\right)}\right)$. Find the value of $1+\left(\sum\alpha_i^2\right)$ where α_i represents the integers in the range of $f(x)$. If there are no integers in the range of $f(x)$, then enter your answer as zero.

704. If $\lim_{n\to\infty}\left(\frac{\binom{5n}{3n}}{\binom{3n}{2n}}\right)^{\frac{1}{n}}=\frac{a^a}{b^{2b}}$ where a and b are co-prime, then find the value of $(a+b)$.

705. Let $I(n)=\int_0^{\pi}\ln(1-2n\cos x+n^2)\,dx$. Find the value of $\frac{I(100)}{I(10)}$.

706. Consider a function $f(x)$ which satisfies $f'(x)=\tan^2 x+\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} f(x)\,dx$. Also $f\left(\frac{\pi}{4}\right)=\frac{-\pi}{4}$.

If the value of $\frac{8+\pi^2}{\pi}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} f(x)\,dx=m$, then find the value of m^2.

707. Let $P(x)=100x^n+a_{n-1}x^{n-1}+\ldots+a_1x+a_0\,\forall a_i\in R;\ 0\le i\le n-1$. Let $\alpha_1,\alpha_2,\alpha_3,\ldots\alpha_n$ be its n roots such that $\prod_{i=2}^{n}(\alpha_1-\alpha_i)=K\,(K\in R-\{0\})$. If $L=\lim_{x\to\alpha_1}(1+P(x))^{\frac{1}{x-\alpha_1}}$ where α_1 is a real root, then find the value of $\frac{\ln L}{K}$.

708. Let $f(x) = x^2 + \alpha x + \beta$ where $\alpha, \beta \in R$ and $f(f(x)) = 0$ has 2 roots 1 and 2. Find the value of $2|f(0)|$.

709. Let $(2x^2 + 3x + 4)^{10} = \sum_{r=0}^{20} a_r x^r$ where $a_0, a_1, a_2, \ldots\ldots, a_{20}$ are constant, then find the value of $\frac{a_7}{a_{13}}$.

710. If α is a real root of the equation $x^5 - x^3 + x \quad 2 = 0$, then find the value of $[\alpha^6]$.
[**Note:** $[\cdot]$ denotes the greatest integer function.]

711. If $\int_0^{\pi} \frac{\sin x(\sin x + 1)e^{\sin x + \cos x}}{e^{\cos x} + 1} dx = a + b\int_0^{\pi} e^{\sin x} dx$ where a and b are positive rational numbers, then find the value of $100(a^2 + b^2)$.

712. Let $S_n = \sum_{x=1}^{n} x!; n \geq 6, T - arc\sin\left(\sin\left(S_n - 7\left[\frac{S_n}{7}\right]\right)\right)$. If $\int_0^1 \frac{T}{\sqrt{1-x^2}} dx - \frac{a\pi}{b} - \pi^c$ where $a, b, c \in W; b \neq 0$, then find $\left(\frac{b}{c} + a\right)$.

713. If $\cos^4\theta + \alpha$ and $\sin^4\theta + \alpha$ are the roots of the equation $x^2 + b(2x+1) = 0$ and $\cos^2\theta + \beta$ and $\sin^2\theta + \beta$ are the roots of the equation $x^2 + 4x + 2 - 0$, then find the value of b where $b \in N$.
[**Note:** θ can be non-real number also.]

714. If $\cot(\theta - \alpha), 3\cot\theta, \cot(\theta + \alpha)$ are in A.P. and θ is not an integral multiple of $\frac{\pi}{2}$, then find the value of $\frac{2\sin^2\theta}{\sin^2\alpha}$.

715. Let x, y, z and t be real number such that (x, y) lies on a circle having radius 3; (z, t) lies on a circle having radius 2 and $xt - yz = 6$. Find the greatest value of $P = xz$.
[**Note:** Both circles have centre at origin.]

716. If $\forall h \in R - \{0\}$ 2 distinct tangents can be drawn from the points $(2+h, 3h-1)$ to the curve $y = x^3 - 6x^2 - a + bx$. Find the value of $\frac{a}{b}$ (where a and b are co-prime).

717. A hyperbola has the equation $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. A tangent and normal to the hyperbola is drawn at the same point. The tangent has y-intercept α and normal has y-intercept β. If $\alpha = -4$ and $\beta = 9$ then find the x coordinate of the focus c of hyperbola. (Given c lies on the positive x-axis.)

718. Let E and M be 3×3 matrices satisfying the system of equations

$$EM^T = (EM)^T = 20I$$

and $$(E+M)^T = 17(E-M)^T$$

where I denotes identity matrix of order 3.

If $E^2 + M^2 = \frac{a}{b} I$ (where a and b are co-prime), then find the value of $(a+b)$.

719. If the largest possible value of x such that $0 < x < \pi$ satisfying the equation $\frac{2\sin 2x + 1}{\cos x + \sin x} = \sqrt{2}$ is $\frac{a\pi}{b}$. Find $(a+b)$.

720. Let $f: R \to R$ satisfy the equation $(x-y) f(x+y) - (x+y) f(x-y) = 4xy(x^2 - y^2)\ \forall$ $x, y \in R$. If $f(1) = -2$, find the absolute difference between maximum and minimum value of $f(x)$ on the interval $[-\sqrt{3}, \sqrt{3}]$.

721. Let $y^2 = x$ be a given parabola and a variable chord cuts the parabola at P and Q. Let C be the vertex of parabola. If the locus of the point of intersection of tangents at P and Q is $x + 1 = 0$, then the minimum area of triangle PCQ be M. Find $4M$.

722. Tangents are drawn from a point on the line $x - y + 3 = 0$ on the curve $y^2 - 6x$. From some point P on the line, the area of triangle Δ formed by tangents and chord of contact is minimum. Find $\frac{4\Delta}{9}$.

723. Let a_k denotes the coefficient of x^k in the expansion of $(1+2x)^n$, $n \in N$.

If $\sum_{k=0}^{n} (3k+1) a_k = (px+q) r^n$, $p, q, r \in N$. Find $p + q + r$.

724. Let a function $f: R \to R$ be defined as $f(x) = \begin{cases} \sin x, & a < x \le b \\ |x - c| - d, & x \le a \text{ or } x > b \end{cases}$

If $f(x)$ is derivable $\forall\, x \in R$, find the minimum value of $|[a + b + c + d]|$.

[**Note:** $[\cdot]$ denotes the greatest integer function.]

725. If the curve $f(x) = 3x^3 + ax^2 + bx$ where a, b are non negative integers, cuts the x-axis at 3 distinct points. Find the minimum value of $(a+b)$.

726. $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ are 3 non-coplanar unit vectors inclined at an angle $\alpha \left(\le \frac{\pi}{2}\right)$ to each other. If the volume of tetrahedron formed by these vectors is $\frac{1}{\sqrt{360}}$, then find the value of $10(3\cos^2\alpha - 2\cos^3\alpha)$.

727. Let $f(x)$ be monotonically strictly increasing function in $[3, 5]$ such that $\int_3^5 f^2(x)dx = 9$; $f(1) = 3$; $f(4) = 5$. Find the value of $2\int_1^4 x(5 - f^{-1}(x))\,dx$.

728. Let $a_1, a_2, \ldots\ldots, a_n$ be the roots of the polynomial $\sum_{k=1}^{n} kx^k = 0$. If $\sum_{k=1}^{n} \frac{1}{(1-a_k)^2} = -13$. Find n.

729. Suppose a function $f: [0,10] \to R$ is continuous and differentiable everywhere in its domain. If $f(10) = 19$ and $|f'(x) - 5| \le 4\ \forall x$ in domain. Find maximum value of $f(0)$.

730. Let z ($z \in$ complex number) be one of the roots of the equation $x^2 - (\log_2 \alpha - \log_2 \beta)x + \cos\alpha - \sin\beta = 0$. If the harmonic mean of the roots is 2 and $|z| = 1$, find the sum of all values of β in degrees when $0 < \beta < 360°$.

731. Let $y = y(x)$ satisfy the differential equation $\left(2xy + x^2 y + \frac{y^3}{3}\right)dx + (x^2 + y^2)dy = 0$. If $y(1) = 1$ and the value of $(y(0))^3 = ke\ (k \in N)$. Find k.

732. If $\int \frac{\operatorname{cosec}^2 x - 2019}{(\cos x)^{2019}}\,dx = \frac{f(x)}{(g(x))^{2019}} + C$, then find the value of $\left|f\left(\frac{\pi}{4}\right) + g(0)\right|$.

(where C is constant of integration.)

733. Let $f(x) = \left[x - \frac{1}{4}\right] + x[x] + |x(x-4)\sin x| + (2x-1)^{1/3}$. Find the number of points in $[0, 2\pi]$ where $f(x)$ is non-derivable.

[Note: $[\cdot]$ denotes the greatest integer function.]

734. If the value of $\lim_{x\to 0}\left(\frac{1}{x^5}\left(\int_0^x e^{-t^2}dt\right) - \frac{1}{x^4} + \frac{1}{3x^2}\right)$ is equal to $\frac{m}{n}$ (m, n are co-prime), find $(m+n)$.

735. A continuous function $f(x) = \begin{cases} x + a & ;\ \text{for } |x| < 2 \\ bf(x/2) + c & ,\ \text{for } |x| \ge 2 \end{cases}$ is defined for some non-zero constants a, b and c. Find the value of $\frac{a}{c} + b$.

736. Let $f(x) = (x-1)^{100}(x-2)^{2(99)}(x-3)^{3(98)}\ldots\ldots(x-100)^{100}$ where $k = \frac{f'(101)}{f(101)}$.

Find the value of $\frac{k}{50} - 97$.

737. The value of the expression

$$\tan\left(\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{2}{9}\right)+\tan^{-1}\left(\frac{1}{8}\right)+\tan^{-1}\left(\frac{2}{25}\right)+\tan^{-1}\left(\frac{1}{18}\right)+\ldots\ldots\infty\right)$$, is:

738. If the matrix $=\begin{pmatrix}0.3 & b & c\\ l & m & n\\ 0 & p & q\end{pmatrix}$ is an orthogonal matrix, find the sum of all possible value of $10(mq-np)$.

739. Find the sum of all integral values of a for which all the roots of the equation $x^4-4x^3-8x^2+a=0$ are real.

740. Let $f:(0,1)\to R$ be a function defined by $f(x)=10x^{\sin x+\cos x}$. If $I=\int_0^1 f(x)\,dx$, then find the value of $[I]$.

[**Note:** Where $[k]$ denotes the greatest integer function less than or equal to k.]

741. Consider a point P_1 on the curve $y=x^3$ such that the tangent on $P_1(1,1)$ meets the curve again at P_2. And the tangent at P_2 meets the curve at P_3 and so on.

Let (x_n, y_n) be the coordinates of P_n then the value of $\dfrac{\lim\limits_{n\to\infty}\sum\limits_{r=1}^{n}\frac{1}{x_r}}{\lim\limits_{n\to\infty}\sum\limits_{r=1}^{n}\frac{1}{y_r}}$ is equal to $\frac{m}{n}$, where m and n are co-prime positive integers, find the value of $(m+n)$.

742. Let S_k be the area bounded by the curve $y=x^2(1-x)^k$ and the lines $x=0$, $y=0$ and $x=1$. If $\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}S_k$ is equal to $\frac{p}{q}$, where p and q are co-prime positive integers, find $(p+q)$.

743. Let

$$A=\lim_{x\to 0}(1+arc\tan(arc\sin x)+arc\tan(arc\sin 2x)+\ldots\ldots+arc\tan(arc\sin nx))^{1/arc\sin nx},$$

then find the value of $\ln A$ at $n=11$.

744. For all x, define the functions, $f(x)=x^2$, $g(x)=-x^2+1$, $p(x)=|x|+1$. Consider the piecewise function $h(x)$ where $h(x)=\begin{cases}\max(f(x), g(x), p(x)), & x\geq\frac{-81}{100}\\ \min(f(x), g(x), p(x)), & x<\frac{-81}{100}\end{cases}$

If $h(x)$ is non-differentiable at m number of points and discontinuous at n number of points, then find the value of $(m+n)$.

745. Let $f(x)$ be a continuous, periodic and bounded function with period 3 such that $\int_0^3 f(t)\,dt = 6$. Also $g'(x) = f(x)$, such that $g(0) = 0$. Find the value of $\lim_{x\to 0} xg\left(\frac{1}{x}\right)$.

746. Let $f:(0,\infty)\to R$ be a differentiable function satisfying $f(x) + e^{f(x)} = \frac{2}{x} - \ln x - 1$. Find the number of integers in the range of x satisfying the inequality $f(2x^2+1) - f(x^2+5) \geq f(1), x > 0$.

747. Let α, β are the roots of the quadratic equation $2x^2 - 5x + 1 = 0$. If $S_n = (\alpha)^{2n} + (\beta)^{2n}$ then find the value of $\frac{4S_{2021} + S_{2019}}{S_{2020}}$.

748. If $\lim_{x\to 0}\left(1 + \int_0^{\sqrt{a^x - 1}} (\sin(2\,arc\tan t))(1+t^2)^{\ln a}\,dt\right)^{\frac{1}{x}} = 5$, then find the value of a. where $a \in N$ and $a > 1, x > 0$.

749. Let a denotes the number of non-negative values of p for which the equation $p2^x + 2^{\,x} = 5$ possess a unique solution. If $a, \alpha_1, \alpha_2, \ldots\ldots, \alpha_{20}, 6$ are in H.P. and $a, \beta_1, \beta_2, \ldots\ldots, \beta_{20}, 6$ are in A.P. find $\alpha_{18}\beta_3$.

750. Let $f(x)$ be a differentiable function satisfying $f(y) - f(x) = \frac{x^x}{y^y} f\left(\frac{y^y}{x^x}\right)$ for all $x, y \in R^+$. If $f'(1) = 1$, then find the value of $\left|f(e)f\left(\frac{1}{e}\right)\right|$.

751. If the equation $x^2 + 2ax + a = \sqrt{a^2 + x - \frac{1}{16}} - \frac{1}{16}$ has no real roots, then the range of a is (p, q) where p and q are rational numbers. If $p^2 + q^2 = \frac{c}{d}$ (where c and d are co-prime) then find the value of $\sqrt{4c^2 - d^2}$.

752. A curve passing through (1, 2) has its slope at any point (x, y) equal to $\frac{2}{y-2}$ Find the area of the region bounded by the curve and the line $2x - y \quad 4 = 0$.

753. Consider the following statements about positive function $f(x)$ and $g(x)$ whose limits to infinity exists.

Statement-A : $\lim_{x\to\infty} f(x) = \lim_{x\to\infty} g(x)$

Statement-B : $\lim_{x\to\infty} (f(x) - g(x)) = 0$

Statement-C : $\lim_{x\to\infty} \sqrt{f(x)} = \lim_{x\to\infty} \sqrt{g(x)}$

How many of the following six statements are true: $A \Rightarrow B, B \Rightarrow C, C \Rightarrow A, A \Rightarrow C, B \Rightarrow A, C \Rightarrow B$.

754. Let $\vec{k}, \vec{l}, \vec{m}, \vec{n}$ are four distinct units vectors in space such that $\vec{k} \cdot \vec{l} = \vec{l} \cdot \vec{m} = \vec{m} \cdot \vec{k} = \vec{n} \cdot \vec{l} = \vec{n} \cdot \vec{m} = \frac{-1}{11}$. The value of $\vec{k} \cdot \vec{n}$ can be expressed as $\frac{-A}{B}$, where A, B are co-prime positive integer.

Find the value of $A + B$.

755. Let $a, b > 0$ be real numbers satisfying the relation

$$arc\tan(3+a) + arc\tan(3+b) = \frac{\pi}{2} + arc\cot\left(\frac{1}{3}\right).$$

If the minimum value of $(a+b)$ is $\sqrt{N}$, find the value of N.

756. If x, y and z are real numbers that satisfy the three equations

$$\begin{cases} \tan(x) + \tan(y) + \tan(z) = 6 - (\cot(x) + \cot(y) + \cot(z)) \\ \tan^2(x) + \tan^2(y) + \tan^2(z) - 6 - (\cot^2(x) + \cot^2(y) + \cot^2(z)) \\ \tan^3(x) + \tan^3(y) + \tan^3(z) = 6 - (\cot^3(x) + \cot^3(y) + \cot^3(z)) \end{cases}$$

Find the value of the expression $\left(\frac{\tan(x)}{\tan(y)} + \frac{\tan(y)}{\tan(z)} + \frac{\tan(z)}{\tan(x)} + 3\tan(x)\tan(y)\tan(z)\right)$.

757. If $[\vec{b}\ \vec{c}\ \vec{d}] = 48$ and $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) + (\vec{a} \times \vec{c}) \times (\vec{d} \times \vec{b}) + (\vec{a} \times \vec{d}) \times (\vec{b} \times \vec{c}) + k\,\vec{a} = \vec{0}$

(where $\vec{a}$ is a non zero vector), then find value of k.

758. Let P_0 is the parabola $y^2 = 4x$ with vertex $K(0,0)$, A and B are points on P_0 where tangents drawn intersect at right angles. Let C be the centroid of ΔABK. The locus of C is another parabola P_1. Now the process is repeated with P_1 then P_2, P_3..... etc. Then the length of latus rectum of P_{10} can be expressed as $\frac{a}{b}$ where a, b are co-prime natural numbers. Find the value of $(a + \log_3 b)$.

759. Let the solution of the equation $\frac{dy}{dx} - y + \int_0^2 y\,dx$ is $y(x)$. If $y(0) - 1$,

then find the absolute value of $[y(2)]$.

[**Note:** $[k]$ denotes greatest integer function less than or equal to k.]

760. Let $D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ and $P = \begin{bmatrix} 7 & 0 & 2 \\ 0 & 1 & 0 \\ 2 & 0 & 5 \end{bmatrix}$. Consider $A = P^{-1}DP$. Find det.$(A^2 + A)$.

761. Let $f(x)$ be a function continuous for all $x \in R$ except at $x = 0$ such that

- $f'(x) < 0,\ \forall\, x \in (-\infty, 0)$
- $f'(x) > 0,\ \forall\, x \in (0, \infty)$

- $\lim\limits_{x\to 0^+} f(x)=2$
- $\lim\limits_{x\to 0^-} f(x)=3$
- $f(0)=4$

If $$\begin{cases} 2\lim\limits_{x\to 0} f(x^3-x^2)=\mu \lim\limits_{x\to 0} f(2x^4-x^5) \\ \lim\limits_{x\to 0^+} \dfrac{f(-x)x^2}{\left\{\dfrac{1-\cos x}{[f(x)]}\right\}}=\lambda \\ \lim\limits_{x\to 0^+}\left(\left[3f\left(\dfrac{x^3-\sin^3 x}{x^4}\right)\right]-f\left(\left[\dfrac{\sin x^3}{x}\right]\right)\right)=\gamma \end{cases}$$

Then find the value of $\mu\cdot\lambda\cdot\gamma$.

[Note: Where $[k]$ denotes greatest integer function less than or equal to k, $\{k\}$ denotes the fractional part function.]

762. Let f be a continuous and differentiable function in (x_1, x_2) and the following conditions hold for it.

$$\begin{cases} f(x)f'(x)\ge x\sqrt{1-(f(x))^4} \\ \lim\limits_{x\to x_1^+}(f(x))^2=1 \\ \lim\limits_{x\to x_2^-}(f(x))^2=\dfrac{1}{2} \end{cases}$$

If the minimum value of $(x_1^2-x_2^2)$ is equal to $\dfrac{\pi}{k}$ where $k\in N$, then find the value of k.

[Note: $f'(x)=\dfrac{df(x)}{dx}$]

763. Consider a series of n concentric circles $C_1, C_2, \ldots\ldots, C_n$ with radii $r_1, r_2, r_3, \ldots\ldots, r_n$ respectively satisfying $r_1>r_2>r_3>\ldots\ldots>r_n$ and $r_1=10$

The circles are such that the chord of contact of tangents from any point on C_i to C_{i+1} is a tangent to C_{i+2} where $i=1, 2, 3, \ldots\ldots$

Find the value of $\lim\limits_{n\to\infty}\sum\limits_{r=1}^{n} r_i$, if the angle between the tangents from any point of C_1 to C_2 is $60°$.

764. Let $S = \sum_{n=1}^{\infty} \dfrac{1}{\sum_{r=1}^{n} a_r a_{r+1} a_{r+2} a_{r+3}}$ where $a_n = \int_0^{\frac{\pi}{2}} \dfrac{1-\cos 2nx}{1-\cos 2x}\,dx$. Then find the value of $24\pi^4 \cdot S$.

765. If $f(x) = (x-a)(x-b)$ for $a, b \in R$, then find the minimum number of roots of equation

$$\pi(f'(x))^2 \cos(\pi(f(x))) + \sin(\pi(f(x)))f''(x) = 0$$

in $[\alpha, \beta]$ where $f(\alpha) = 3 = f(\beta)$ and $\alpha < a < b < \beta$.

766. Positive number x, y and z satisfy $xyz = 10^{81}$ and $(\log_{10} x)(\log_{10} yz) + (\log_{10} y)(\log_{10} z) = 468$ Find $\sqrt{(\log_{10} x)^2 + (\log_{10} y)^2 + (\log_{10} z)^2}$.

767. If there exist a straight line which intersect the curve $y = x^4 + 2x^3 + cx^2 + 9x + 4$ $(c \in R)$ at four distinct points, then the range of c is (a, b), find the value of $100(b)$.

768. There is a cubic polynomial $f(x)$ with values of x lying in the interval $[-1, 2]$. Given the condition.

(i) $f'''(x) = 24$

(ii) An extreme of $f'(x)$ lies at $x = \dfrac{-1}{6}$

(iii) The coefficient of x and x^0 in $f(x)$ are 0 and 6 respectively.

Find the greatest value of $f(x)$.

769. Let $f(x)$ be a function $f(x) = \dfrac{arc\sin(1-\{x\})arc\cos(1-\{x\})}{\sqrt{2\{x\}}(1-\{x\})}$. Find the value of

$$\left(\frac{\lim\limits_{x \to 0^+} f(x)}{\lim\limits_{x \to 0^-} f(x)}\right)^2.$$

[**Note:** where $\{k\}$ denotes fraction part function of k.]

770. If the inequality $1 + \log_5(x^2+1) \geq \log_5(ax^2 + 4x + a)$ holds $\forall\, x \in R$, then the range of a is $(p, q]$, find $(p+q)$.

771. Let $f: R \to R$ be a function defined by $f(x) = \begin{cases} x^3 + 2x^2 + x + c, & \text{if } x \leq b \\ e^x, & \text{if } x > b \end{cases}$, where b and c are integers. If $f(x)$ is differentiable $\forall\, x \in R$, then find the value of $(b+c)$.

772. Two parallel planes are given by, $x + y + z = 1$ and $x + y + z = \dfrac{9}{2}$. A third plane that intersects them is given by $2x - 5y + z = -5$, resulting in two parallel lines of intersection. If the distance 'd' between these two parallel lines can be expressed as $d = \sqrt{\dfrac{a}{b}}$, where a and b are co-prime positive integers, then find the value of $[d]$.

[**Note:** Where $[k]$ denotes greatest integer function less than or equal to k.]

773. If A_E is the area of an ellipse with an eccentricity of $e=\frac{7}{25}$ and A_F is the area of the shape bounded by the set of points for which two tangents of that ellipse meet at a right angles, then $\frac{A_E}{A_F}=\frac{p}{q}$, where p and q are positive co-prime integers. Find $(p+q)$.

774. Find the value of $\log_{\sqrt{5}}\left(\frac{\sqrt{(5\sqrt{5}+5)\sqrt{(5\sqrt{5}+5)^2\sqrt{(5\sqrt{5}+5)^3\sqrt{\ldots}}}}}{(6+2\sqrt{5})\sqrt[3]{215-18\sqrt[3]{215-18\sqrt[3]{215-18\sqrt[3]{\ldots}}}}}\right)$.

775. Let $f(x)$ and $g(x)$ be continuous on R (set of real numbers).

If $\lim\limits_{x\to 0}\frac{f(x)}{\sin^2 x}=8$, $\lim\limits_{x\to 0}\frac{g(x)}{2\cos x-xe^x+x^3+x-2}=\lambda$ and $\lim\limits_{x\to 0}(1+2f(x))^{1/g(x)}=\frac{1}{e}$, then find the value of λ.

776. The value of $\int_0^{\frac{\pi}{4}} e^{\sec x}\frac{\sin\left(x+\frac{\pi}{4}\right)}{(1\quad\sin x)\cos x}dx$ can be expressed as $\frac{(a+\sqrt{b})e^{\sqrt{c}}-e}{\sqrt{d}}$, then find the value of $(a+b+c+d)$.

777. Let $J_n=\int_0^{\pi/2}(1-\sin x)^n\sin 2x\,dx$. Find $\sum\limits_{n=0}^{\infty}J_n$.

778. If the value of $\lim\limits_{x\to 0}\left(\frac{1}{x^4}\cdot\frac{\int_0^{x^2}e^{-y^2}dy}{x^6}\right)=\frac{a}{b}$ (where a and b are co-prime)

then find the value of $(a+b)$.

779. Find the value of $S=\frac{2+6}{4^{100}}+\frac{2+2(6)}{4^{99}}+\frac{2+3(6)}{4^{98}}+\ldots\ldots+\frac{2+99(6)}{4^2}+\frac{2+100(6)}{4}$.

780. If $a_1, a_2, \ldots\ldots\ldots, a_{4001}$ are in arithmetic progression and

$\frac{1}{a_1a_2}+\frac{1}{a_2a_3}+\ldots\ldots+\frac{1}{a_{4000}a_{4001}}=10$ and $a_2+a_{4000}=50$. Find the value of $|a_1-a_{4001}|$.

781. Let a, b be the roots of the quadratic equation $x^2-(k\quad 2)cx-(k-1)d=0$ and c, d be the roots of the quadratic equation $x^2-(k-2)ax-(k-1)b=0$. If a, b, c and d are distinct, non-zero real numbers such that $a+b+c+d=100$, then find the integral value of k.

782. Find the number of integral values of x satisfying the inequality

$$\frac{\left(2^{\frac{\pi}{\tan^{-1}x}}-4\right)(x-4)(x-10)}{x|-(x-1)|}<0$$

783. Let $f(x)=\dfrac{\{x\}+2}{2\{x\}+1}$. If different integral values of $[f(x)]$ are the roots of the equation $3x^2-2(k+1)x+\mu=0$ then find the value of $(2k+\mu)$.

[Note: $[y]$ and $\{y\}$ denote greatest integer function and fractional part function of y respectively.]

784. Let $f(x)=\begin{cases}\dfrac{\tan^3 x+\tan^2 x-a\tan x-a}{e^{(\tan x-2)}-1}, & x\in(0,\pi/2)-x_0\\ b, & x=x_0\end{cases}$.

If f is continuous in $(0,\pi/2)$ then find the value of $[a+b+x_0]$.

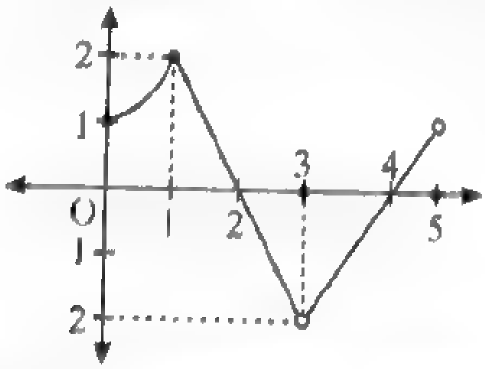
[Note: $[y]$ denotes greatest integer function less than or equal to y.]

785. Let $\alpha=\sum\limits_{k=1}^{20}\dfrac{\sin\left(\frac{\pi}{3}k\right)}{4\cos^2\left(\frac{k\pi}{9}\right)-1}$ and $\beta=\sum\limits_{k=1}^{20}\dfrac{\cos\left(\frac{\pi}{3}k\right)}{1-4\sin^2\left(\frac{k\pi}{9}\right)}$.

If $\beta^2-\alpha^2=1+\cos\lambda^\circ$ then find the value of λ.

786. Let $f(n)=\ln(n^2-1)-\ln(n^2+2n)$, $n\in N$, $n\geq 2$. If $L=\lim\limits_{m\to\infty}\left(e^{e^{\sum\limits_{n=2}^{m}f(n)}}-1\right)m^\alpha$ exists and has non-zero finite value, then find the value of $(\alpha+L)$.

787. Graph of a function $y=f(x)$ is shown as



If $g(x)=|f(|x|)|$, then find number of solution(s) of the equation $g(g(x))=\text{sgn}(x^2-(k+1)x+(k^2+1))$, $k\in R$.

[Note: sgn(y) denotes the signum function of y.]

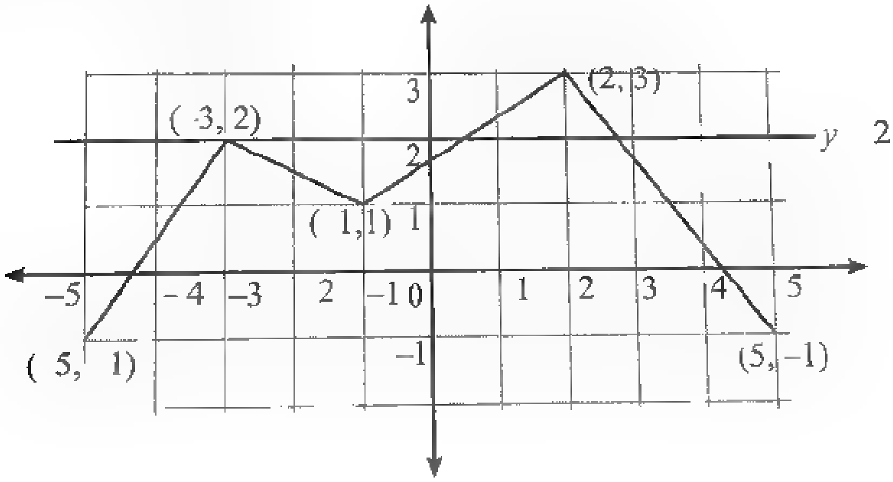
788. If $T_r = \sqrt{r}\sqrt{r+1}\left(\dfrac{4r+5}{(r+2)+\sqrt{r^2-1}}\right)$, then find the value of $\dfrac{1}{\sqrt{68}}\sum_{r=1}^{16} T_r$.

789. Let f be a polynomial function of degree 3 satisfying $f(1)=3$, $f(2)=5$ and $f(3)=7$. If product of the roots of the equation $(f(x))^2+4xf(x)+3x^2=0$ is 4 and the sum of all possible values of $f(4)$ is k then find $[k]$.

[**Note:** $[y]$ denotes greatest integer function less than or equal to y.]

790. If $f(x)=\{x+\sin x\}+[x-\sin x]+[x]$ where $[y]$ and $\{y\}$ denote greatest integer function and fractional part function of y respectively, then find the number of points of discontinuity in $[0, \pi]$.

791. Consider the graph of $y=f(x)$.



Find the number of solution(s) of x satisfying $f(f(x))=2$.

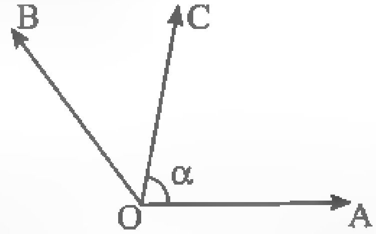
792. If the value of $\lim\limits_{x\to 0}\dfrac{\ln(1+\sin^3 x\cos^2 x)\cot(\ln^3(1+x))\tan^4 x}{\sin(\sqrt{x^2+2}-\sqrt{2})\cdot\ln(1+x^2)}=\sqrt{n}$ where $n\in N$, then find the value of n.

793. If a, b and c are side lengths of a triangle ABC such that

$x^2-2(a+b+c)x+3k(ab+bc+ca)=0$, where $k<\dfrac{p}{q}$ (where p and q are relatively prime), has real roots, find $(p+q)$.

794. The equation $\sum\limits_{r=1}^{\infty}\dfrac{r^3+(r^2+1)^2}{(r^4+r^2+1)(r^2+r)}=\dfrac{a}{b}$ holds true for co-prime positive integers a and b. Find $a+b$.

795. Vector $\overrightarrow{\mathbf{OA}}, \overrightarrow{\mathbf{OB}}, \overrightarrow{\mathbf{OC}}$ are on the same plane, $\left|\overrightarrow{\mathbf{OA}}\right|=1, \left|\overrightarrow{\mathbf{OB}}\right|=1, \left|\overrightarrow{\mathbf{OC}}\right|=\sqrt{2}$, the relative position is shown in the figure.



If $\tan\alpha = 7$ and $\overrightarrow{\mathbf{OB}} \wedge \overrightarrow{\mathbf{OC}} = \dfrac{\pi}{4}$ and $\overrightarrow{\mathbf{OC}} = m\overrightarrow{\mathbf{OA}} + n\overrightarrow{\mathbf{OB}}$, $(m, n \in R)$, find the value of $(m+n)$.

796. If $\displaystyle\int \frac{\sin^2 x\cos^2 x}{(\sin^3 x+\cos^3 x)^2}\,dx = -\frac{1}{3}\frac{\cos^3 x}{(A\cos x + B\sin x)(1-\sin x\cos x)} + C$

for constants A and B, where C denotes the arbitrary constant of integration. Then find the value of $(A+B)$.

797. Let $f(x)=x$, $g(x)=|1-f(x)|$, $h(x)=2-g(x)$, $L(x)=h(|x|)+|h(x)|$.

Find the number of points where $L(x)$ is non-differentiable.

798. Find the sum of squares of the solution of the equation

$$\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)=\frac{\pi}{2}-2\sin^{-1}\left(\frac{2x}{1+x^2}\right).$$

799. Let $I_1 = \displaystyle\int_1^3 (x^2+x+3)f(x^3-2x^2-5x+2020)\,dx$ and

$I_2 = \displaystyle\int_1^3 (4x^2-3x-2)f(x^3-2x^2-5x+2020)\,dx$. If the value of $\dfrac{2I_1}{3I_2}=\dfrac{a}{b}$ where a and b are co-prime, then find the value of $(a+b)$.

800. The value of the definite integral $\displaystyle\int_0^{\pi/2} \sin^2 t\cdot\ln(\sin t)\,dt$ can be expressed as $\dfrac{\pi^a}{b}(c-\ln d)$, where a, b, c and d are integers. Find the smallest possible value of $a+b+c+d$.

801. If the equation $\displaystyle\lim_{\theta\to\frac{\pi}{4}} \frac{(\cos\theta)^{\frac{\sin\theta}{\cos\theta-\sin\theta}}}{(\sin\theta)^{\frac{\cos\theta}{\cos\theta-\sin\theta}}} = a\sqrt{b}$ holds true for square-free positive integer b, find $[a]+b$.

[**Note:** $[k]$ denotes greatest integer function less than or equal to k.]

802. Two cubic function $f(x)=x^3+ax^2+bx+c$ and $g(x)=cx^3+bx^2+ax+1$ satisfy the following.

(i) $f(3)=0,\ g(4)=0$

(ii) The value of $\lim\limits_{x\to p}\dfrac{f(x)}{g(x)}$ exists for all $p\in R-\{4\}$

Find the value of $\lim\limits_{x\to -1}\dfrac{f(x)+g(x)}{x+1}$.

803. For some function $f(x)$ and $g(x)$ which are differentiable $\forall\, x>0$ satisfy the following condition.

(i) $\left(\dfrac{f(x)}{x}\right)'=x^2e^{-x^2}$ (ii) $g(x)=\dfrac{4}{e^4}\displaystyle\int_1^x e^{t^2}\cdot f(t)\,dt$ (iii) $f(1)=\dfrac{1}{e}$

Find the value of $3e^4(f(2)-g(2))$.

804. If $\vec{\mathbf{a}},\ \vec{\mathbf{b}},\ \vec{\mathbf{c}}$ are three vectors such that $\begin{cases}|\vec{\mathbf{a}}|=|\vec{\mathbf{c}}|=1\\ |\vec{\mathbf{b}}|=4\\ |\vec{\mathbf{b}}\times\vec{\mathbf{c}}|=2\\ 2\vec{\mathbf{b}}=\vec{\mathbf{c}}+\lambda\vec{\mathbf{a}}\end{cases}$ where λ is a scalar. If the value of λ is equal to $\sqrt{\alpha-\beta\sqrt{3}}$ where α and β are natural numbers, then find the value of $\alpha+\beta$.

805. Let τ be a circle with centre $C(3,5)$ and PA and PB are pair of tangents drawn from an external point $P(9,11)$ to the circle τ. Find the distance between the origin and the point inside the quadrilateral $ACBP$ which is equidistant from its four vertices.

806. Let A be $m\times m$ matrix with all elements equal to 1 such that $A^n=16^{17}A,\ m,n\in N$, find sum of possible values of n.

807. The area bounded by $y=f(x),\ x=\dfrac{1}{2},\ x=\dfrac{\sqrt{3}}{2}$ and the x-axis is A sq. units, where $f(x)=x+\dfrac{2}{3}x^3+\dfrac{2}{3}\cdot\dfrac{4}{5}x^5+\dfrac{2}{3}\cdot\dfrac{4}{5}\cdot\dfrac{6}{7}x^7+\ldots\ldots\infty$ and $|x|<1$.

If $A=\dfrac{\pi^a}{b}$, where $a,b\in N$, then find the value of $(a+b)$.

808. If the equation $|2x+\sin^2 a|+|2x+3+2\sin a|=0$ has exactly one solution $x=\lambda$ (where 'a' is a constant) then find the value of $4\lambda^2$.

809. If the sum of the infinite geometric series $\dfrac{a}{b}+\dfrac{a}{b^2}+\dfrac{a}{b^3}+\ldots\ldots\infty$ is 4, then find the sum of $\dfrac{a}{(a+b)}+\dfrac{a}{(a+b)^2}+\dfrac{a}{(a+b)^3}+\ldots\ldots\infty=\dfrac{p}{q}$ where p and q are co-prime, then find the value of $(p+q)$.

810. In ΔABC if inradius $r = 1$, circumradius $R = 3$ and semiperimeter $s = 7$, then find the value of $(a^2+b^2+c^2)$, where a, b, c are the sides of triangle ABC.

811. In ΔABC, if $\sin A \sin B \sin C + \cos A \cos B = 1$ then the value of $\cos^2 A + \sin^2 B + 2\sin^2 \frac{C}{2}$ is:

812. A letter is known to have either from "TATA NAGAR" or from "CALCUTTA". On the envelope, just two consecutive letters TA are visible. If the probability that the letter came from 'TATA NAGAR" is in the form $\frac{p}{q}$, where p and q are co-prime positive integers, then find the value of $(p+q)$.

813. Let (a, b) be the outcome of throwing a pair of fair dice. If the probability for which $\lim\limits_{x\to 0}\frac{\ln((\cos x)^a)}{x^b}$ exist and is finite can be expressed as $\frac{p}{q}$, where p and q are co-prime positive integers, then find the value of $(p+q)$.

814. If the function $f(x)=2x^3 \quad (8 \quad a)x^2+\left(a^2+\frac{16}{9}\right)x-12$ has a local minima at some $x\in R^-$, then find the number of integers in the range of a.

815. Let the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ contains the circle $(x-1)^2+y^2 \quad 1$ and has least area. If $a^2+b^2=2n$, then find the value of $n\in N$.

816. Consider a family of n children. Let two events A and B are defined as follows:

A : is the event that the family has both boys and girls

B : is the event that the family has atmost one girl

If the events A and B are independent, then find the value of n.

[**Note:** Probability that a randomly selected child is a boy or girl is same.]

817. If $x\int_0^x \sin(f(t))\,dt=(x+2)\int_0^x t\sin(f(t))\,dt$ where $x>0$, then find the value of $f'(x)\cot(f(x))+\frac{3}{1+x}$.

818. Mr. A either walks to school or take bus to school everyday. The probability that he takes a bus to school is 1/4. If he takes a bus, the probability that he will be late is 2/3. If he walks to school, the probability that he will be late is 1/3. The probability that Mr. A will be on time for at least one out of two consecutive days is $\frac{p}{q}$, where p and q are co-prime, find the value of $(q-p)$.

819. The contents of three urns are 1 white, 2 red, 3 green balls; 2 white, 1 red and 1 green balls; 4 white, 5 red and 3 green balls. Two ball are drawn from an urn chosen at random and are found to be one white and one green. If the probability that the balls so drawn came from the third urn can be expressed as $\frac{a}{b}$, where a and b are co-prime positive integers, find $(a + b)$.

820. Find the value of the definite integral

$$\left(\frac{1}{\pi}\int_0^{\frac{\pi}{2}} \frac{\cos^4 x+\sin x\cos^3 x+\sin^2 x\cos^2 x+\sin^3 x\cos x}{\sin^4 x+\cos^4 x+2\sin x\cos^3 x+2\sin^2 x\cos^2 x+2\sin^3 x\cos x}\,dx\right)^{-1}$$

821. If $a^{\log_5 11}=25$ and $b^{\log_{11} 25}=\sqrt{11}$, then last digit of $N=a^{(\log_5 11)^2}+b^{(\log_{11} 25)^2}$ is equal to:

822. Let $x=(\text{antilog}_2 3)\cdot\log_3 2$, $y=\log_2(\log_3(\log_2 512))$ and $z=\log_5 3\ \log_7 5\cdot\log_2 7$, then xyz is equal to:

823. Let $\left|\log_{\sqrt{2}} 30-\left|\log_2 9+\log_4 9-\left|\log_{1/2} 5\right|\right|\right|=x$. Then the smallest integer greater than or equal to x, is:

824. Let $\sqrt{x}-\frac{1}{\sqrt{x}}=3$ and $x^3+\frac{1}{x^3}=k$, then characteristic of k with base 10 is:

825. If x_1 and x_2 are the solutions of the equation $5^{(\log_5 x)^2}+x^{\log_5 x}=1250$, then $x_1\cdot x_2$ is equal to.

826. Number of value(s) of x satisfying $\log_{(x^2+2)}(5+\sqrt{x})+\log_{(2+\sqrt{x})}(5+x^2)=0$ is/are:

827. Number of real values of x, such that $\log_{(x^2+2x+5)}(\log_{(2x^2+2x+3)}(x^2-2x))=0$ is:

828. Find the value of $16(\sin^2 18°+\sin^2 36°+\sin^2 54°+\sin^2 72°)$.

829. In ΔABC, if $\tan^2 A+\tan^2 B+\tan^2 C=\tan A\tan B+\tan B\tan C+\tan C\tan A$ and perimeter = 6, then square of the area of the triangle is:

830. Let $\alpha>1$ is a root of the equation $\frac{1-2(\log_{27} x^2)^2}{\log_{27} x-2(\log_{27} x)^2}=1$, then α is:

831. Let $0<\theta<\frac{\pi}{2}$, such that $\frac{\sin^2 2\theta+4\sin^4\theta-4\sin^2\theta\cos^2\theta}{4-4\sin^2\theta-\sin^2 2\theta}=\frac{7-4\sqrt{3}}{7+4\sqrt{3}}$, then $\theta=\frac{\pi}{n}$ where n is:

832. Let $x=\frac{4\sin 80°\sin 65°\cos 55°}{\sin 20°+\sin 50°+\sin 70°}$, then x is:

833. If $(\sin 25°+\sqrt{3}\cos 85°+\sin 85°)^2=a+b\cos 50°$, then $(a+b)=$

834. Let $0 \le \theta \le 2\pi$ and $x = |\cos\theta + 1| + |\cos\theta - 1| + |\cos\theta - 2| + |\cos\theta - 3|$, then product of the maximum and minimum values of x is:

835. Number of real values of λ such that $(\lambda^2 - 4\lambda + 3)x^2 + (\lambda^2 - 5\lambda + 6)x + (\lambda^2 - 9) = 0$ has more than 2 roots is:

836. If $9^{1+\log x} = 3^{1+\log x} + 210$ (where base of log is 3) then integral value of x is:

837. If $x^3 - x^2 + 3x + 5 = 0$ and $ax^2 + bx + 5 = 0$ have two common roots, then $|a + b| =$

838. Let x and y are real numbers satisfying $x^2 + y^2 = 4$ then find the number of integers in the range of $(x^2 - xy + y^2)$.

839. Let $f(x) = x^4 - 8x^3 + 18x^2 - 6x + 1 - 2\sqrt{3}$, then $f\left(x = \cot\dfrac{\pi}{12}\right)$ is equal to:

840. Let $\lambda = \left(\dfrac{\cos 65° + \sqrt{3}\cos 85° + \sin 85°}{\sin 65°}\right)^2$, then $\lambda =$

841. If sum of the roots of the equation $2\ln(4^x - 2) = \ln 8 + \ln\left(4^x - \dfrac{31}{8}\right)$ lies between two consecutive natural numbers a and b, then find the value of $(a + b)$.

842. Find the number of integral values of x satisfying the equation:

$$|\log_2^2 x - 5\log_2 x + 6| = |\log_2^2 x - 7\log_2 x + 12| - |2\log_2 x - 6|.$$

843. If $\left(\dfrac{1}{4}\cos 36° \sin 54°\right)^2 - (\sin 12° \sin 36° \sin 48° \sin 72°)^2 = \dfrac{\sqrt{a} - b}{c}$ where a, c are relatively prime numbers, then find the value of $(a + b + c)$.

844. Let $S_n = \displaystyle\sum_{r=1}^{n} \frac{6r + 9}{(r+1)^2(r+2)^2}$. If $S_\infty = \dfrac{p}{q}$ where $p, q \in N$, then find the least value of $|p - q|$.

845. Let $f(x) = x^2 - 2px + 3p^2 - 5$ and $g(x) = -x^2 + 2px + 2p - 3q$, $p, q \in R$. If $f(x)$ and $g(x)$ do not intersects at two distinct points $\forall\ p \in R$, then find the least value of q.

846. If $M = (\cos^2\theta - 2\cos\theta)\sec^2\phi + 9\operatorname{cosec}^2\phi + 5\sec^2\phi$ where $\theta \in [0, \pi]$ and $\phi \in \left(0, \dfrac{\pi}{2}\right)$, then find the least value of M.

847. If $8\alpha^3 + \beta^3 - \gamma^3 + 6\alpha\beta\gamma = 0$ and $\alpha^2 + 3\gamma = 2\beta$ where $\alpha, \beta, \gamma \in R$ and $\beta + \gamma \ne 0$, then find the largest integral value of γ.

848. Let 3, $7 - b$ and $\dfrac{-3a^2 + 10}{a + 2}$ be three natural numbers, which are first three terms (in order) of an A.P.. If $(a, b) \in I$, then find the number of possible such arithmetic progressions.

849. If $\log 4, \log(2^x+2), \log(2^{x+2}+1)$ are 3 consecutive terms of an arithmetic progression then x lies between 2 consecutive natural numbers p and $q(p<q)$ find the value of $(p+q)$.

850. If $0<\alpha<30°$, then find the value of $\log_2\left(\dfrac{8\cos(30°-2\alpha)-\dfrac{8}{\cot\alpha+\cot(30°-\alpha)}}{\sqrt{3}}\right)$.

851. Let S be a circle of radius 1 with centre at A. Two circles with radius r and R and centre at C and D are externally tangent to each other and internally tangent to S. If $\angle DAC=120°$, find the value of $Rr+3R+3r$.

852. If the sum of all solutions of the equation $2\sin 2\theta\cos 2\theta+\cos^2\theta=\dfrac{1-\cos 2\theta}{2}$ in $[0,\pi]$ is $\dfrac{a\pi}{b}$ where a and b are co-prime, then find the value of $(a+b)$.

853. In ΔABC, circumradius is 3 and inradius is 1.5 units. If the value of $a\cot^2 A+b^2\cot^3 B+c^3\cot^4 C$ is $m\sqrt{n}$ where m and n are prime numbers, then find the value of $\left(\dfrac{m-1}{n}\right)$.

854. Let $\log_2\left(-2+\sum_{r=1}^{100} r\cdot 2^r\right)=a+\log_c b$ where a, b, c are integers and $a>b>c>0$. Then find the value of $(a+b+c)$.

855. Let ABC be a triangle with $\angle A=45°$. If P be a point of contact of the inscribed circle of ΔABC on side BC such that $PB=3$ and $PC=5$, then find the value of $\dfrac{\Delta R}{2+\sqrt{2}}$ (where Δ denotes area of triangle and R is the circumradius of ΔABC respectively.)

856. Two parallel chords of a circle S have length 10 and 14 and are 6 units apart. If a regular polygon of 12 sides is inscribed in a circle S, then find the area of regular polygon.

857. The extremities of a diagonal of a rectangle are (0, 0) and (4, 4). The locus of the extremities of the other diagonal is $x^2+y^2-\lambda_1 x-\lambda_2 y=0$, then find the value of $(\lambda_1+\lambda_2)$.

858. Let $\sin^2\theta$ and $\tan^2\theta$ are roots of the quadratic equation $ax^2+bx+c=0$ and $\dfrac{b^2-c^2}{ac}=\lambda$. If solution set of the inequality $\log_\lambda(8\sin x)<1$ in $(0,2\pi)$ is $(\alpha,\beta)\cup(\gamma,\delta)$, then find the value of $\left(\dfrac{\gamma}{\beta}+\dfrac{\alpha}{\delta}\right)$.

859. An incident ray is reflected by the line mirror $y-1$ at the point $(2, 1)$ If the reflected ray touches the circle $x^2+y^2=1$, then slope of the reflected ray is $\frac{m}{n}$ (where m and n are co-prime integers), then $(m+n)$ is equal to:

860. Consider two circles of radii r_1 and r_2 passing through vertex A of ΔABC and touching side BC at points B and C respectively. If $a=5$ and $\angle A=30°$, then $\sqrt{r_1 r_2}$ is equal to:

861. A line with positive rational slope, which passes through the point $(6, 0)$ and at a distance of 5 units from the point $(1, 3)$ is equal to $\frac{p}{q}$ (where p, q are co-prime), then find the value of $(p+q)$.

862. If the point $M(h, k)$ lie on the line $2x+3y=5$ such that $|MA-MB|$ is maximum where $A(2, 3)$ and $B(1, 2)$, then find the value of $(3h+2k)$.

863. Let $f(x)=x^2+3x+1$ and $g(x)=x+1$. If $f(x)+\lambda g(x)>10\ \forall x \in R$ then find the sum of all possible positive integral values of λ.

864. Find the number of three digit numbers which can be formed using the digits 0, 1, 2, 2, 3, 3, 3.

865. Let (x, y) be satisfy the curve $x^2+y^2-6x-8y+21=0$ and m_1 and m_2 are minimum and maximum values of $\left(\frac{x}{y}\right)$. If $m_1^2+m_2^2=\frac{p}{q}$ where $p, q \in N$, then find the least value of $(p+q)$.

866. Let maximum value of the expression $y=|k-3|\cos 2x+|t-4|\sin 2x+3$ where $0 \le k \le 6$ and $1<t \le 7$ is equal to 6 then find the minimum value of (k^2+t^2)

867. If sum of the roots of the equation

$$\sqrt{2}\sin\left(x+\frac{\pi}{4}\right)(\sec^2 x-(1+\sqrt{3})\tan x+2+\sqrt{3})=\sqrt{9+18\sin x\cos x} \text{ in } [0, 2\pi] \text{ is } \frac{p}{q}\pi$$

where $p, q \in N$, then find the least value of $|p-q|$

868. Let three positive numbers a, b, c (in order) be in HP such that $a+c=8$. If $<t_n>$ is a geometric progression with common ratio 3 where $t_1=a-\frac{b}{2}$, $t_2=\frac{b}{2}$ and $t_3=c-\frac{b}{2}$ then find the value of $t_7\left(\frac{2}{3}\right)^6$.

869. In ΔABC, let $|a-5|+(7\cos B-3)^2+\log_3(1+|c-7|)=0$. A circle drawn with the altitude AD as diameter which meets the sides AB and AC at E and F respectively. If $(BE)(CF)=\frac{p}{q\sqrt{11}}$ where $p, q \in N$, then find the least value of $(p+q)$.

870. Let points A and C be lying on straight lines $4y = 3x$ and $x = 0$ respectively. If point $(6, 2)$ is lying on the straight line AC and rhombus $OABC$ is completed where O is the origin and B lies in first quadrant, then find the area of the rhombus (in sq. units).

871. A straight line L with the slope $\frac{3}{4}$ touches a circle whose radius is 5 units and centre lies on the x-axis. If the length of this tangent from a point on the x-axis is $\frac{p}{q}$ where $p, q \in N$, then find the least value of $(p + q)$.

872. Let ABC be a triangle with $\angle B = 45°$ and $c = 5$. A circle with center A and radius AB meets BC at D. If altitude from vertex A to the side BC meet the circle at E then area of ΔBED is $\frac{p}{2}(\sqrt{q} - 1)$. Find the value of $(p + q)$.

873. Let $A = \{x \; x^3 + x^2 \quad px + q = 0, p, q, \in R\}$ and $B - \{x | x^2 \quad qx + 2 = 0, q \in R\}$ be the sets. If $n(A \cap B) = 2$ and $x_0 \in (A - B)$, then find the value of $|p - q + x_0|$.
[**Note:** $n(P \cap Q)$ denotes number of common elements in set P and set Q and $a \in (P \quad Q)$ denotes elements 'a' lies in set P not in set Q.]

874. Let there are 6 shirts of different colours and 6 trousers of same colours as that of shirts. If the number of ways in which these can be put on by 5 men such that no men wear the shirt and the trouser of the same colour is $(k)6!$, then find the value of k.

875. Let S be infinite sum of the series $2 + 3\cos x + 4\cos^2 x + 5\cos^3 x + \ldots\ldots\ldots\ldots \infty$, where x satisfies the equation $|5\cos x + 4| + |5\cos x - 2| = 6$. If the least value of S is equal to $\left(\frac{a}{b}\right)$ where a and b are co-prime numbers, then find the value of $(a + b)$.

876. If the coefficient of x^8 in the expansion of $(2 + 9x^3 + 6x^4 + x^5)^{10}$ is $5 \cdot 2^p \quad 3^q$ where $p, q \in N$, then find the value of $(p + q)$.

877. Consider, $f(x) - \frac{|x - 4|}{|x| + 1}$. If sum of all distinct possible values of $\sin^{-1}(\sin[f(x)])$ is $a\pi + b$ then find the absolute value of $(a + b)$.
[**Note:** $[z]$ denotes greatest integer function less than or equal to z.]

878. Find the number of integral values of k for which $e^{\lambda^2 - 2\lambda + 1 + \ln 3}$ and $e^{-(\lambda^2 - 2\lambda + 1) + \ln 2}$, where $\lambda \in R - \{1\}$ are the roots of the equation $x^2 - (3k + 1)x + 3k^2 - k + 2 = 0$.

879. Let $f : R^+ \to (-7, \infty)$ be a function defined by $f(x) = \frac{x^3 - 7}{x^2 + 1}$ and g be the inverse of f such that $\frac{1}{\alpha} g\left(\frac{1}{\alpha^2 + 1}\right) = 1$ for some α. If $k = \frac{g(\alpha + 1)}{g(g(\alpha))}$ then find the value of k.

880. If the number of ways in which 5 Apples, 5 Bananas, 5 Chickoos and 5 Oranges (fruits of the same species are alike) can be distributed equally among five persons so that exactly 2 of them get all 4 identical fruits and each of the remaining persons gets exactly 2 kind of fruits, is N then find the sum of the digits in N.

881. Consider $f(x)=\{[x]+||x-1|-2|\}$. Find the number of solution(s) of the equation $3f(f(x))-1=0$ in $[-2, 4]$.

[Note: $[y]$ and $\{y\}$ denotes greatest integer function less than or equal to y and fraction part function of y respectively.]

882. Let $f(x)=\begin{cases}\dfrac{ax^3+bx^2+cx+d}{x}, & x\neq 0\\ 2, & x=0\end{cases}$ be a continuous function where a, b, c, d are in arithmetic progression. Then find the number of points where $|f(|x|)|$ is non derivable.

883. The least integral value of α for which the function $f(x)=\begin{cases}x^{\alpha}\left(\dfrac{e^{1/x}-e^{-1/x}}{e^{1/x}+e^{-1/x}}\right), & x\neq 0\\ 0 & x=0\end{cases}$

is differentiable at $x=0$ is:

884. Let $L=\prod_{n=3}^{\infty}\left(1-\dfrac{4}{n^2}\right)$, $M=\prod_{n=2}^{\infty}\left(\dfrac{n^3-1}{n^3+1}\right)$ and $N=\prod_{n=1}^{\infty}\dfrac{(1+n^{-1})^2}{1+2n^{-1}}$, then find the value of $(L^{-1}+M^{-1}+N^{-1})$.

885. Let $f:R\to R, f(x)=\cos^{-1}\left(\dfrac{1-x^2}{1+x^2}\right)+\sin^{-1}\left(\dfrac{2x}{1+x^2}\right)$.

If the value of $f(\sqrt{3})+f(-\ln 2)+f(1)+f(\ln 3)$ is equal to $k\pi (k\in W)$, then find the value of k.

886. Let $P=\{x|x^2+(n-1)x-2(n+1)=0\}$ and $Q=\{(n-1)x^2+nx+1=0\}$. Then find the number of values of n such that $P\cup Q$ has exactly 3 distinct elements (where x is a real number).

887. Let $f:R\to R$ be defined as $f(x)=(2x-3\pi)^3+\dfrac{4x}{3}+\cos x$ and $g=f^{-1}$, then find the value of $7g'(2\pi)+3g''(2\pi)$.

888. Let $k(x)$ be a continuous function satisfying the equation $\int_0^{x^3} k(t)\,dt=x^{1+x^2}$, find the value of $3k(1)$.

889. Let α, β are the roots of the equation $ax^2+bx+c=0$ where $\beta=4\alpha\ (\alpha>0)$. If $3a=2(c-b)$ and $S=\sum_{r=0}^{\infty}\beta(\alpha^r)$, then find the value of $3S$.

890. Let $L = \lim_{n\to\infty} \frac{1}{n^3}\sum_{k=1}^{n} k^2 e^{\frac{k}{n}}$, then find the value of $e - L$.

891. For some positive numbers a and b, if

$$\frac{\cos 3x}{\sin 5x} - \frac{\sin 3x}{\cos 5x} = a(\sin(2x) + \cos(2x)\cot(bx)),\ x \in \left(0, \frac{\pi}{20}\right)$$

Then find the value of $\frac{b}{a}$.

892. Let f be a continuous and even function such that $\int_0^a f(x)\,dx = 10$. If $g(x)$ is a continuous positive function such that $g(x)g(-x) = 1$ and $\int_0^a g(x)\,dx = 5$, then find the value of $\int_{-a}^{a} \frac{f(x)}{1+g(x)}\,dx$.

893. If $\left(x - 2 + \frac{1}{x}\right)^{30} = n_0 x^{30} + n_1 x^{29} + \ldots\ldots + n_{29}x + n_{30} + n_{31}x^{-1} + \ldots\ldots + n_{60}x^{-30}$ and $C = n_0 + n_1 + n_2 + \ldots\ldots + n_{60}$. Find the value of $(a + b)$ if $C - n_{30} = -\binom{a}{b}$.

[**Note:** $\binom{n}{r}$ denotes ${}^nC_r$.]

894. Let $f(1) + g(1) = 9e$; $f(x) = -x^2 g'(x)$; $g(x) = -x^2 f'(x)$. If $\int_1^4 \frac{f(x) + g(x)}{x^2}\,dx = k\left(e - e^{\frac{1}{4}}\right)$, then find the value of k.

895. If $f(x) = \begin{cases} (x+1)(x+2), & \text{if } x > 0 \\ a\sin x + b\cos x, & \text{if } x \le 0 \end{cases}$ is differentiable at 0. Find the value of $a - b$.

896. Let $f(x) = x^2 - 2px + p^2 - 1$, where $p \in R - \{-1, 1\}$.

If α and β are distinct real roots of the equation $f(x) = 0$ such that $\left|\frac{\alpha^2 + \beta^2 + 3\alpha\beta}{\alpha\beta}\right| \le 5$, then set of values of $p \in [a, b]$. The value of $[2(a^2 + b^2)]$ is:

[**Note:** $[k]$ denotes greatest integer less than or equal to k.]

897. Let A_r, $r = 1, 2, \ldots.29$ be arithmetic means between 303 and -57 where $A_r > A_{r+1}\ \forall$ $r = 1, 2, \ldots.28$. If S be the sum of these means, then the value of $\left[\frac{S}{(A_{14} - 12)|A_r|_{\min}}\right]$.

[**Note:** $[k]$ denotes greatest integer less than or equal to k and $|A_r|_{\min}$, denotes the minimum value of $|A_r|$.]

898. If $I_r = \int_a^{\pi+a} \left| \frac{1}{r}\sin x + \frac{1}{r+1}\cos x \right| dx$ where $a \in R$ and $r \in N$, then $\lim_{n\to\infty} \sum_{r=1}^{n} \left(I_r^2 - \frac{8}{(r+1)^2} \right)$ equals:

899. If a circle of radius 2 unit touches the y-axis at the origin, 'O' and intersects the lines $y=(2-\sqrt{3})x$ and $y=-(2+\sqrt{3})x$ in the I and IV quadrants at A and B respectively, then area of ΔAOB (in square units) is:

900. If sum of all the solutions of the equation $(4\tan x + \tan^2 x + 1) = 2\sqrt{2}\sin\left(x+\frac{\pi}{4}\right)(1+\tan^2 x)$ in $[0, \pi]$ is $\left(\frac{p\pi}{q}\right)$ where p, q are relatively prime number, then find the value of $(p+q)$.

901. Let f be a differentiable function defined in $[0, 1]$ such that $f(f(x)) = x$ and $f(0)=1$. If the value of $\int_0^1 (x-f(x))^{2018} dx = \frac{p}{q}$ where p and q are relatively prime then find the value of $(p+q)$.

902. If $\int_0^{1/2} \tan^{-1}\left(\frac{1}{2}\left(\sqrt{\frac{1+x}{x}} - \sqrt{\frac{x}{1+x}}\right)\right) dx = p\ln(2+\sqrt{3}) - \frac{\pi}{q}$, then pq is equal to:

903. If the functions $g(x) = x^2 + ax + b$ and $h(x) = cx - x^2$ intersect and have the same tangent line at the point (1, 0), then find the value of $(b+c-a)$.

904. Let $f(x) = \begin{cases} (x-a)^2 + b, & x \geq k \\ [2x], & 0 < x < k \\ \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right), & x \leq 0 \end{cases}$

If $f(x)$ is non-derivable at exactly 6 points, then the value of $a \times b \times k$ is:

[**Note:** $[k]$ denotes greatest integer less than or equal to k and $k \in N$.]

905. Let $f(x) = x^2 + 4x + a$ and $g(x) = x^2 + 6x + 2a$ be two functions and another function $h(x) = \frac{f(x)}{g(x)}$ $\forall\, g(x) \neq 0$. Then find the sum of all integral values of a for which $y = h(x)$ is an onto function.

906. A curve passes through (2, 0) and the slope of tangent at any point (x, y) is $x^2 - 2x$ $\forall\, x \in R$. The point of minimum ordinate on the curve where $x > 0$ is (a, b), then find the value of $(a+6b)$.

907. Find the number of polynomials $P(x)$ with integer coefficients such that $P'(x) > 0$ and $(P(x))^2 + 4 \leq 4P(x^2)$ for all x.

908. Suppose f and g are differentiable functions such that $xg(f(x))f'(g(x))g'(x) = f(g(x))g'(f(x))f'(x)$ for all real x. Also f is non negative and g is positive. If $\int_0^a f(g(x))\,dx = \frac{1}{2} - \frac{e^{-2a}}{2}$ for all reals a and $g(f(0)) = 1$ then the value of $g(f(4))$ is equal to $e^{-\lambda}$ where $\lambda \in N$, find the value of λ.

909. Let $A = \begin{bmatrix} x^3+1 & x+5 & 3x+2 \\ y+1 & -6x^2+2 & z-1 \\ 2 & 3 & 9x+6 \end{bmatrix}$ and $|A| = 3$

If $f(x) = \text{tr.}(B^{-1})$ and $B = adj(A)$, then global maximum value of $f(x)$ in $x \in [0, 6]$ is:

910. If the value of $\lim_{n\to\infty} \sum_{k=0}^{n-1} \frac{k}{n}\left[\sqrt[m]{\frac{k+1}{n}} - \sqrt[m]{\frac{k}{n}}\right]$, $(m \in N)$ is equal to $\frac{1}{10}$ then find the value of m.

911. If f and g are two functions such that $2f(1) = g(2) = 4$ and $2f(9) = g(10) = 20$ and $\int_0^2 (x^2 g(f(x^3+1))f'(x^3+1) - 3x^2)\,dx = 0$, then find the value of $\int_4^{20} g^{-1}(x)\,dx$.

912. Let $A = \begin{bmatrix} a & x & p \\ y & q & b \\ r & c & z \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ where $a, b, c, x, y, z, p, q, r$ are natural numbers.

If $\text{tr.}(AB + AB^3 + AB^5 + \ldots + AB^{19}) = 210$, then find number of ordered triplets (p, q, r).

[**Note:** $\text{tr.}(P)$ denotes the trace of matrix P.]

913. Let f be monic cubic polynomial such that $f(1) = 1^4 - 1$, $f(2) = 2^4 - 2$ and $f(3) = 3^4 - 3$. If $f(4) = N$, then find the number of prime factors of N.

914. Let the circle $S: x^2 + y^2 \quad ax - by + c = 0$ intersects the pair of straight lines $xy - 4x - 3y + 12 = 0$ orthogonally and the circle lies in the first quadrant. If S touches the circle $S_1: (x-3)^2 + y^2 = r^2$, $r \in N$, then find the sum of all possible values of r.

915. If $\int_{-n}^{n} \frac{3\{x\}+1}{\{3x\}+1}\,dx = 6\ln(4e^2)$, then find the value of n.

[**Note:** $\{k\}$ denotes the fractional part function of k.]

916. Let $f: R \to R$ be a function defined by $f(x) = x^3 + 2x^2 + 3x + 2$ and g be the inverse function of f. If $\left.\frac{d}{dx}(g(g(g(x))))\right|_{x=24} = \frac{p}{q}$, where p, q are co-prime, then find the value of $(p+q)$.

917. If sum of reciprocal of radii of all the circles which touches all the lines represented by the equation $x^2y - 2xy^2 \quad 4xy = 0$ is $k\left(1+\cos\dfrac{2\pi}{p}\right)$, where $k,\ p \in N$, then find the value of $(k+p)$.

918. Let a, b, c and d be four roots of the equation $x^4 - 10x^3 + 37x^2 - 60x + 32 = 0$.
If $\dfrac{1}{a^2-5a+10} + \dfrac{1}{b^2-5b+10} + \dfrac{1}{c^2-5c+10} + \dfrac{1}{d^2-5d+10} = \dfrac{p}{q}$, where p and q are co-prime numbers, then find the value of $(p+q)$.

919. Let $I = \displaystyle\int \frac{(e^x-1)(\sin x - \cos x) + x\cos x}{\sin^2 x + (e^x - 1 - x)^2}\,dx - \tan^{-1}(f(x)) + C$, where '$C$' is the constant of integration and $\lim\limits_{x\to 0} f(x) = 0$. If $\lim\limits_{x\to 0}\dfrac{f(x)}{x} = \dfrac{p}{q}$, where $p, q \in N$ then find the least value of $(p+q)$.

920. Let $\vec{v}_1 = \sin\theta\,\hat{i} - 2\hat{j} + a\hat{k}$, $\vec{v}_2 = 2\hat{i} + \cos\theta\,\hat{j} - \hat{k}$ and $\vec{v}_3 = \hat{i} + \hat{k}$ be three vectors such that the value of 'a' is maximum for $\vec{v}_1 \cdot \vec{v}_2 = 0$ where $\theta \in [0, \pi]$. If for these values of 'a' and 'θ'

$$\sqrt{\begin{vmatrix} \vec{v}_1\cdot\vec{v}_1 & \vec{v}_1\cdot\vec{v}_2 & \vec{v}_1\cdot\vec{v}_3 \\ \vec{v}_2\cdot\vec{v}_1 & \vec{v}_2\cdot\vec{v}_2 & \vec{v}_2\cdot\vec{v}_3 \\ \vec{v}_3\cdot\vec{v}_1 & \vec{v}_3\cdot\vec{v}_2 & \vec{v}_3\cdot\vec{v}_3 \end{vmatrix}} = \frac{p}{q}$$ where $p, q \in N$, then find least value of $|p - 4q|$.

921. If A and B are square matrices of order 3 such that $2(A+B) = A^T + B^T + 3I$ and $AA^T = 4I$, then find the value of det. $(12A^{-1} - BA^T + I)$.

[**Note:** I is an identity matrix of order 3 and P^T denotes the transpose matrix of matrix P.]

922. In ΔABC, let a and b be minimum and maximum values of the function $f(x) = \sin^{-1}x + \cos^{-1}x + \tan^{-1}x$ respectively and $\cos A = \dfrac{3}{\sqrt{10}}$. If $r = \dfrac{p\pi}{q(q+\sqrt{10})}$ where p, q are co-prime numbers, then find the value of $(p+q)$.

[**Note:** Symbols used have usual meaning in ΔABC]

923. Let $f: R - \{1\} \to R - \{1\}$ be a function satisfying the differential equation $2x(y+x)dx - x^2(dx+dy) = (x+y)^2dx$ with $f(2) = 2$. If area enclosed by $y = f(x)$ and x-axis from $x = 2$ to $x = 3$ is $(a + \ln b)$ where $a, b \in N$, then find the value of $(a+b)$.

924. If the area of the region $\left\{(x, y) \in R^2 : y^2 \geq 4x, |y| \leq \frac{x}{2}+2\right\}$ is Δ, then find the value of $[\Delta]$.

[**Note:** $[S]$ denotes greatest integer less than or equal to S.]

925. Let $f:[-\alpha, \beta] \to [-4, 2]$ be a continuous decreasing function such that $f(0)=0$ and $\alpha, \beta > 0$. If area enclosed by $f(x)$ and x-axis from $x=-\alpha$ to 0 and area enclosed by $f(x)$ and x-axis from $x=0$ to $x=\beta$ are 1 sq. units and 3 sq. units respectively, then area enclosed by $f^{-1}(x)$ and x-axis from $x=-4$ to $x=2$ is $(p+q\beta+r\alpha)$. Find the value of $(p+q+r)$.

926. Let an octahedral dice (8 faces) marked the numbers 1 to 8 on its faces. On throwing two such dice three events A, B, C are defined as:

A : getting a sum 10 or more.

B : getting a sum divisible by 2.

C : getting a sum divisible by 3.

If $P\left(\frac{C-B}{A}\right)=\frac{a}{b}$ where $a, b \in N$, then find the least value of $|a-b|$.

927. Let A and B be two sets of complex numbers such that $A=\{z \,|\, |z|^3-2|z|^2+3i|z|-6i=0\}$ and $B=\{z \,|\, |z^2-4|+|z^2+4| \leq 4|z|\}$. Find the area of the figure enclosed by joining the points lying in $A \cap B$.

928. Consider $A=\begin{bmatrix}1 & 2 & 4\\ 0 & 1 & 3\\ 0 & 1 & 1\end{bmatrix}$ and $B=\begin{bmatrix}2 & 4 & 5\\ 4 & 0 & 0\\ 5 & 0 & 0\end{bmatrix}$. If $C=[c_{ij}]_3=A^{20}+B$ and $c_{22}c_{33}-c_{23}c_{32}=2^m$, then find the value of m.

929. Let $f(x)=\begin{cases} e^x\left(\dfrac{e^{nx}-1}{e^x-1}+x^3\right), & x \neq 0 \\ k, & x=0 \end{cases}$

(where $n \in N$) be a differentiable function and if $f'''(0)=1302$, then find the value of $(k+n)$.

930. Let E_1, E_2, E_3 be three independent events associated with a random experiment such that $3P(E_1 \cap \bar{E}_2 \cap \bar{E}_3)=P(\bar{E}_1 \cap E_2 \cap \bar{E}_3)=9P(\bar{E}_1 \cap \bar{E}_2 \cap E_3)=3-3P(E_1 \cup E_2 \cup E_3)$, where $P(E_1), P(E_2), P(E_3) \neq 1$ and $P(A)$ denotes probability of event A.

If absolute value of $\begin{vmatrix} P(E_1) & P(E_2) & P(E_3)\\ P(E_2) & P(E_3) & P(E_1)\\ P(E_3) & P(E_1) & P(E_2)\end{vmatrix}=\frac{a}{b}$, where $a, b \in N$, then find the least value of $(a+b)$.

931. If tangent drawn to the parabola $y^2 = -ax(a > 0)$ from a point $A(1, 0)$ which also touches the hyperbola $\frac{x^2}{4} - \frac{y^2}{b^2} = 1$ at B such that $\angle ASB = 90°$, where S is the focus of the hyperbola then find the value of $(a + b^2)$.

932. If the value of $\sum_{r=2}^{\infty} \tan^{-1}\left(\frac{1}{r^2 - 5r + 7}\right)$ is equal to $\frac{a\pi}{b}$, where a and b are co-prime, then find the value of $(a + b)$.

933. Let $f(x)$ is a monic polynomial of degree $= 5$ such that $f(1) = 1, f(2) = 2, f(3) = 3, f(4) = 4$ and $f(5) = 5$. If $f(6) = 5! + \lambda$, then find the value of λ.

934. Find the number of integers not in the domain of $f(x) = \cos^{-1}\left(\frac{2 - x}{2x}\right)$.

935. Let $\lim_{x \to \frac{\pi}{2}} \left(2x \tan x - \frac{\pi}{\cos x}\right) = \lambda$, then find the value of $|\lambda|$.

936. Let $f : \left[\frac{-1}{2}, 0\right] \to B$ defined by $f(x) = \cos^{-1}(4x^2 + 3x)$ is onto, then the set B is $\left[\frac{\pi}{2}, \pi - \cos^{-1}\frac{a}{b}\right]$, where a and b are co-prime, then find the value of $(a + b)$.

937. Let $x, y, z \in R^+$ such that $x + y + z = 27$. If maximum value of $x^2 y^3 z^4$ is $\lambda \cdot 6^{10}$, then find the value of λ.

938. If the solution set of the equation $[\sin x] + [2\sin x] + [3\sin x] = 1$ in $x \in \left[0, \frac{\pi}{2}\right]$ is written as $\alpha \le x < \beta$, then the value of $\cos(\alpha + \beta)$ is $\frac{\sqrt{a}}{3} - \frac{1}{a}$ where $a \in N$, find a.

[Note: $[\cdot]$ denotes the greatest integer function.]

939. If $x = 4t^3 + 3$, $y = 4 + 3t^4$ and $\dfrac{\left(\frac{d^2x}{dy^2}\right)}{\left(\frac{dx}{dy}\right)^n}$ is a constant, then find the value of $\frac{4}{5} + \frac{4}{5n} + \frac{4}{5n^2} + \ldots\ldots$ upto infinity.

940. Let $f(x)$ be a derivable function satisfying $f(x + y) = f(x) + f(y)\ \forall\ x, y \in R$ and $f'(0) = 1$. If $A = \lim_{x \to 0} \frac{2^{f(\tan x)} - 2^{f(\sin x)}}{x^3}$, then find the value of e^{4A}.

941. Let $I = \int_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} \sqrt{\frac{1-x}{1+x}} \sin^{-1} x \, dx$. If $I = \frac{\pi}{M} - \sqrt{N}$, then find the value of $(M+N)$.

942. If tangent at a point P_1 (other than (0, 0)) on the curve $y^2 = ax^3$ meets the curve again at P_2. The tangent at P_2 meets the curve again at P_3 and so on, then find $\lim_{n\to\infty} \sum_{i=1}^{n} x_i$, where x_i's are abscissae of P_i with $x_1 = 3$.

943. If the greatest value of $\frac{x^2 - x + c}{x^2 + x + c}$ is $\frac{5}{3}$, then find the value of c

944. Let x_1, x_2, x_3, x_4 and x_5 be 5 positive numbers such that $x_1 + 2x_2 + 3x_3 + 4x_4 + 5x_5 = 15$ and $x_1 \cdot x_2^2 \cdot x_3^3 \cdot x_4^4 \cdot x_5^5 = 1$, then find the value of $x_1 + x_2 + x_3 + x_4 + x_5$.

945. Let $y = f(x)$ be a differentiable function such that $f(x) = \int_0^x \left(\frac{f(t)}{t} + \ln t \right) dt \; \forall x > 0$ and $f(1) = 0$. If $L = \lim_{x\to 1} \frac{f(x)}{\sin^2 \pi x}$, then find the value of $\left[\frac{1}{\pi L} \right]$.

[Note: Where [$\cdot$] denotes greatest integer function.]

946. If A is the area bounded by $x + 2y^2 = 0$ and $x + 3y^2 = 1$, then find the value of $3A$.

947. Let $f(x) = 6 - 3^{(\sqrt{3}\sin x - \cos x)}$ and $g(x) = \text{sgn}(x^2 + 2px + 4p + f(x))$; if $g(x)$ is continuous $\forall x \in R$, then find the sum of all possible integral values of p.

948. Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ be the vertices of a triangle such that algebraic sum of perpendicular distance from A, B and C to the variable line $ax + by + c = 0$ is always '0'. If $3a + 2b + c = 0$, then find the value of $\sum_{i=1}^{3} (x_i + y_i)$.

949. Let $f(x) = \frac{(x-1)(2x-215)}{(x-c)}$ be an onto-function, then find the greatest integral value of c.

950. Consider a curve passing through (1, 1) such that perpendicular distance of normal drawn at any point P on the curve from origin is equal to ordinate of the point P. If area enclosed by the curve is A, then find $[2A]$.

[Note: $[k]$ denotes greatest integer less than or equal to k.]

951. Let $f(x) = \begin{cases} \{x^2\}, & -1 \le x < 1 \\ |1-2x|, & 1 \le x < 2 \\ (1-x^2)\,\text{sgn}(x^2 - 3x - 4), & 2 \le x \le 4 \end{cases}$, where $\{x\}$ and sgn(x) denote fractional part function and signum function of x respectively.

If number of points where $f(x)$ is discontinuous in $[-1, 4]$ is m and number of points where $f(x)$ is non-derivable in $(-1, 4)$ is n, then find the value of $(m+n)$.

952. Let line L_1 be the reflection of a tangent to the parabola $(y-2)^2=4(x-1)$ drawn at P in the line $x=1$.
If area of the triangle formed by the line L_1, the tangent and straight line $y=2$ is 64 sq. units, then find the abscissa of point P.

953. If the set of values of x satisfying the equation $[2x]+[-2x]=\dfrac{\log_{10}(x^2-2x+2)-1}{|\log_{10}(x^2-2x+2)-1|}$ is:

$(a, b)-\{p_1, p_2, \ldots\ldots, p_n\}$, then find the value of $\left(a+b+\sum_{i=1}^{n} p_i\right)$.

[Note: $[k]$ denotes greatest integer function less than or equal to k.]

954. Let a line parallel to z-axis passing through a point $P(3, 4, a)$ intersects the plane $x-2y+2z=a^2+4a+1$ at Q where $a\in R$

If least area of ΔOPQ is equal to $\left(\dfrac{p}{q}\right)$ where p and q are co-prime numbers, then find the value of $(p+q)$.

955. If $\displaystyle\int_{-39}^{59}\frac{\sin(2(\{x\}+\{-x\}))}{e^{-\{x\}}}\left(\frac{\tan x-\tan[x]}{1+\tan x\tan[x]}+\sec^2\{x\}\right)dx=p\cdot e\cdot\sin^2 q$ where $p, q\in N$ and e is Napier's constant, then find the value of $(p+q)$.

[Note: $[k]$ and $\{k\}$ denotes greatest integer function less then or equal to k and fractional part function of k respectively.]

956. Let $z_1, z_2\in C$ and satisfy the equation $|z+1|^2+|z-1|^2+2|z|=6$.
If maximum value of $(2|z_1-2+|2z_2+1|)$ is equal to λ, then find the value of 4λ.

957. Consider $f(x)=x^4+ax^3+bx^2+cx+d$ If straight line $y=3x+2$ is tangent to the curve $y=f(x)$ at $P(1, f(1))$ which again intersects $f(x)$ at $Q(2, 8)$ and $f''(1)=0$ then find the value of $f(3)$.

958. On the coordinate plane, ellipse $C_1:\dfrac{x^2}{a_1^2}+\dfrac{y^2}{b_1^2}=1\ (a_1>b_1>0)$ and hyperbola $C_2:\dfrac{x^2}{a_2^2}-\dfrac{y^2}{b_2^2}=1\ (a_2, b_2>0)$ has the same focus points F_1, F_2. Point P is one of the intersection points of C_1 and C_2 and $PF_1\perp PF_2$. If e_1 is the eccentricity of C_1 and e_2 is the eccentricity of C_2. Then find the minimum value of $9e_1^2+e_2^2$

959. Let the domain of the function f be all the real numbers. It is known that for all x in this domain, $f(x+1)=2f(x)$. Also, for $x\in(0,1]$, $f(x)=x^2-x$. If $f(x)\geq\dfrac{-8}{9}$ for $x\in(-\infty, m]$

Find $3m$.

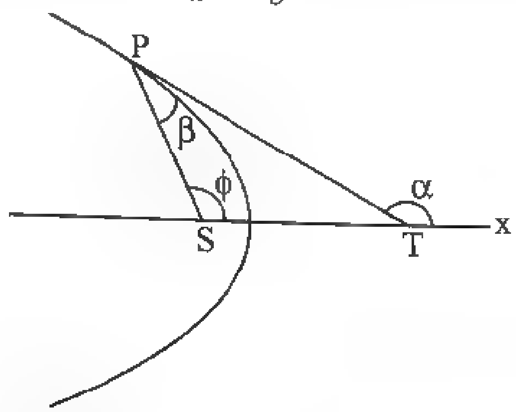
960. Let $f : A \to A$ where $A = \{1, 2, 3, 4, 5, 6, 7\}$, then number of functions f such that $f(f(f(x))) = x \ \forall\, x \in A$, is:

961. Let $f : A \to A$ where $A = \{1, 2, 3, 4, 5\}$, then number of functions f such that $f(f(x)) = x \ \forall\, x \in A$, is:

962. Let f be quadratic function such that: $f(x) = 0$ has 2 real solutions and $f(f(x)) = 0$ has 3 real solutions. What is the maximum number of solutions for $f(f(f(x))) = 0$?

963. Let a function f is defined as $f : \{1, 2, 3, 4, 5\} \to \{1, 2, 3, 4, 5\}$. If f satisfy $f(f(x)) = f(x), \forall\, x \in \{1, 2, 3, 4\}$ then find the number of such functions.

964. If the tangent at any point of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where $a > b$, makes an angle $\alpha = \frac{5\pi}{6}$



with the major axis and angle $\beta = \frac{\pi}{3}$ with the focal radius of the point of contact then find the eccentricity e of the ellipse.

965. $$\left|2\left(x^2 + \frac{1}{x^2}\right) + |1 - x^2|\right| = 4\left(\frac{3}{2} - 2^{x^2-3} - \frac{1}{2^{x^2+1}}\right)$$

If x_1 and x_2, where $x_1 < x_2$, are two values of x satisfying the equation above, find the value of $\int_{x_1-x_2}^{3x_2-x_1} \left\{\frac{x}{4}\right\}\left(1 + \left[\tan\left(\frac{\{x\}}{1+\{x\}}\right)\right]\right) dx$

[Note: $|\cdot|$ denotes the absolute value function,

$\{\cdot\}$ denotes the fraction part function,

$[\cdot]$ denotes the floor function]

966. Let $f(x) = (e^x - a)(3ax + 1)$. Number of possible values of a satisfying $f(x) \geq 0$ for $\forall\, x \in R$.

967. Let $f(x)$ be a thrice differentiable function $[a, b]$ and $\alpha < \alpha_1 < \alpha_2 < \alpha_3 < \alpha_4 < \alpha_5 < b$ and if $f(a) = f(b) = -2$, $f(\alpha_1) = f(\alpha_3) = 3$, $f(\alpha_2) = f(\alpha_4) = -3$, $f(\alpha_5) = -1$, then what is the minimum number of roots of the equation $f(x)f'''(x) + f'(x)f''(x) = 0$ for $x \in [a, b]$?

968. The number of real solutions of the equation $\sqrt{1+\cos 2x} = \sqrt{2}\sin^{-1}(\sin x)$ where $-\pi \le x \le \pi$.

969. Let $S = S_1 \cap S_2 \cap S_3$ where :

- $S_1 = \{z \mid z \in C, |z| < 4\}$
- $S_2 = \left\{z \mid z \in C, I_m\left(\dfrac{z-1+i\sqrt{3}}{1-i\sqrt{3}}\right) > 0\right\}$
- $S_3 = \{z \mid z \in C, R(z) > 0\}$

If the area of S can be expressed as $\dfrac{a}{b}\pi$, where a and b are positive integers that are relatively prime. Find $a + b$.

970. A regular heptadecagon $P_1P_2P_3.....P_{17}$ is inscribed in a unit circle. Find $\prod_{n=2}^{7} P_1P_n$.

971. A variable point P on an ellipse of eccentricity $e = \dfrac{1}{8}$, is joined to it's focii S_1 and S_2. Given that the locus of the incentre of the triangle ΔPS_1S_2 comes out to be a conic; evaluate its eccentricity e'. Now e' is of the form $\dfrac{\sqrt{a}}{b}$, where a and b are co-prime positive integers, find $a \times b$.

972. Let $H : x^2 + y^2 + 4xy + 8x + 8y + 8 = 0$ be a hyperbola. A line $L : x + y + 1 = 0$ intersects the hyperbola H at two distinct points. If radius of the circle which touches the hyperbola at the points where H meets the line L is R, then find the value of R^2.

973. $a^2 \cot 9° + b^2 \cot 27° + c^2 \cot 63° + d^2 \cot 81°$

Let a, b, c and d be real numbers satisfying $a + b + c + d = 5$. If the minimum value of the expression above is equal to $\dfrac{\sqrt{x}}{y}$, where x and y are co-prime positive integers, find $x + y$.

974. Given that $x \in R$, find the minimum value of $(3\sqrt{5-4\cos x} + \sqrt{13-12\sin x})^2$.

975. Let ABC is obtuse triangle ($\angle C \ne 3\angle A$) such that $\begin{cases} \dfrac{3BC - AB}{4BC} = \sin^2 A \\ \dfrac{1}{2}\cot\dfrac{A}{2} = \sin A + \sin B + \sin C \\ \cos^2 A + \cos^2 B + \cos^2 C = p \end{cases}$

If the value of $p = \dfrac{m}{n}$, where m and n are co-prime, then find the value of $(m + n)$.

976. Find the least value of the function $y = x^2 e^{-x} + 4 - \sqrt{4 - x^2}$.

977. Let f be a polynomial of degree 2018 such that $f(1)=1, f(2)=0, f(3)=-5, f(-4)=2$. If $f(x)$ is an even function then find the minimum number of points where $f''(x)=0$.

978. If $\lim_{t\to x}\dfrac{\int_x^t \sin^{-1}(nz)dz}{t^2-x^2}=f_n(x)$ then find the value of $\lim_{x\to 0}([f_2(x)]+[f_4(x)])$

[Note: $[k]$ denotes greatest integer function less than or equal to k]

979. If $\lim_{x\to 0}\left(\sum_{r=1}^{n}\cos\frac{r\pi}{2n}\right)\left(\sum_{r=1}^{n}\cos^2\frac{r\pi}{2n}\right)\left(\sum_{r=1}^{n}\cos^3\frac{r\pi}{2n}\right)\left(\sum_{r=1}^{n}\cos^4\frac{r\pi}{2n}\right)\left(e^{\frac{1}{n}}-1\right)^4=\frac{k}{\pi^2}$ then find the value of $\frac{1}{k}$.

980. If $S_1: x^2+y^2=4$ and $S_2: x^2+y^2-2ax-2by+2=0$ touches each other, then 4 times of radius of the circle S_2 is:

981. If term independent of x in the expansion of $\left(x^2-\frac{3}{x}\right)^6$ is $\lambda 3^5$, then find the value of λ.

982. If number of words which can be formed using all the letters of the word "MIMIXA" in which no two alike letters are together is (12 m), then find the value of m.

983. If $\alpha=2\beta=3\gamma$, then find the value of $\log_{\left(\frac{7}{6}\right)}\left(\dfrac{\alpha^2+\beta^2+\gamma^2}{\alpha\beta+\beta\gamma+\gamma\alpha}\right)$.

984. If $2[x+32]=3[x-64]$ and $y=\prod_{j=1}^{9}\sin\left(\frac{2j-1}{18}\right)\pi$, then find the value of $\left[\frac{1}{x}\right]+\left[\frac{1}{16y}\right]$.

[Note: Where $[k]$ denotes greatest integer function less than or equal to k.]

985. Let $f(x)$ be a continuous function satisfying $\begin{cases} f(x)=x^2-6x+8 \\ f(x)=f(x+a) \end{cases}$ for $-1\le x\le a-1$ where 'a' is a constant, then find the sum of all possible values of 'a'.

986. Let M be the greatest and m be the least value of $\sqrt{\sin^{-1}x}+\sqrt{\cos^{-1}x}$, then find the value of $(M/m)^4$.

987. If $\lim_{n\to\infty}\left(\frac{1}{\sqrt{n}\sqrt{n+1}}+\frac{1}{\sqrt{n}\sqrt{n+2}}+\ldots\ldots+\frac{1}{\sqrt{n}\sqrt{n+n}}\right)$ is equal to $a\sqrt{b}-\frac{c}{d}$, where a, b, c and d are positive integer, c and d are co-prime. Find the value of $(a^4+b^3+c^2+d)$.

988. In a triangle ABC, angle A and B and angle C are in arithmetic progression and $\sin A$, $\sin^2 B$ and $\sin C$ are also in arithmetic progression, where A, B and C are in degrees. Find the value of $|A+B|$.

989. Let $T(n)=\cos^2(30^\circ-n^\circ)-\cos(30^\circ-n^\circ)\cos(30^\circ+n^\circ)+\cos^2(30^\circ+n^\circ)$. Find the value of $4\sum_{n=1}^{30} nT(n)$.

990. $R-\{0\}\to R$ is a differentiable function such that:

$\int_1^{xy} f(t)\,dt = y\int_1^{x} f(t)\,dt + x\int_1^{y} f(t)\,dt \ \forall x, y, \in (R)-\{0\}$ and $f(1)-1$.

Function g is defined as:

$g(x)=-\left(e^{f(x^2)-1}+\left(e^{f(1/x^2)-1}\right)\right)$

If $I(P)-\int_P^{1/P} e^{g(x)}\,dx$ then, the value of $\lim_{P\to 0^+} I(P)=\frac{\sqrt{a}}{b}$ (a and $b\in R$). Find the value of $[a+b]$.

(You may use $\int_{-\infty}^{\infty} e^{-t^2}\,dt=\sqrt{\pi}$)

[**Note:** $[k]$ denotes greatest integer function less than or equal to k.]

991. Let $f:R\to R$ be a continuous function which satisfies $f(2x)=3f(x)\ \forall x\in R$. If $\int_0^1 f(x)\,dx-1$, then find the value of $\frac{1}{2}\int_1^2 f(x)\,dx$.

992. For the differential equation $\frac{dy}{dx}+iy=2\sin(x)$, with initial condition $y(0)=\frac{3}{2}$. If $y(\pi)=\frac{a}{b}+c\pi i$, for co-prime integers a and b, find abc.

[**Note:** $i=\sqrt{-1}$ denotes the imaginary unit.]

993. Given the function $f(x)=e^{2x}+(1-ax^2)ex-ax^2$ $(a\in R)$. If equation: $f(x)=0$ has three distinct roots, find the range of a.

994. Given that the graph of function $f(x)=(ax+\ln x+1)(x+\ln x+1)(a\in R)$ has at least 3 intersection points with the graph of function $g(x)=x^2$, find the range of a.

995. If complex number z satisfies $(z-\bar{z})^2=12\ z|^2-4$ then find the maximum value of $3\sqrt{3}\,\text{Re}(z)+8\,\text{Im}(z)$.

996. Let P be a 2×2 matrix such that $P\begin{bmatrix}1\\-1\end{bmatrix}=\begin{bmatrix}-1\\2\end{bmatrix}$ and $P^2\begin{bmatrix}1\\-1\end{bmatrix}=\begin{bmatrix}1\\0\end{bmatrix}$. If x_1 and x_2 are two values of x for which $|P-xI|=0$, where I is an identity matrix of order 2 then find the value of $x_1^2+x_2^2$.

997. Let f be a real valued derivable function such that $f(x)f(y)=f(x)y+xf(y), \forall x, y \in R$. If $f'(0)=2$ then find $\lim_{x\to 0}\left[\frac{f(x)}{\sin x}\right]$.

[**Note:** [ ] represents greatest integer function.]

998. Let circle $C_1: x^2+(y-4)^2=12$ intersects circle $C_2:(x-3)^2+y^2=13$ at A and B. A quadrilateral $ACBD$ is formed by tangents at A and B to both circles. Find the diameter of circumcircle of quadrilateral $ACBD$.

999. If p is a positive integer and f be a function defined for positive numbers and attains only positive values such that $f(xf(y))=x^p y^4$ then find p.

1000. Consider the set of complex number A, B, C and S defined as

$$A=\{z:||z+2|-|z-2||=2\}$$

$$B=\left\{z:\arg\left(\frac{z-1}{z}\right)=\frac{\pi}{2}\right\}$$

$$C=\{z:\arg(z-1)=\pi\}$$

$$S=\left\{z:\operatorname{Re}\left(\frac{z-1}{z+1}\right)=0\right\}$$

If $z_1, z_2, z_3 \in S$, then find the minimum value of $|z_1+z_2|^2+|z_2+z_3|^2+|z_3+z_1|^2$.

ANSWERS KEY

SINGLE CORRECT TYPE QUESTIONS

1.	(a)	2.	(c)	3.	(c)	4.	(b)	5.	(a)	6.	(d)	7.	(d)	8.	(d)
9.	(d)	10.	(c)	11.	(d)	12.	(c)	13.	(c)	14.	(a)	15.	(b)	16.	(d)
17.	(b)	18.	(c)	19.	(b)	20.	(b)	21.	(c)	22.	(d)	23.	(d)	24.	(c)
25.	(c)	26.	(c)	27.	(d)	28.	(c)	29.	(c)	30.	(c)	31.	(d)	32.	(a)
33.	(c)	34.	(a)	35.	(c)	36.	(d)	37.	(a)	38.	(a)	39.	(b)	40.	(b)
41.	(c)	42.	(b)	43.	(a)	44.	(b)	45.	(c)	46.	(b)	47.	(a)	48.	(c)
49.	(c)	50.	(b)	51.	(a)	52.	(c)	53.	(b)	54.	(a)	55.	(c)	56.	(c)
57.	(b)	58.	(d)	59.	(c)	60.	(c)	61.	(c)	62.	(a)	63.	(a)	64.	(c)
65.	(d)	66.	(c)	67.	(c)	68.	(b)	69.	(b)	70.	(a)	71.	(d)	72.	(a)
73.	(a)	74.	(d)	75.	(d)	76.	(c)	77.	(d)	78.	(a)	79.	(a)	80.	(b)
81.	(c)	82.	(c)	83.	(c)	84.	(c)	85.	(d)	86.	(b)	87.	(c)	88.	(d)
89.	(b)	90.	(c)	91.	(d)	92.	(a)	93.	(c)	94.	(c)	95.	(b)	96.	(b)
97.	(b)	98.	(a)	99.	(d)	100.	(d)	101.	(b)	102.	(b)	103.	(b)	104.	(c)
105.	(a)	106.	(c)	107.	(b)	108.	(c)	109.	(a)	110.	(c)	111.	(a)	112.	(c)
113.	(a)	114.	(d)	115.	(c)	116.	(d)	117.	(c)	118.	(c)	119.	(b)	120.	(a)
121.	(c)	122.	(a)	123.	(b)	124.	(a)	125.	(a)	126.	(d)	127.	(c)	128.	(c)
129.	(c)	130.	(a)	131.	(c)	132.	(b)	133.	(a)	134.	(b)	135.	(b)	136.	(d)
137.	(d)	138.	(c)	139.	(a)	140.	(c)	141.	(a)	142.	(a)	143.	(b)	144.	(c)
145.	(b)	146.	(c)	147.	(a)	148.	(c)	149.	(c)	150.	(a)	151.	(c)	152.	(b)
153.	(a)	154.	(b)	155.	(b)	156.	(d)	157.	(a)	158.	(c)	159.	(b)	160.	(d)
161.	(b)	162.	(a)	163.	(c)	164.	(d)	165.	(a)	166.	(b)	167.	(b)	168.	(c)
169.	(a)	170.	(b)	171.	(a)	172.	(d)	173.	(b)	174.	(c)	175.	(d)	176.	(d)
177.	(b)	178.	(c)	179.	(b)	180.	(a)	181.	(c)	182.	(b)	183.	(b)	184.	(b)
185.	(b)	186.	(c)	187.	(a)	188.	(a)	189.	(d)	190.	(c)	191.	(b)	192.	(c)
193.	(b)	194.	(a)	195.	(a)	196.	(c)	197.	(a)	198.	(b)	199.	(a)	200.	(b)
201.	(d)	202.	(d)	203.	(c)	204.	(a)	205.	(a)	206.	(a)	207.	(a)	208.	(d)

209.	(b)	210.	(d)	211.	(d)	212.	(d)	213.	(c)	214.	(c)	215.	(c)	216.	(c)
217.	(d)	218.	(b)	219.	(d)	220.	(b)	221.	(d)	222.	(b)	223.	(d)	224.	(a)
225.	(c)	226.	(d)	227.	(c)	228.	(b)	229.	(c)	230.	(b)	231.	(a)	232.	(d)
233.	(b)	234.	(a)	235.	(b)	236.	(b)	237.	(d)	238.	(b)	239.	(d)	240.	(a)
241.	(c)	242.	(d)	243.	(d)	244.	(c)	245.	(d)	246.	(a)	247.	(d)	248.	(a)
249.	(a)	250.	(d)	251.	(b)	252.	(c)	253.	(c)	254.	(d)	255.	(b)	256.	(b)
257.	(c)	258.	(a)	259.	(a)	260.	(d)	261.	(a)	262.	(a)	263.	(c)	264.	(c)
265.	(c)	266.	(c)	267.	(a)	268.	(b)	269.	(a)	270.	(d)	271.	(c)	272.	(d)
273.	(d)	274.	(d)	275.	(c)	276.	(c)	277.	(b)	278.	(b)	279.	(a)	280.	(a)
281.	(a)	282.	(d)	283.	(d)	284.	(a)	285.	(a)	286.	(c)	287.	(c)	288.	(c)
289.	(b)	290.	(a)	291.	(d)	292.	(c)	293.	(a)	294.	(c)	295.	(b)	296.	(d)
297.	(d)	298.	(c)	299.	(c)	300.	(c)								

MORE THAN ONE CORRECT TYPE QUESTIONS

301.	(a,c,d)	302.	(a,c)	303.	(c,d)	304.	(a,b,c)	305.	(a,d)	306.	(a,c)
307.	(a,c)	308.	(a,b,c,d)	309.	(a,b,c,d)	310.	(b,c,d)	311.	(a,b,c,d)	312.	(a,b,c)
313.	(a,b,c,)	314.	(a,b,c)	315.	(a,c,d)	316.	(a,c,d)	317.	(a,d)	318.	(a,b,d)
319.	(a,b)	320.	(a,c,d)	321.	(a,b,c)	322.	(a,c)	323.	(c,d)	324.	(a,b,c)
325.	(a,d)	326.	(a,b,d)	327.	(b,c)	328.	(b,c,d)	329.	(a,b,c)	330.	(a,c,d)
331.	(a,b)	332.	(a,b)	333.	(a,d)	334.	(a,b,c)	335.	(a,c,d)	336.	(a,c,d)
337.	(a,d)	338.	(b,c)	339.	(b,d)	340.	(b,c,d)	341.	(a,b,c)	342.	(a,c)
343.	(b,c,d)	344.	(b,c,d)	345.	(a,c,d)	346.	(a,b,c)	347.	(a,c)	348.	(a,b,d)
349.	(a,b,c)	350.	(a,b,c,d)	351.	(a,b,d)	352.	(b,d)	353.	(c,d)	354.	(b,c,d)
355.	(a,b,d)	356.	(a,b,c)	357.	(a,b,d)	358.	(a,b,c)	359.	(a,b,c,d)	360.	(a,b,c,d)
361.	(b,c,d)	362.	(a,d)	363.	(a,b)	364.	(a,b,c)	365.	(a,b)	366.	(a,b,d)
367.	(a,b,d)	368.	(b,c,d)	369.	(b,d)	370.	(a,d)	371.	(a,b,c)	372.	(a,d)
373.	(a,b)	374.	(a,b)	375.	(a,c,d)	376.	(a,c)	377.	(a,b,d)	378.	(a,b,d)
379.	(b,c,d)	380.	(a,c,d)	381.	(a,b,c)	382.	(a,b,d)	383.	(a,b,c)	384.	(c,d)
385.	(a,b)	386.	(b,c)	387.	(a,b,c)	388.	(a,b,d)	389.	(a,c)	390.	(a,b,d)
391.	(a,b,c)	392.	(a,b,d)	393.	(a,b)	394.	(a,d)	395.	(a,b,c)	396.	(a,b,c)
397.	(a,c,d)	398.	(a,b,d)	399.	(b,c,d)	400.	(a,b)	401.	(a,b,c)	402.	(a,b,c,d)

403.	(b,c,d)	404.	(b,c,d)	405.	(a,b)	406.	(b,c,d)	407.	(a,d)	408.	(a,b,c,d)
409.	(a,b,d)	410.	(b,c)	411.	(a,b,c)	412.	(c)	413.	(a,c)	414.	(a,c,d)
415.	(a,c)	416.	(a,b,c,d)	417.	(a,b,d)	418.	(a,b,d)	419.	(b,d)	420.	(b,c)
421.	(a,c,d)	422.	(a,c)	423.	(b,c,d)	424.	(a,c)	425.	(a,b)	426.	(a,c)
427.	(b,c,d)	428.	(a,b,c)	429.	(a,b)	430.	(a,c)	431.	(a,b,c)	432.	(a,c,d)
433.	(b,d)	434.	(a,b,c)	435.	(b,d)	436.	(a,b,c,d)	437.	(b,c,d)	438.	(a,b,c)
439.	(a,b)	440.	(a,c)	441.	(a,b,c)	442.	(c,d)	443.	(a,b,c,d)	444.	(a,d)
445.	(b,c)	446.	(b,c,d)	447.	(a,b,d)	448.	(b,c)	449.	(a,c,d)	450.	(b,c)
451.	(a,c)	452.	(a,d)	453.	(a,c)	454.	(a,b,c)	455.	(a,b,d)	456.	(b,c)
457.	(a,c)	458.	(c,d)	459.	(a,b,c)	460.	(a,c)	461.	(a,b,d)	462.	(a,c)
463.	(a,b,d)	464.	(a,c)	465.	(b,c,d)	466.	(c,d)	467.	(a,b)	468.	(b,c)
469.	(a,b,d)	470.	(b,c)	471.	(a,d)	472.	(a,b,c)	473.	(a,d)	474.	(a,b,c)
475.	(b,c)	476.	(a,b,c)	477.	(a,b,d)	478.	(b,c,d)	479.	(b,c)	480.	(a,b,c,d)
481.	(a,b,d)	482.	(a,b,c)	483.	(a,c,d)	484.	(a,b,d)	485.	(b,d)	486.	(a,c,d)
487.	(a,b,c)	488.	(a,c,d)	489.	(b,d)	490.	(a,b)	491.	(a,b,c,d)	492.	(a,d)
493.	(a,c)	494.	(b,c)	495.	(b,c,d)	496.	(a,b)	497.	(b,d)	498.	(a,d)
499.	(a,b,c)	500.	(a,c)	501.	(a,b,d)	502.	(b,c,d)	503.	(b,c,d)	504.	(b,c)
505.	(a,d)	506.	(a,c,d)	507.	(a,b,c,d)	508.	(a,b,c,d)	509.	(a,b,c,d)	510.	(a,b,c,d)
511.	(a,b,d)	512.	(a,b,d)	513.	(a,c)	514.	(a,c)	515.	(b,c)	516.	(b,d)
517.	(b,c,d)	518.	(a,b,c)	519.	(a,c)	520.	(a,b,c,d)	521.	(a,b,c)	522.	(a,b,c)
523.	(b,c)	524.	(a,d)	525.	(a,b,c,d)	526.	(a,b,c)	527.	(a,c,d)	528.	(a,b)
529.	(a,b,d)	530.	(a,b,c)	531.	(b,c)	532.	(a,b,c)	533.	(a,b,c,d)	534.	(a,d)
535.	(b)	536.	(a,c)	537.	(a,c)	538.	(a,d)	539.	(a,c,d)	540.	(a,c,d)
541.	(a,b,c,d)	542.	(a,c)	543.	(b,c,d)	544.	(b,c)	545.	(a,d)	546.	(a,b,c)
547.	(b,c,d)	548.	(a,b,c,d)	549.	(a,b)	550.	(b,d)	551.	(b,d)	552.	(a,d)
553.	(a,c,d)	554.	(b,c,d)	555.	(a,b)	556.	(b,c,d)	557.	(a,d)	558.	(a,b,d)
559.	(a,c)	560.	(a,d)	561.	(a,c)	562.	(c,d)	563.	(a,b)	564.	(a,b)
565.	(a,c,d)	566.	(a,c)	567.	(a,b)	568.	(a,d)	569.	(b,c)	570.	(a,d)
571.	(a,b,c,d)	572.	(a,b,c,d)	573.	(a,b)	574.	(a,b,c,d)	575.	(a,d)		

PARAGRAPH TYPE QUESTIONS

576.	(b)	577.	(c)	578.	(c)	579.	(b)	580.	(a)	581.	(a)	582.	(b)	583.	(a)
584.	(b)	585.	(b)	586.	(b)	587.	(b)	588.	(a)	589.	(b)	590.	(a)	591.	(d)
592.	(b)	593.	(c)	594.	(b)	595.	(c)	596.	(d)	597.	(b)	598.	(d)	599.	(c)
600.	(b)	601.	(c)	602.	(c)	603.	(d)	604.	(b)	605.	(a)	606.	(b)	607.	(c)
608.	(b)	609.	(c)	610.	(a)	611.	(c)	612.	(d)	613.	(b)	614.	(b)	615.	(a)
616.	(a)	617.	(d)	618.	(d)	619.	(c)	620.	(c)	621.	(d)	622.	(d)	623.	(a)
624.	(a)	625.	(d)	626.	(b)	627.	(d)	628.	(b)	629.	(c)	630.	(c)	631.	(b)
632.	(c)	633.	(d)	634.	(d)	635.	(b)	636.	(b)	637.	(a)	638.	(a)	639.	(d)
640.	(d)	641.	(c)	642.	(c)	643.	(c)	644.	(c)	645.	(b)	646.	(c)	647.	(a)
648.	(d)	649.	(a)	650.	(c)	651.	(b)	652.	(b)	653.	(d)	654.	(b)	655.	(c)
656.	(c)	657.	(d)	658.	(c)	659.	(a)	660.	(b)	661.	(c)	662.	(d)	663.	(c)
664.	(b)	665.	(d)	666.	(b)	667.	(d)	668.	(c)	669.	(b)	670.	(b)	671.	(c)
672.	(c)	673.	(c)	674.	(c)	675.	(b)								

MATCH THE COLUMN TYPE QUESTIONS

676. (a) → (P,Q,S); (b) → (P,R); (c) → (P); (d) → (P,R,T)
677. (a) → (S); (b) → (R); (c) → (P)
678. (a) → (P,Q,R); (b) → (Q); (c) → (P)

										679.	(d)	680.	(b)	681.	(a)
682.	(c)	683.	(d)	684.	(a)	685.	(b)	686.	(c)	687.	(b)	688.	(c)	689.	(d)
690.	(d)	691.	(b)	692.	(a)	693.	(c)	694.	(d)	695.	(b)	696.	(a)	697.	(d)
698.	(a)	699.	(b)	700.	(c)										

INTEGER TYPE QUESTIONS

701.	100	702.	3	703.	0	704.	8	705.	2	706.	16	707.	100	708.	3
709.	8	710.	3	711.	125	712.	6	713.	2	714.	3	715.	3	716.	1
717.	6	718.	743	719.	37	720.	4	721.	4	722.	4	723.	6	724.	4
725.	4	726.	9	727.	7	728.	22	729.	9	730.	720	731.	4	732.	2
733.	15	734.	11	735.	1	736.	4	737.	3	738.	18	739.	6	740.	4
741.	7	742.	7	743.	6	744.	4	745.	2	746.	2	747.	21	748.	5
749.	12	750.	1	751.	6	752.	9	753.	6	754.	108	755.	40	756.	6
757.	96	758.	14	759.	5	760.	144	761.	72	762.	3	763.	20	764.	20

765.	8	**766.**	75	**767.**	150	**768.**	46	**769.**	2	**770.**	5	**771.**	1	**772.**	2
773.	1801	**774.**	2	**775.**	8	**776.**	7	**777.**	2	**778.**	4	**779.**	600	**780.**	30
781.	6	**782.**	5	**783.**	13	**784.**	17	**785.**	20	**786.**	5	**787.**	16	**788.**	36
789.	20	**790.**	9	**791.**	3	**792.**	8	**793.**	7	**794.**	5	**795.**	3	**796.**	2
797.	3	**798.**	14	**799.**	5	**800.**	14	**801.**	4	**802.**	4	**803.**	20	**804.**	73
805.	10	**806.**	132	**807.**	26	**808.**	1	**809.**	9	**810.**	72	**811.**	2	**812.**	18
813.	4	**814.**	0	**815.**	3	**816.**	3	**817.**	0	**818.**	25	**819.**	74	**820.**	4
821.	6	**822.**	8	**823.**	6	**824.**	3	**825.**	1	**826.**	0	**827.**	2	**828.**	32
829.	3	**830.**	3	**831.**	12	**832.**	1	**833.**	3	**834.**	45	**835.**	1	**836.**	5
837.	1	**838.**	5	**839.**	4	**840.**	3	**841.**	5	**842.**	5	**843.**	70	**844.**	1
845.	2	**846.**	25	**847.**	4	**848.**	4	**849.**	7	**850.**	2	**851.**	3	**852.**	7
853.	4	**854.**	202	**855.**	60	**856.**	150	**857.**	8	**858.**	5	**859.**	7	**860.**	5
861.	23	**862.**	4	**863.**	10	**864.**	35	**865.**	25	**866.**	4	**867.**	25	**868**	32
869.	25	**870.**	20	**871.**	23	**872.**	27	**873.**	3	**874.**	309	**875.**	151	**876.**	14
877.	5	**878.**	0	**879.**	1	**880.**	9	**881.**	6	**882.**	1	**883.**	2	**884.**	8
885.	3	**886.**	7	**887.**	3	**888.**	2	**889.**	4	**890.**	2	**891.**	5	**892.**	10
893.	90	**894.**	9	**895.**	1	**896.**	2	**897.**	9	**898.**	4	**899.**	2	**900.**	7
901.	2020	**902.**	6	**903.**	6	**904.**	45	**905.**	6	**906.**	2	**907.**	0	**908.**	16
909.	21	**910.**	9	**911.**	168	**912.**	190	**913.**	3	**914.**	28	**915.**	6	**916.**	139
917.	7	**918.**	7	**919.**	3	**920.**	7	**921.**	125	**922.**	7	**923.**	3	**924.**	10
925.	2	**926.**	13	**927.**	8	**928.**	20	**929.**	16	**930.**	41	**931.**	28	**932.**	7
933.	6	**934.**	2	**935.**	2	**936.**	25	**937.**	9	**938.**	6	**939.**	1	**940.**	4
941.	6	**942.**	4	**943.**	4	**944.**	5	**945.**	6	**946.**	4	**947.**	6	**948.**	15
949.	107	**950.**	6	**951.**	3	**952.**	17	**953.**	13	**954.**	29	**955.**	197	**956.**	36
957.	19	**958.**	8	**959.**	7	**960.**	351	**961.**	26	**962.**	6	**963.**	196	**964.**	$\frac{1}{\sqrt{3}}$
965.	2	**966.**	3	**967.**	4	**968.**	2	**969.**	23	**970.**	17	**971.**	6	**972.**	14
973.	129	**974.**	40	**975.**	9	**976.**	2	**977.**	2	**978.**	3	**979.**	2	**980.**	2
981.	5	**982.**	7	**983.**	2	**984.**	16	**985.**	8	**986.**	4	**987.**	29	**988.**	30
989.	1395	**990.**	10	**991.**	2.5	**992.**	6	**993.**	$a > \frac{e^2}{4}$		**994.**	$\left(-\frac{1}{2}, 1\right)$	**995.**	5	**996.**	5
997.	2	**998.**	5	**999.**	2	**1000.**	3								

HINTS & SOLUTIONS

Hints & Solutions

Single Correct Type Questions

1. (a) $$I=\int_{\frac{-1}{\sqrt{3}}}^{\frac{1}{\sqrt{3}}}\frac{\cos^{-1}\left(\frac{2x}{1+x^2}\right)+\tan^{-1}\left(\frac{2x}{1-x^2}\right)}{e^x+1}dx$$

$$I=\int_{\frac{-1}{\sqrt{3}}}^{\frac{1}{\sqrt{3}}}\frac{\cos^{-1}\left(\frac{2x}{1+x^2}\right)+\sin^{-1}\left(\frac{2x}{1+x^2}\right)}{e^x+1}dx$$

$$I=\frac{\pi}{2}\int_{0}^{\frac{1}{\sqrt{3}}}\left(\frac{1}{e^x+1}+\frac{e^x}{e^x+1}\right)dx=\frac{\pi}{2\sqrt{3}}$$

2. (c) $$L=\lim_{n\to\infty}\sqrt[n]{\frac{n^{2n}}{((n^2+1^2)(n^2+2^2)\ldots\ldots(n^2+n^2))}}$$

$$\ln L=\lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^{n}\ln\left(\frac{1}{1+\frac{i^2}{n^2}}\right)=-\int_0^1\ln(1+x^2)\,dx\,x$$

$$\therefore \quad L=\frac{1}{2}e^{2-\frac{\pi}{2}}$$

3. (c) Solution is $x^2-y^2=1$

4. (b) Both LHL = RHL = -1

5. (a) $$\lim_{\alpha\to 0}\frac{e^{\cos(\alpha^n)-1}-1}{\alpha^m}=-\frac{1}{2}$$

$$\lim_{\alpha\to 0}\frac{\cos(\alpha^n)-1}{\alpha^{2n}(\alpha^{m-2n})}=-\frac{1}{2}$$

Hence, $m-2n=0$

$\therefore \qquad \frac{m}{n} = 2$

6. (d) $\lim_{x\to 0} \frac{x^2 \sin\left(\frac{1}{x}\right) + 2x}{x} \cdot \frac{x}{(1+x)^{1/x} - e} = 2\left(\frac{-2}{e}\right) = -\frac{4}{e}$

7. (d) $f(x) = 2e^{\frac{x^2}{2}} - 2$

8. (d) Do yourself.

9. (d) $\lim_{x\to 0} \frac{(1-\cos x) + (1-\cos 2x) + \ldots\ldots + (1-\cos 10x)}{x^2} = \frac{1}{2}(1^2 + 2^2 + \ldots\ldots + 10^2) = \frac{385}{2}$

10. (c) $I = \int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x} dx$

Using King and add

$$2I = \int_{-\pi}^{\pi} \frac{2x(1+\sin x) + 2(-x)(1-\sin x)}{1+\cos^2 x} dx = 4\int_{-\pi}^{\pi} \frac{x\sin x}{1+\cos^2 x} dx$$

$$I = 2\int_{-\pi}^{\pi} f(x)\, dx$$

$$I = 4\int_{0}^{\pi} \frac{x\sin x}{1+\cos^2 x} dx$$

Using Kind and add

$$2I = 4\pi \int_{0}^{\pi} \frac{\sin x}{1+\cos^2 x} dx$$

Put $\cos x = t$

$$2I = 4\pi \int_{-1}^{1} \frac{dt}{1+t^2}$$

$$\therefore \qquad I = 4\pi \times \frac{\pi}{4} = \pi^2$$

11. (d) $S_K = \frac{k^2 - 1}{1 - \frac{1}{k}} = k(k+1) \qquad (k \neq 1)$

$S_1 = T_1 = 0$

Now, $S = \sum_{k=2}^{\infty} \frac{k(k+1)}{2^{k-1}}$

$$S = \frac{2\cdot 3}{2} + \frac{3\cdot 4}{2^2} + \frac{4\cdot 5}{2^3} + \ldots\ldots + \infty \qquad \ldots(1)$$

$$\frac{S}{2}=\frac{2\cdot 3}{2^2}+\frac{3\cdot 4}{2^3}+\ldots\ldots+\infty \qquad \ldots(2)$$

Eqn. (1) – eqn. (2)

$$\frac{S}{2}=3+\underbrace{3\cdot\frac{1}{2}+4\cdot\frac{1}{2^2}+5\cdot\frac{1}{2^3}+\ldots\ldots+\infty}_{S'}$$

$$S'=\frac{3}{2}+\frac{4}{2^2}+\frac{5}{2^3}+\ldots\ldots+\infty \qquad \ldots(3)$$

$$\frac{S'}{2}=\frac{3}{2^2}+\frac{4}{2^3}+\ldots\ldots+\infty \qquad \ldots(4)$$

Eqn. (3) eqn. (4)

$$\frac{S'}{2}=\frac{3}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\ldots\ldots+\infty=\frac{3}{2}+\frac{1/4}{1-(1/2)}=2$$

$\Rightarrow \qquad S'=4$

Now $\qquad \frac{S}{2}=3+4$

$S=14$

12. (c) $$y=\tan^{-1}\left(\frac{5x-x}{1+5x(x)}\right)+\tan^{-1}\left(\frac{x+\frac{2}{3}}{1-\frac{2x}{3}}\right)=(\tan^{-1}5x-\tan^{-1}x)+\left(\tan^{-1}x+\tan^{-1}\frac{2}{3}\right)$$

$$y'=\frac{5}{1+25x^2}$$

$\therefore \quad \alpha=5$

13. (c) $$T_1=\sin^{-1}\left(\frac{1}{\sqrt{10}}\right)=\tan^{-1}\left(\frac{1}{3}\right)=\tan^{-1}2-\tan^{-1}1$$

$$T_2=\sin^{-1}\left(\frac{1}{\sqrt{50}}\right)=\tan^{-1}\left(\frac{1}{7}\right)=\tan^{-1}3-\tan^{-1}2$$

$\vdots$

$$T_{10}=\tan^{-1}101-\tan^{-1}100$$

$$\text{Sum}=\tan^{-1}101-\tan^{-1}1=\tan^{-1}\left(\frac{100}{102}\right)=\tan^{-1}\left(\frac{50}{51}\right)$$

$\therefore \quad p+q=50+51=101$

14. (a) $S_1: x(x-at_1^2)+y(y-2at_1)=0$

$S_2: x(x-at_2^2)+y(y-2at_2)=0$

Equation of the line joining the vertex of parabola to the intersection of the two circles is

$$L : S_1 - S_2 = 0 \quad \Rightarrow \quad L : y = -\left(\frac{t_1 + t_2}{2}\right)x$$

Using this we have

$$\tan C = -\left(\frac{t_1 + t_2}{2}\right) \quad \Rightarrow \quad \tan C = -\left[\frac{(\tan A)^{-1} + (\tan B)^{-1}}{2}\right]$$

$$\Rightarrow \quad \tan C = -\left(\frac{\cot A + \cot B}{2}\right)$$

$$\Rightarrow \quad \cot A + \cot B + 2\tan C = 0$$

$$\Rightarrow \quad m = 2$$

15. (b) $\lim\limits_{n\to\infty}\left[\ln\left(\sqrt[n]{\frac{4}{n^2}}\right) + \ln\left(\sqrt[n]{\frac{16}{n^2}}\right) + \ln\left(\sqrt[n]{\frac{36}{n^2}}\right) + \ldots\ldots + \ln\left(\sqrt[n]{\frac{4n^2}{n^2}}\right)\right]$

$$= \lim_{n\to\infty}\sum_{k=1}^{n}\ln\left(\sqrt[n]{\frac{4k^2}{n^2}}\right) = \lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{n}\ln\left(\frac{4k^2}{n^2}\right)$$

$$= \int_0^1 \ln(4x^2)\,dx = \int_0^1 (2\ln 2 + 2\ln x)\,dx$$

$$= 2x\ln 2 + 2x\ln x - 2x\big|_0^1 = 2\ln 2 + 2\ln 1 - 2x - 0 - 2\lim_{x\to 0} x\ln x - 0$$

$$-2\ln 2 - 2$$

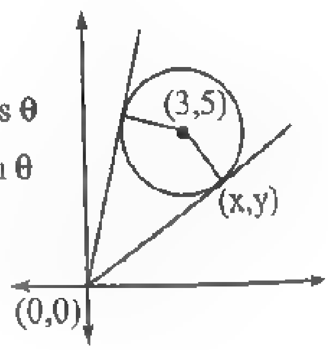
16. (d) $f(x) = \dfrac{e^x}{x^2}$

17. (b) Do yourself.

18. (c) $y = mx$

$x = 3 + \sqrt{6}\cos\theta$

$y = 3 + \sqrt{6}\sin\theta$



$$\left|\frac{3m-3}{\sqrt{1+m^2}}\right| = \sqrt{6}$$

$$9(m-1)^2 = 6(1+m^2)$$

$$3m^2 - 18m + 3 = 0$$

$$m^2 - 6m + 1 = 0$$

$$m = \frac{6 \pm \sqrt{31-4}}{2} = \frac{6 \pm 4\sqrt{2}}{2} = 3 \pm 2\sqrt{2}$$

19. (b) $f'(0) = \sin^2(\sin 1)$; $\quad f''(0) = 2\sin(\sin 1)\cdot\cos(\sin 1)\cos 1$

Now, $\quad g''(y) = \dfrac{-1}{[f'(x)]^3} f''(x)$

$$g''(3) = \frac{-f''(0)}{[f'(0)]^3} = \frac{-2\sin(\sin 1)\cos(\sin 1)\cos 1}{\sin^6(\sin 1)}$$

20. (b) (Slope of OM) (Slope of AN) $= -1$

$$\frac{k-2}{h-1} = \frac{k}{h} = -1$$

$\therefore$ ...... is $x(x-1) + y(y-2) = 0$

$$x^2 + y^2 - 2y - x = 0$$

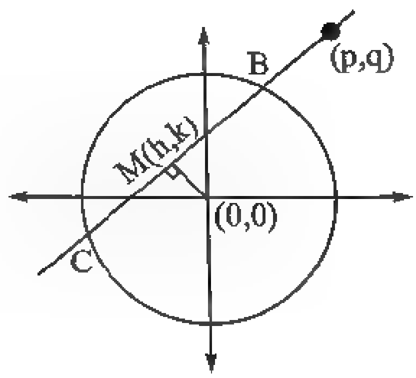
$$\left(x - \frac{1}{2}\right)^2 + (y-1)^2 = \frac{5}{4}$$

$$C \equiv (1/2, 1)$$

$$r = \frac{\sqrt{5}}{2}$$

21. (c) Do yourself.

22. (d) $\frac{k}{h} - \frac{k-q}{h-p} = -1$



$x^2 + y^2 - px - qy = 0$

$h = 0$

$g = \frac{-p}{2}$

$f = \frac{-q}{2}$

$c = 0$

23. (d) $$x_1^2 + \sqrt{x_2^2 - 2x_2} = 2x_1 - 1 \quad \ldots(1)$$

$$\Rightarrow \quad (x_1 - 1)^2 = -\sqrt{x_2(x_2 - 2)} \quad \ldots(2)$$

Notice that $(x_1 - 1)^2 \geq 0$ and $-\sqrt{x_2(x_2-2)} \leq 0$

So, this will force to deduce that $(x_1 - 1)^2 = 0$ and $-\sqrt{x_2(x_2-2)} = 0$

For equation (1)

$$(x_1 - 1)^2 = 0 \quad \Rightarrow \quad x_1 = 1$$

For equation (2)

$$-\sqrt{x_2(x_2-2)} = 0 \quad \Rightarrow \quad x_2 = 0 \quad \text{or} \quad x_2 = 2$$

But x_2 is positive real solution $\Rightarrow$ $x_2 = 2$

Thus, $x^2 - bx + c = (x-1)(x-2) = 0 \Rightarrow x^2 - 3x + 2 = 0 \Rightarrow b = 3$ and $c = 2$

$\Rightarrow$ $b + c = 3 + 2 = 5 \Rightarrow (b+c)_{\min} = 5$

24. (c) $$I = \int_{e^{\pi/6}}^{e^{\pi/2}} \frac{\sin(\ln(\sin(\ln x)))\cos(\ln x)}{x\sin(\ln x)}\,dx$$

Put $\ln(\sin(\ln x)) = t$

$$I = \int_{-\ln 2}^{0} \sin t\, dt = \cos(\ln 2) - 1$$

Hence, $\cos^{-1}(I+1) = \ln 2$

25. (c) $y = (x+\sqrt{1+x^2})^n$

$$\frac{dy}{dx} = n(x+\sqrt{1+x^2})^{n-1}\left(1+\frac{x}{\sqrt{1+x^2}}\right) = \frac{n(x+\sqrt{1+x^2})^n}{\sqrt{1+x^2}}$$

$$\rightarrow \quad \frac{dy}{dx} = \frac{ny}{\sqrt{1+x^2}} \quad ; \quad (1+x^2)\frac{dy}{dx} = ny\sqrt{1+x^2}$$

$$(1+x^2)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} = n\sqrt{1+x^2} + \frac{nxy}{\sqrt{1+x^2}} = n\sqrt{1+x^2}\ \frac{ny}{\sqrt{1+x^2}} + x\frac{dy}{dx}$$

$$\rightarrow \quad (1+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = n^2 y$$

26. (c) $$\int_{\pi/4}^{\pi/3} e^x\left(\frac{2}{2\cos^2 x} + \frac{2\sin x\cos x}{2\cos^2 x}\right)dx - \int_{\pi/4}^{\pi/3} e^x(\sec^2 x + \tan x)\,dx$$

$$\int e^x(f(x)+f'(x))dx = e^x f(x) + C$$

$$e^x \tan x\Big|_{\pi/4}^{\pi/3} = e^{\pi/4}(\sqrt{3}e^{\pi/12} - 1)$$

$$\Rightarrow \quad a=4; \quad b=\sqrt{3}; \quad c=12 \quad \Rightarrow \quad \frac{b^2c}{a} = 9$$

27. (d) $\lim_{x\to N} f(x)$ will exist iff $\sin(\pi N) = \tan(\pi\sqrt{N})$

Hence, $\tan(\pi\sqrt{N}) = 0$

$$\Rightarrow \quad \pi\sqrt{N} = k\pi \quad ; \quad k \in N$$

The possible values of N are $1^2, 2^2, 3^2, 4^2, 5^2, 6^2, 7^2, 8^2$ and 9^2.

$\therefore$ Sum = 1 + 4 + 9 + 16 + 25 + 36 + 49 + 64 + 81 = 285

28. (c) $$\cot^{-1}\left(\frac{1+2^{2n+1}}{2^n}\right) = \tan^{-1}\left(\frac{2^n}{1+2^{2n+1}}\right) = \tan^{-1}\left(\frac{2^{n+1} - 2^n}{1+2^n\cdot 2^{n+1}}\right)$$

$$= \tan^{-1}(2^{n+1}) - \tan^{-1}(2^n)$$

$$S = \tan^{-1}(2^1) - \tan^{-1}(2^0) \ldots\ldots \tan^{-1}(2^{11}) - \tan^{-1}(2^{10})$$

$$= \tan^{-1}(2^{11}) - \tan^{-1}(1) = \cot^{-1}\left(\frac{a}{b}\right)$$

$$= \tan^{-1}\left(\frac{2^{11}-1}{1+2^{11}}\right) = \tan^{-1}\left(\frac{b}{a}\right)$$

$$= \log_2\left(\frac{b+a}{a-b}\right) = \log_2\left(\frac{2\cdot 2^{11}}{2}\right) = 11$$

29. (c) Do yourself.

30. (c) $x^{2x} - 2x^x \cot y - 1 = 0$ when $x = 1$

$1 - 2\cot(y(1)) - 1 = 0 \Rightarrow \cot(y(1)) = 0$; $y(1) = \frac{\pi}{2}$

Now, we have $x^{2x} - 2x^x \cot y - 1 = 0$

Differentiate both sides with respect to x.

$$2(\ln x + 1)x^{2x} - 2x^x(\ln x + 1)\cot y + 2x^x \sec^2 y \frac{dy}{dx} = 0 \qquad \text{when } x = 1$$

$$2 - 0 + \sec^2\frac{\pi}{2}\cdot y'(1) = 0$$

$$y'(1) = -1$$

31. (d) Let $f(x) = e^{\sqrt{x}} \sin\left(\frac{\pi x}{3}\right) dx$ and $F(x) = \int_0^x f(t)dt$. Then we have

$$L = \lim_{h\to 0}\frac{1}{h}\int_1^{1+2h} e^{\sqrt{x}} \sin\left(\frac{\pi x}{3}\right) dx = \lim_{h\to 0}\frac{F(1+2h) - F(1)}{h}$$

$$= 2\lim_{h\to 0}\frac{F(1+2h) - F(1)}{2h} \qquad \text{By definition of differentiation}$$

$$= 2f(1) = 2e\sin\frac{\pi}{3}$$

32. (a) Do yourself.

33. (c) Differentiating, $f(x+1) - f(x) = e^x$

Putting $x = 0$, $f(1) - f(0) = 1$

Putting $x = 1$, $f(2) - f(1) = e$

$\therefore \quad f(2) - f(0) = e + 1$

34. (a) $\lim_{n\to\infty}\sum_{k=0}^{n}\frac{{}^nC_k}{n^k}\int_0^1 x^{k+2}dx = \int_0^1\left[\lim_{n\to\infty}\sum_{k=0}^{n}{}^nC_k\left(\frac{x}{n}\right)^k x^2\right]dx = \int_0^1\left[\lim_{n\to\infty}\left(1+\frac{x}{n}\right)^n x^2\right]dx$

$$= \int_0^1 e^x x^2 dx = \int_0^1 e^x(x^2+2x)dx - 2\int_0^1 e^x(x+1)dx + 2\int_0^1 e^x dx$$

$$= e^x(x^2 - 2x + 2)]_0^1 = e - 2$$

35. (c) $xf(x)=x+\int_1^x f(t)dt \quad \Rightarrow \quad xf'(x)+f(x)=1+f(x) \quad \Rightarrow \quad f'(x)=\frac{1}{x}$

$\Rightarrow \quad f(x)=\ln x+C \quad \Rightarrow \quad f(x)=\ln x+1 \quad \Rightarrow \quad f(e^k)=k+1$

$\Rightarrow \quad \sum_{k=1}^{10} f(e^k)=\sum_{k=1}^{10}(k+1)=\frac{10\times 11}{2}+10=65$

36. (d) $y=\underbrace{\frac{-2p}{\sqrt{1-p^2}}}_{m}x+\underbrace{\frac{1}{\sqrt{1-p^2}}}_{c}$;

$4-4b^2=4a^2-b^2 \quad \Rightarrow \quad 4=4a^2+3b^2$

$\frac{1}{1-p^2}=\frac{4a^2p^2}{1-p^2}+b^2-b^2p^2 \quad ; \quad (1-b^2)=(4a^2-b^2)p^2$

$\frac{1}{1-p^2}=\frac{4a^2p^2}{1-p^2}+b^2$

$1=4a^2p^2+b^2(1-p^2) \quad ; \quad (4a^2-b^2)p^2+(b^2-1)=0 \quad \forall\ p\in(-1,1)-\{0\}$

$4a^2=b^2=1$

$a^2=1/4$

$b^2=1$

$e^2=1-\frac{1}{4}=\frac{3}{4}$

37. (a) $T_n=\tan^{-1}\left(\frac{3m^2-3m+1}{m^6-3m^5+3m^4-m^3+1}\right)=\tan^{-1}\left[\frac{(m^3-(m-1)^3)}{1+(m^3(m-1)^3)}\right]$

$=\tan^{-1}m^3-\tan^{-1}(m-1)^3$

$\therefore \quad \sum_{m=1}^{\infty}\left[\tan^{-1}\left(\frac{3m^2-3m+1}{m^6-3m^5+3m^4-m^3+1}\right)\right]=\frac{\pi}{2}$

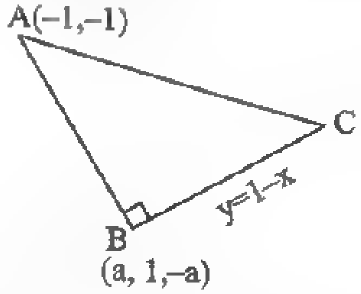
38. (a) Here $x\neq 0,1,3,6$

$x\in$	$(-\infty,0)$	$(0,1)$	$(1,3)$	$(3,4)$	$(4,5)$	$(5,6)$	$(6,+\infty)$
x	–	+	+	+	+	+	+
π^x-7^x	+	–	–	–	–	–	–
$x-1$	–	-	+	+	+	+	+
$x-3$	–	–	–	+	+	+	+
$x-6$	–	–	-	–	–	–	+
$\log_{10}(x-4)$	NA	NA	NA	NA	–	+	+
Product	NA	NA	NA	NA	–	+	–

Hence, $x\in(4,5)\cup(6,\infty)$.

39. (b) $x^3 + y^3 + (-1)^3 - 3(-1)(x)(y) = 0$

$(x + y - 1)[(x - y)^2 + (y + 1)^2 + (x + 1)^2] = 0$



Hence, $A \equiv (-1, -1)$ and the equation of side BC is $x + y = 1$

$$m_{AB} \cdot m_{BC} = -1 \quad \Rightarrow \quad \left(\frac{2-a}{a+1}\right)(-1) = -1$$

$\therefore \quad 2 - a = a + 1$

$\Rightarrow \quad a = \frac{1}{2} \quad ; \quad \therefore \; B = \left(\frac{1}{2}, \frac{1}{2}\right)$

Hence, equation of AB is $y = x$

$m = 1$ and $c = 0$

Hence, $4 - m - c = 4 - 1 - 0 = 3$

40. (b) (a) False because if $g(x)$ is $\sin(2\pi x)$.

(b) Take logarithm on both sides and differentiate once to get the expression.

(c) Obviously, false.

(d) Statement is correct for $|f(x)|$ but not for $f(x)$.

41. (c) We know tangent to a conic is given by $T = 0$

i.e., Tangent to parabola at (p, q) is :

$$2ax - yq = -2ap$$

Substitute x and y in this equation by (x_1, y_1),

Where (x_1, y_1) are points where tangent

$$2ax_1 - y_1 q = -2ap \qquad \ldots(1)$$

Now, equation of chord of contact of circle is $T = 0$, passing through (r, s) and (x_1, y_1).

Therefore, $\quad rx_1 + sy_1 = a^2 \qquad \ldots(2)$

Since eqn. (1) and eqn. (2) are identical (in x_1 and y_1).

$$r/2a = -s/q = -a/2p$$

Now, let each of these ratios be R.

Now, we get

$$a = -2pR$$

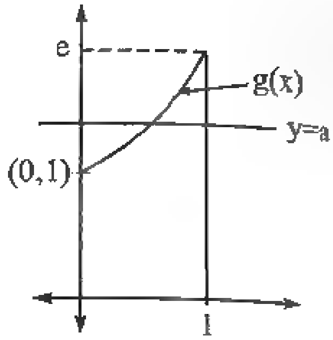
Since, $r = 2aR = -4p(R)^2$

And $s=-qR$

Eliminating R, we get

$$rq^2=-4ps^2$$

42. (b) Given $f(x)=\sqrt{e^x+x-a}$; $a \in R$



$f'(x)>0 \, \forall \, x \in R$

$\therefore \quad f(x)$ is increasing

Given $f(f(x_0))=x_0$

$[f(x)\geq 0 \, \forall \, x, \ \therefore \ f(f(x_0))\geq 0 \ \rightarrow \ x_0 \geq 0; \ x_0 \in [0,1]]$

$\Rightarrow \quad f(x_0)=f^{-1}(x_0)$ will have solution on $y=x$

Hence $f(x_0)=f^{-1}(x_0)=x_0$

$\Rightarrow \quad \sqrt{e^{x_0}+x_0-a}=x_0 \qquad$ where $x_0 \in [0,1]$

$e^{x_0}+x_0-a=x_0^2$

$\therefore \quad a=e^{x_0}+x_0-x_0^2 \qquad$ where $x_0 \in [0,1]$

Let $g(x)=e^x+x-x^2$; $a \in [0,1]$

$g'(x)=e^x+1-2x>0 \, \forall \, x \in [0,1]$

$\therefore \quad a \in [1,e]$

43. (a) Using limit of substitution, Put $\dfrac{x^n}{e^x}=t$.

Now, as $x \to \infty, \dfrac{x^n}{e^x} \to 0$

So, $\lim\limits_{t\to 0}\dfrac{2^t-3^t}{t}$

Add and subtract 1

$$\lim_{t\to 0}\frac{2^t-1-(3^t-1)}{t}=\lim_{t\to 0}\frac{2^t-1}{t}-\lim_{t\to 0}\frac{3^t-1}{t}$$

Using the relation, $\quad \lim\limits_{f(x)\to 0}\dfrac{a^{f(x)}-1}{f(x)}=\ln a$

The expression is equal to $\ln 2-\ln 3$.

44. (b) Do yourself.

45. (c) $\qquad \log_{12}(18)=a \qquad \log_{24}(54)=b$

$$\Rightarrow \frac{\log 18}{\log 12}=a \Rightarrow \frac{2\log 3+\log 2}{2\log 2+\log 3}=a \Rightarrow \frac{\log 54}{\log 24}=b \Rightarrow \frac{3\log 3+\log 2}{2\log 2+\log 3}=b$$

Let $\log 3$ be y and $\log 2$ be x.

$\therefore \quad a = \dfrac{x+2y}{y+2x}$

$\Rightarrow \quad a(y+2x) = x+2y \quad \Rightarrow \quad 2ax - x = 2y - ay$

$\Rightarrow \quad x(2a-1) = y(2-a) \quad \ldots(1)$

$\therefore \quad b = \dfrac{x+3y}{y+3x} \quad \Rightarrow \quad b(y+3x) = x = 3y$

$\Rightarrow \quad 3bx - x = 3y - by$

$\Rightarrow \quad x(3b-1) = y(3-b) \quad \Rightarrow \quad x\left(\dfrac{3b-1}{3-b}\right) = y \quad \ldots(2)$

Putting values of y in eqn. (1),

$$x(2a-1) = \frac{x(3b-1)(2-a)}{(3-b)}$$

$\Rightarrow \quad (2a-1)(3-b) = (3b-1)(2-a)$

$\Rightarrow \quad 6a - 2ab - 3 + b = 6b - 3ab - 2 + a$

$\Rightarrow \quad ab + 5(a-b) = 1$

$\Rightarrow \quad (a+b)^2 - a^2 - b^2 + 10(a-b) = 2$

$\Rightarrow \quad (a+b)^2 + a(10-a) - b(10+b) = 2$

46. (b) $\quad a_n = 16\left(\dfrac{1}{4}\right)^{n-1}; \quad \therefore \quad P_n = \prod_{k=1}^{n} 16\left(\dfrac{1}{4}\right)^{k-1}$

$$= 16^n \prod_{k=1}^{n}\left(\frac{1}{4}\right)^{k-1} = 16^n\left(\frac{1}{4}\right)^{0+1+2+\ldots\ldots+(n-1)}$$

$$= 16^n\left(\frac{1}{4}\right)^{\sum_{i=0}^{n-1} i} = 16^n\left(\frac{1}{4}\right)^{\frac{n(n-1)}{2}}$$

$$= 2^{4n}\cdot 2^{-n(n-1)} = 2^{n(5-n)}$$

$\Rightarrow \quad P_n^{1/n} = 2^{5-n}$

$\therefore \quad \sum_{n=1}^{\infty} P_n^{1/n} = \sum_{n=1}^{\infty} 2^{5-n} = 2^5 \sum_{n=1}^{\infty} 2^{-n} \ldots\ldots\ldots$

47. (a) Do yourself.

48. (c) Do yourself.

49. (c) Do yourself.

50. (b) $(f(x)-1)^2(f(x)-x^3) = 0$

$\therefore \quad f(x) = x^3$

$\quad f'(x) = 3x^2$

$\Rightarrow \quad f'(8)=192 \quad \text{and} \quad g'(8)=\frac{1}{f'(2)}=\frac{1}{12}$

$\therefore \quad f'(8)\times(f^{-1})'(8)=192\times\frac{1}{12}=16$

51. (a) If A and B are equivalence then $A\cap B$ is also equivalence.

52. (c) Let $f(x)=x^{10}+x^9+\ldots\ldots+x+1=(x-x_1)(x-x_2)\ldots\ldots(x-x_{10})$

$\ln f(x)=\sum_{i=1}^{10}\ln(x-x_i)$

On differentiating and put $x=1$, we get

$$\sum_{n=1}^{10}\left(\frac{1}{1-x_n}\right)=\frac{f'(1)}{f(1)}=5$$

53. (b) $\quad e^x=\frac{f'(x)}{1-f'(x)} \quad ; \quad f'(x)=\frac{e^x}{1+e^x} \quad ; \quad f(x)=\ln(1+e^x)+C$

$f(0)=0 \quad \Rightarrow \quad \ln(2)+C=0 \quad \Rightarrow \quad C=-\ln 2$

$f(x)=\ln(1+e^x)-\ln 2 \quad \Rightarrow \quad \lim_{x\to 0}(1+f(x))^{1/x}=\sqrt{e}$

54. (a) $\sum_{n=2}^{2021}\frac{1}{(\alpha_n+1)(\beta_n+1)}=\sum_{n=2}^{2021}\frac{1}{n(n-1)}=\sum_{n=2}^{2021}\frac{1}{(n-1)}-\frac{1}{n}=1-\frac{1}{2021}=\frac{2020}{2021}=\frac{a}{b}$

$\therefore \quad b-a=1$

55. (c) $f'(x)=\begin{cases} e^{-x} & , \quad x<0 \\ e^x & , \quad 0<x<1 \\ 2e^x & , \quad x>1 \end{cases}$

f will be one-one and if $f(0^+)\geq f(0)$

$\Rightarrow \quad 2\geq k-1 \quad \Rightarrow \quad k\leq 3$

and $\quad f(1^+)\geq f(1^-) \Rightarrow \lambda+e\geq e+1 \quad \Rightarrow \quad \lambda\geq 1$

56. (c) Given limit $=\pi\int_0^1\frac{1}{\sin\left(\frac{\pi}{4}(x+1)\right)}dx$

Put $(x+1)\frac{\pi}{4}=t$

$\therefore \quad I=4\int_{\pi/4}^{\pi/2}\operatorname{cosec} t\, dt$

$\Rightarrow \quad I=4\ln\left(\tan\frac{t}{2}\right)\Big]_{\pi/4}^{\pi/2}$

$I=4[0-\ln(\sqrt{2}-1)]$

$I=4\ln(\sqrt{2}+1)$

57. (b) $\int_0^\infty f(x)dx = \int_0^{\tan 1} \cos x\, dx + \int_{\tan 1}^\infty 0\, dx = \sin(\tan 1)$

58. (d) $\because \quad f''(x) = f''(5-x) \quad \Rightarrow \quad f'(x) = -f'(5-x) + C$

$\Rightarrow \quad f'(x) + f'(5-x) = C = 8$

Let $I = \int_1^4 f'(x)\,dx$

Using King

$$I = \int_1^4 f'(5-x)\,dx$$

$$\therefore \quad 2I = \int_1^4 8\,dx \quad \Rightarrow \quad I = 12$$

59. (c) $$I - \int \frac{3(\tan x - 1)\sec^2 x}{(\tan x + 1)\sqrt{\tan^3 x + \tan^2 x + \tan x}}\,dx = 3\int \frac{(t-1)}{(t+1)\sqrt{t^3 + t^2 + t}}\,dt$$

$$= 3\int \frac{\left(1 - \frac{1}{t^2}\right)}{\left(t + \frac{1}{t} + 2\right)\sqrt{t + \frac{1}{t} + 1}}\,dt \qquad \text{Let } t + \frac{1}{t} + 1 = z^2 \Rightarrow \left(1 - \frac{1}{t^2}\right)dt = 2z\,dz$$

$$= 6\int \frac{dz}{(z^2+1)} = 6\tan^{-1}\sqrt{1 + \frac{1}{t} + 1} + C$$

60. (c) Differentiating both sides, we get

$$x^{26}(x-1)^{17}(5x-3) = \frac{1}{k}\{x^{27} \cdot 18(x-1)^{17} + (x-1)^{18} 27x^{26}\}$$

$$= \frac{x^{26}(x-1)^{17}}{k}(18x + 27(x-1))$$

$$\therefore \quad k = 9$$

61. (c) $f(x)$ is discontinuous in [1, 7] at two points *i.e.*, $x = 5, 7$.

62. (a) $\because \quad f'(x) = \int_0^x \ln(1+t^2)\,dt + x\ln(1+x^2)$

$$f''(x) = 2\ln(1+x^2) + \frac{2x^2}{1+x^2}$$

$\therefore \quad f''(0) = 0$

63. (a) $\because \quad f(1) = 3 \quad \Rightarrow \quad g(3) = 1$

$\therefore \quad$ Point = (3, 1)

$$g'(f(x)) = \frac{1}{f'(x)} \quad \Rightarrow \quad g'(3) = \frac{1}{f'(1)} = \frac{1}{4}$$

$\therefore$ Tangent $\Rightarrow$ $y-1=\frac{1}{4}(x-3)$ $\Rightarrow$ $x-4y+1=0$

64. (c) $\because$ Slope of tangent $=-1$

Let point of contact be (x_1, y_1)

$$\therefore \quad \frac{dy}{dx}=\frac{x}{4y}=-1 \quad \Rightarrow \quad x_1=-4y_1$$

$$\because \quad x_1^2-4y_1^2=4 \quad \Rightarrow \quad y_1=\pm\frac{1}{\sqrt{3}}$$

$$\therefore \quad \text{Length of sub-tangent}=\left|y_1\frac{dx}{dy}\right|=\left|\frac{1}{\sqrt{3}}\times 1\right|=\frac{1}{\sqrt{3}}$$

$\therefore$ $k=3$

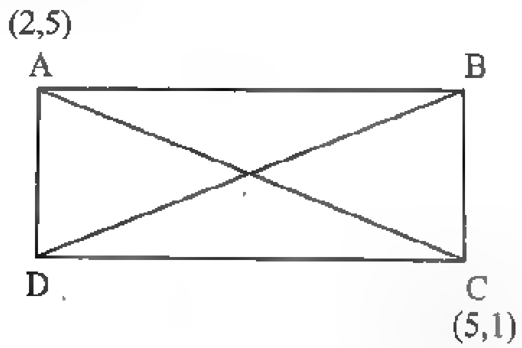
65. (d) $\frac{dr}{dt}=5\,\text{cm/sec}; r=8$

$$A=\pi r^2 \quad \Rightarrow \quad \frac{dA}{dt}=2\pi r\frac{dr}{dt}=80\pi\,\text{cm}^2/\text{sec}$$

66. (c) $\because$ Diagonal bisect each other

$\therefore$ mid-point on AC *i.e.*, $\left(\frac{7}{2},3\right)$ will be on $y=2x+k$

$\therefore$ $3=7+k \Rightarrow k=-4$



67. (c) Equation of line parallel to $3x-y=7$ is $3x-y=\lambda$

$\because$ (1, 2) is on it.

$\therefore$ $\lambda=1$

Now, point of intersection of $3x-y=1$ and $x+y+5=0$ is $(-1,-4)$.

$\therefore$ Distance between (1, 2) and $(-1,-4)=\sqrt{2^2+6^2}=\sqrt{40}$

68. (b) $A\in[4,5]$, $f(x)$ is increasing in [4, 5]

$$\therefore \quad f(5)|_{\max}=7$$

69. (b) $\because$ λ lies between the roots

$\therefore$ $f(\lambda)<0$

$\Rightarrow$ $2\lambda^2-2(2\lambda+1)\lambda+\lambda(\lambda+1)<0$

$\Rightarrow \quad -\lambda^2 - \lambda < 0 \Rightarrow \lambda^2 + \lambda > 0$

$\Rightarrow \quad \lambda < -1 \quad$ or $\quad \lambda > 0$

$\therefore \quad$ Least non-negative integral value of $\lambda = 1$.

70. (a) $S = -d\,(a_1 + a_2 + \ldots\ldots + a_{2k})$, where d is common difference

$$S = -d \cdot \frac{2k}{2} \cdot (a_1 + a_{2k}) = -kd\,(a_1 + a_{2k}) \quad \ldots(1)$$

Now, $a_2 - a_1 = d$

$a_3 - a_2 = d$

$\vdots$

$a_{2k} - a_{2k-1} = d$

Add $\quad a_{2k} - a_1 = (2k-1)d \quad \ldots(2)$

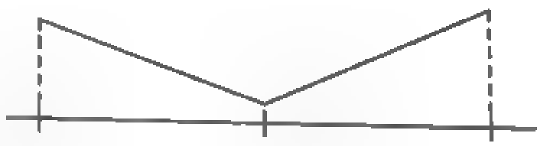
From eqn. (1) and eqn. (2)

$$S = \frac{k}{2k-1}(a_1^2 - a_{2k}^2)$$

71. (d) $f(t) = t^2 + t, \; t \in [-1, 1]$

Maximum value $= 2$ at $t = 1$

Minimum value $= \dfrac{-1}{4}$ at $t = \dfrac{-1}{2}$.



72. (a) $\because \quad$ L.H.S. $< \dfrac{\pi}{2}$ and R.H.S. $> \dfrac{\pi}{2}$

$\therefore \quad$ No solution.

73. (a) $\because \quad \lim\limits_{x \to 0} \dfrac{f(x) - 5}{x} = f'(0) = 4$

$\because \quad g(x) = (x^2 + 2x + 3)\,f(x)$

$\Rightarrow \quad g'(x) = (2x+2)\,f(x) + (x^2 + 2x + 3)\,f'(x)$

$\Rightarrow \quad g'(0) = 2f(0) + 3f'(0) = 10 + 12 = 22$

74. (d) Using LMVT in $[-2, 5]$

$$f'(c) = \frac{f(5) - f(-2)}{7}$$

$\therefore \quad -4 \le \dfrac{f(5) - f(-2)}{7} \le 3$

$\Rightarrow \quad -28 + f(-2) \le f(5) < 21 + f(-2)$

$\therefore \quad$ Difference $= 49$

75. (d) $\because \quad f'(x) = ax(x-1) \quad \Rightarrow \quad f'(2) = 6 \quad \Rightarrow \quad a = 3$

$f'(x) = 3(x^2 - x) \quad \Rightarrow \quad f(x) = x^3 - \dfrac{3x^2}{2} + C$

$\therefore \quad f(x) = x^2\left(x - \dfrac{3}{2}\right)$

$f(2) = 2 \;\Rightarrow\; C = 0$

76. (c) $\because \quad f'(x) \geq 0 \quad \Rightarrow \quad p^2 \geq \dfrac{2^4}{2^{x^2}} \quad \Rightarrow \quad p^2 \geq 16$

$\Rightarrow \quad p \in (-\infty, -4] \cup [4, \infty).$

77. (d) $\because \quad I = \int_1^{\sqrt{3}} (x^{x^2})^2 (x + 2x + \ln x)\, dx$

Let $x^{x^2} = t \Rightarrow x^{x^2}(x + 2x + \ln x)\, dx = dt$

$$= \int_1^{3\sqrt{3}} t\, dt = \left(\frac{t^2}{2}\right)_1^{3\sqrt{3}} = \frac{27}{2} - \frac{1}{2} = 13$$

78. (a) Putting $x = 0$ in given relation

$$0 = 1 - a\int_0^1 f(t)e^{-t}\, dt \quad \Rightarrow \quad a\int_0^1 f(t)e^{-1}\, dt\; t = 1$$

$$\therefore \quad \int_0^x f(t)\, dt = e^x - e^{2x} \quad \Rightarrow \quad f(x) = e^x - 2e^{2x}$$

$$f(1) + 2f(2) = e - 2e^2 + 2e^2 - 4e^4 = e - 4e^4$$

79. (a) $\quad f(3) = \int_2^3 \dfrac{1}{1+t^4}\, dt$

$\therefore \quad 2 < t < 3$

$$\frac{1}{1+t^4} < \frac{1}{17}$$

$$\Rightarrow \quad f(3) < \int_2^3 \frac{1}{17}\, dt \quad \Rightarrow \quad f(3) < \frac{1}{17}$$

80. (b) Clearly, $g(x) = 0 \;\forall\; x \in R$

$\therefore \quad f(x) = -2$

$\therefore \quad f'(x) = 0$

81. (c) $\int_\alpha^\beta f(x)\, dx + \int_\alpha^\beta f^{-1}(x)\, dx = 13$

$\Rightarrow \quad \beta^2 - \alpha^2 = 13$

$$(\beta - \alpha)(\beta + \alpha) = 13 \times 1$$

$\therefore \quad \beta - \alpha = 1 \quad$ and $\quad \beta + \alpha = 13$

$\Rightarrow \quad \beta = 7,\ \alpha = 6$

82. (c) $$\lim_{h\to 0}\frac{f(3+7h)-f(3+4h)}{h}=4$$

Using L' hospital rule

$\Rightarrow \quad 7f'(3)-4f'(3)=4$

$\Rightarrow \quad f'(3)=\frac{4}{3}$

83. (c) $\because \quad 2b=a+a^2$ and $(a^2)^2=ab \Rightarrow a^3=ab$

$\Rightarrow \quad 2a^3=a^2+a \Rightarrow a(2a^2-a-1)=0$

$\Rightarrow \quad a(2a+1)(a-1)=0$

$\therefore \quad a<0$

$\therefore \quad a=\frac{-1}{2},\ b=\frac{\frac{-1}{2}+\frac{1}{4}}{2}=\frac{-1}{8}$

$\therefore$ G.P. is $\frac{-1}{2},\frac{1}{4},\frac{-1}{8},\ldots\ldots\ldots$

$\therefore$ Sum $=\frac{\frac{-1}{2}}{1+\frac{1}{2}}=\frac{-1}{3}$

84. (c) $x\,f(x)=x+\int_1^x f(t)\,dt \quad\Rightarrow\quad xf'(x)+f(x)=1+f(x)$

$\Rightarrow \quad f'(x)=\frac{1}{x} \quad\Rightarrow\quad f(x)=\ln x+C$

$\Rightarrow \quad f(x)=\ln x+1 \quad\Rightarrow\quad f(e^k)=k+1$

$\Rightarrow \quad \sum_{k=1}^{10} f(e^k)=\sum_{k=1}^{10}(k+1)=\frac{10\times 11}{2}+10=65$

85. (d) Do yourself.

86. (b) $$h(x)=fog(x)=\begin{cases}1-\sqrt{x}, & x\in Q\\ (1-x)^2, & x\notin Q\end{cases}$$

$h\left(1+\frac{1}{\sqrt{x}}\right)=h\left(1-\frac{1}{\sqrt{x}}\right) \quad\Rightarrow\quad$ many one

range is not $R \quad\Rightarrow\quad$ into

87. (c) putting $x=y=1$, we get $f(1)=2$

putting $y=1$, $f(x)=x+1$

$\therefore \quad f^{-1}(x)=x-1$

$\therefore \quad f(x)f^{-1}(x)=(x^2-1).$

88. (d) LHL $= \lim_{x\to 0^-} \frac{\tan(\pi \sin^2 x)}{x^2} + \left(\frac{-x+\sin x}{x}\right)^2 = \pi$

RHL $= \lim_{x\to 0^+} \frac{\tan(\pi \sin^2 x)}{x^2} + \left(\frac{x-0}{x}\right)^2 = \pi + 1$

$\therefore$ Limit does not exist.

89. (b) Do yourself.

90. (c) Do yourself.

91. (d) $\because \begin{vmatrix} \lambda-1 & 3\lambda+1 & 2\lambda \\ \lambda-1 & 4\lambda-2 & \lambda+3 \\ 2 & 3\lambda+1 & 3(\lambda-1) \end{vmatrix} = 0 \Rightarrow \lambda = 0$ or 3

If $\lambda = 0$, then $x = y = z$

$\therefore \quad x : y : z = 1 : 1 : 1$

92. (a) Clearly, A is skew symmetric and B is symmetric and $|A| = 0$

$\therefore \quad |A^4 B^3| = 0$

$\therefore$ Singular.

93. (c) $\because \quad (I - \alpha A)(I - 0.4A) = I$

$\Rightarrow \quad I - (\alpha + 0.4)A + 0.4\alpha A^2 = I$

$\Rightarrow \quad -(\alpha + 0.4)A + 0.4\alpha A = 0$

$\Rightarrow \quad 0.6\alpha = -0.4$

$\Rightarrow \quad \alpha = \frac{-2}{3}$

94. (c) $\because \quad A^2 = A \cdot A = ABA = AB = A$

and $\quad B^2 = B \cdot AB = BA = B$

$\therefore \quad A = A^2 = A^3 = \ldots\ldots$ and $B = B^2 = B^3 = \ldots\ldots\ldots$

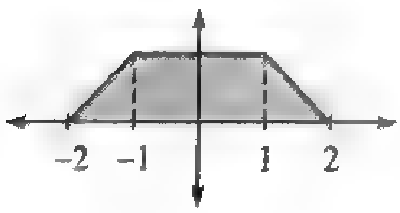
$(A^{2019} + B^{2019})^{2020} = (A + B)^{2020}$

$(A + B)^2 = A^2 + B^2 + AB + BA = 2(A + B)$

and $\quad (A + B)^3 = 2(A + B)(A + B) = 2^2 (A + B)$

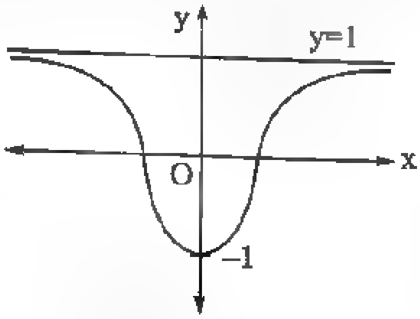
$(A + B)^{2020} = 2^{2019}(A + B)$.

95. (b)



Area bounded $= \frac{1}{2} \times 1 \times 1 + 2 \times 1 + \frac{1}{2} \times 1 \times 1 = 3$

96. (b)



$$\text{Area} = 2\int_0^{\infty}\left(1-\left(\frac{x^2-1}{x^2+1}\right)\right)dx = 4(\tan^{-1}x)_0^{\infty} = 2\pi$$

97. (b) $(1+\tan y)\frac{dx}{dy}+2x=(1+\tan y) \Rightarrow \frac{dx}{dy}+\frac{2x}{1+\tan y}=1$

$$\text{I.F.} = e^{\int\frac{2}{1+\tan y}dy} = e^{\int\left(1+\frac{\cos y-\sin y}{\cos y+\sin y}\right)dy} = e^{y+\ln(\cos y+\sin y)} = (\cos y+\sin y)e^y$$

$\therefore$ Solution is $x e^y(\cos y+\sin y)=\int e^y(\sin y+\cos y)dy - e^y\sin y + C$

98. (a) $\because \frac{dx}{dy}+\frac{x^2}{y^2}-\frac{x}{y}+1=0$

Let $\frac{x}{y}=t \Rightarrow x=ty \Rightarrow \frac{dx}{dy}=y\frac{dt}{dy}+t$

$\therefore y\frac{dt}{dy}+t+t^2-t+1=0 \Rightarrow \frac{dt}{t^2+1}=\frac{-dy}{y}$

$\Rightarrow \tan^{-1}(t)+\ln y+C=0 \Rightarrow \tan^{-1}\left(\frac{x}{y}\right)+\ln y+C=0$

99. (d) $\because \int\left(\frac{y+1}{y}\right)dy=\int e^x(\sin 2x-\cos^2 x)dx$

$\Rightarrow y+\ln y=-e^x\cos^2 x+C$

$\Rightarrow x=0,\ y=1$

$\therefore C=2$

100. (d) $f'(x)=3x^2-3=3(x-1)(x+1)$

$f(1)=-1,\ f(-1)=3$

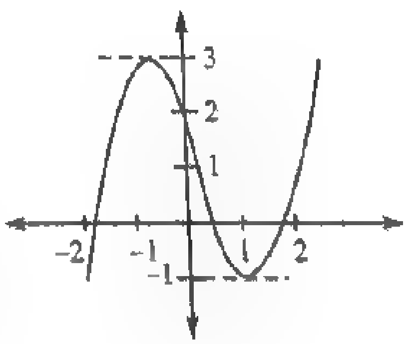
$f(x)=0 \Rightarrow x=x_1,x_2,x_3$

where $-2<x_1<-1, 0<x_2<1, 1<x_3<2$

$\therefore f(f(x))=0 \Rightarrow f(x)=x_1$ has 1 solution

$f(x)=x_2$ has 3 solutions and $f(x)=x_3$ has 3 solutions.

$\therefore$ Number of solutions = 7.



101. (b) $y = e^x \sin\left(\frac{\pi}{2} - x\right) = e^x \cos x$

$\therefore$ Slope of tangent, $S = e^x(\cos x - \sin x)$

$\therefore$ $\frac{dS}{dx} = -2e^x \sin x$

sign of $\frac{dS}{dx}$ = $\underset{0}{|}$ —— $\underset{\pi}{|}$ $\overset{+}{\text{——}}$ $\underset{2\pi}{|}$

$\therefore$ Slope is minimum when $x = \pi$.

102. (b) $\frac{dy}{dx} = (2x^2+1)e^{x^2} \Rightarrow \left.\frac{dy}{dx}\right|_{x=1} = 3e$

$\therefore$ Tangent is $y - e = 3e(x-1)$ passes through $\left(\frac{4}{3}, 2e\right)$.

103. (b) $\because$ $\lim_{x\to 0} f(x) = e^{\lim_{x\to 0}\frac{ab}{x^2}\left(\sin\left(\frac{2x^2}{a}\right) + \cos\left(\frac{3x}{b}\right) - 1\right)} = f(0) = e^3$

$$\Rightarrow \lim_{x\to 0} ab\left[\frac{2}{a}\frac{\sin\left(\frac{2x^2}{a}\right)}{\frac{2x^2}{a}} - \left(\frac{3}{b}\right)^2 \frac{\left(1-\cos\frac{3x}{b}\right)}{\left(\frac{3x}{b}\right)^2}\right] - 3$$

$$\Rightarrow ab\left(\frac{2}{a} - \frac{a}{2b^2}\right) = 3 \Rightarrow 4b^2 - 4a = 6b$$

$$\Rightarrow 4b^2 \quad 6b - 9a = 0$$

$\therefore$ b is real.

$\therefore$ $D \geq 0$

$$\Rightarrow 36 + 144a \geq 0 \Rightarrow a \geq \frac{-1}{4}.$$

104. (c) Do yourself.

105. (a) Let $g(x) = \frac{f(x)}{x}$

Here $g(a) = g(b)$

$\therefore$ According to Rolle's Theorem, $g'(x) = 0$ for some $x_0 \in (a, b)$

$\therefore$ $\frac{xf'(x) - f(x)}{x^2} = 0 \Rightarrow x_0 f'(x_0) = f(x_0)$.

106. (c) $\because$ $f(2x) = f(x)$

replace x by $\frac{x}{2}$

$\therefore \quad f(x)=f\left(\frac{x}{2}\right)=f\left(\frac{x}{2^2}\right)=\ldots\ldots\ldots=f\left(\frac{x}{2^n}\right)$

$\Rightarrow \quad \lim_{n\to\infty} f(x)=\lim_{n\to\infty} f\left(\frac{x}{2^n}\right) \Rightarrow f(x)=f(0)=\text{constant}$

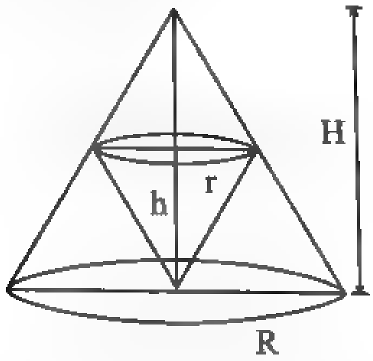
$\because \quad f(1)=3$

$\therefore \quad f(x)=3\ \forall\ x\in R$

$\therefore \quad \int_{-1}^{1} f(f(x))\,dx=\int_{-1}^{1} 3\,dx=6.$

107. (b) $\frac{H-h}{r}=\frac{H}{R} \quad \Rightarrow \quad r=\frac{(H-h)R}{H}$

$\therefore \quad V=\frac{\pi r^2 h}{3}=\frac{\pi R^2}{3H^2}h(H-h)^2$



$$\frac{dV}{dh}=\frac{\pi R^2}{3H^2}[(H-h)^2-2h(H-h)]=\frac{\pi R^2}{3H^2}(H-h)(H-3h)$$

$\therefore \quad V_{max}$ when $h=\frac{H}{3} \Rightarrow \frac{H}{h}=3$

108. (c) $f(x)=\int \frac{(3x^2-x^{-2})}{\left(x^3+1+\frac{1}{x}\right)^2}dx=\frac{-1}{\left(x^3+1+\frac{1}{x}\right)}+C=\frac{-x}{x^4+x+1}+C$

$f(0)=0 \Rightarrow C=0$

$\therefore \quad f(-1)=\frac{1}{1-1+1}=1$

109. (a) $\because$ $f(x)$ is odd function.

$\therefore$ fourth derivative is also odd.

$\therefore \quad f(0)=0$

110. (c) D must be perfect square and $D>0$

$D=(n+1)^2-4n(n+2)=-3n^2-6n+1=4-3(n+1)^2$

$\therefore$ Possible values of n are $-1, 0$.

At $n=-2, D<0$.

111. (a) $\because \quad \frac{1}{p}+\frac{1}{q}+\frac{r}{pq}=\frac{p+q+r}{pq}=\frac{\cos 55^\circ+2\cos 120^\circ \cos 55^\circ}{pq}=0$

112. (c) $\because \quad 4\cos x\cos\left(\frac{\pi}{3}-x\right)\cos\left(\frac{\pi}{3}+x\right)=1$

$\Rightarrow \quad \cos 3x=1 \quad \Rightarrow \quad 3x=2n\pi,\ n\in I$

$\Rightarrow \quad x=\frac{2\pi}{3},\ \frac{4\pi}{3}$

$\therefore \quad$ Sum $=2\pi$

113. (a) Do yourself.

114. (d) Do yourself.

115. (c) $\lim_{x\to 0}\left(\frac{\sin 3x+ax}{x^3}+b\right)=0;\quad \lim_{x\to 0}\left(\frac{3\sin x+ax}{x^3}\right)=4-b;\quad \lim_{x\to 0}\frac{3\left(x-\frac{x^3}{6}\right)+ax}{x^3}$

$a+3=0$ and limit $=\frac{-1}{2}$;

$\therefore \quad 4-b=\frac{-1}{2} \quad ; \quad b=\frac{9}{2}$ and $a=-3$

$\therefore \quad a+b=\frac{9}{2}-3=\frac{3}{2}$

116. (d) $\lim_{x\to 0}f(x)=\lim_{x\to 0}\left(\frac{2^x+3^x+5^x}{3}\right)^{\frac{3}{x}}=\left(\frac{1+1+1}{3}\right)^{\frac{3}{0}}=1^\infty$ form

$\lim_{x\to 0}e^{\left(\frac{2^x+3^x+5^x}{x}\right)}=\lim_{x\to 0}e^{\left(\frac{2^x-1}{x}+\frac{3^x-1}{x}+\frac{5^x-1}{x}\right)}=\lim_{x\to 0}e^{\ln(2\times 3\times 5)}=30$

117. (c) $-1\le \sin x\le 1 \Rightarrow \frac{-\pi}{2}\le\frac{\pi}{2}\sin x\le\frac{\pi}{2} \Rightarrow \sin\left(\frac{-\pi}{2}\right)\le\sin\left(\frac{\pi}{2}\sin x\right)\le\sin\left(\frac{\pi}{2}\right)$

$\Rightarrow \quad \frac{-\pi}{6}\le\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\le\frac{\pi}{6} \Rightarrow \sin\left(\frac{-\pi}{6}\right)\le\sin\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right)\le\sin\left(\frac{\pi}{6}\right)$

$\Rightarrow \quad \frac{-1}{2}\le f(x)\le\frac{1}{2}.$

118. (c) $f(x)$ must be a linear polynomial.

$f(x)=x-1$

$\therefore \quad f(4)=3$

119. (b) $N_r=\frac{5\pi}{4}+\frac{3\pi}{4}=2\pi$

For D_r put $25\sin^2\theta+9\cos^2\theta=t$

$\therefore \quad D_r = \int_9^{25} \frac{1}{32}\sqrt{t}\,dt = \frac{96}{48}$

$\therefore \quad I = \frac{48\pi}{49}$

120. (a) $$\lim_{x\to\frac{\pi}{2}} \frac{4(x-\pi)\sin^2\left(\frac{\pi}{2}-x\right)}{-2\pi\left(\frac{\pi}{2}-x\right)^2 \frac{\tan\left(x-\frac{\pi}{2}\right)}{\left(x-\frac{\pi}{2}\right)}} = \frac{4\left(\frac{-\pi}{2}\right)}{-2\pi} = 1$$

121. (c) Given limit, $$I = \int_0^1 \frac{e^x + e^{-x}}{\sqrt{11 - e^{2x} - e^{2x}}}dx$$

Let $e^x - e^{-x} = t$

$$\therefore \quad I = \left(\sin^{-1}\left(\frac{e^x - e^{-x}}{3}\right)\right)_0^1 = \sin^{-1}\left(\frac{e - e^{-1}}{3}\right)$$

122. (a) $$y'' = \frac{1}{(xy'-y)}(xy''+y'-y') \quad \Rightarrow \quad xy'-y = x$$

$$\Rightarrow \quad \frac{dy}{dx} - \frac{y}{x} = 1$$

$$\text{I.F.} = e^{-\ln x} = \frac{1}{x}$$

$$\therefore \quad y\cdot\frac{1}{x} = \ln x + C \quad \Rightarrow \quad y = x\ln x + Cx$$

123. (b) Clearly function are inverse of each other.

$\therefore \quad \frac{e^x}{a} = \ln ax \Rightarrow \frac{e^x}{a} = x$ will have only one solution.

$\therefore \quad a = e \Rightarrow [a] = 2$

124. (a) $$\frac{dy}{dx} + y = xe^{-x}$$

$\therefore \quad \text{I.F.} = e^x$

Solution is $ye^x = \frac{x^2}{2} + C$, here $C = 0$

$\therefore \quad f(1) = \frac{1}{2e}$

125. (a) $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\sin t}{\cos t}$ and length of tangent $= y\sqrt{1+\left(\frac{dx}{dy}\right)^2} = \sin t\sqrt{1+\left(\frac{\cos t}{\sin t}\right)^2} = 1$

126. (d) $\because \quad x^4+(2-\sqrt{3})x^2+2+\sqrt{3}=(x-\alpha_1)(x-\alpha_2)(x-\alpha_3)(x-\alpha_4)$

Putting $x=1$, we get

$$(1-\alpha_1)(1-\alpha_2)(1-\alpha_3)(1-\alpha_4)=5$$

127. (c) $f'(x)=6x^2-30x+36=6(x^2-5x+6)=6(x-3)(x-2)$

$f'(x)=0$ at $x=2, 3$, so that this is not one-one.

Range of $f(x)$ is $[1, 29]$, this is onto.

128. (c) $\because \quad |a-a|\le 1 \forall a\in R$

$\therefore$ Reflexive

If $|a-b|\le 1$, then $|b-a|\le 1$

$\therefore$ Symmetric

If $|a-b|\le 1$ and $|b-c|\le 1 \not\Rightarrow |a-c|\le 1$

$\therefore$ Not transitive.

129. (c) $\because \quad -3\le |3x+4|\le 5 \Rightarrow 0\le |3x+4|\le 5$

$$\Rightarrow \quad -5\le 3x+4\le 5 \Rightarrow -3\le x\le \frac{1}{3}$$

130. (a) $\because \quad f(x)=(x^2+1)+\dfrac{1}{(x^2+1)}-1\ge 2-1 \Rightarrow f(x)\ge 1$

131. (c) $\because \quad y=(x+2)^2-2 \Rightarrow (x+2)^2=y+2$

$$\Rightarrow \quad x=-2+\sqrt{y+2}=f^{-1}(y)$$

$$\therefore \quad f^{-1}(x)=g(x)=\sqrt{2+x}-2$$

132. (b) $\because$

$$3f(x)+2f\left(\frac{x+59}{x-1}\right)=10x+30 \qquad \text{...(1)}$$

Replace x by $\dfrac{x+59}{x-1}$, we get

$$3f\left(\frac{x+59}{x-1}\right)+2f(x)=10\left(\frac{x+59}{x-1}\right)+30 \qquad \text{.. (2)}$$

From eqn. (1) × 3 – eqn. (2) × 2, we get

$$5f(x)=30x-20\left(\frac{x+59}{x-1}\right)+30$$

$$\therefore \quad 5f(7)=210-20\times 11+30=20 \quad \Rightarrow \quad f(7)=4$$

133. (a)

$$y=\frac{1-x}{1+x} \Rightarrow x=\frac{1-y}{1+y}=f^{-1}(y)$$

$\therefore$ Self inverse.

134. (b) $\because \quad f(0)=2 \Rightarrow f(f(0))=f(2)=8$

$$\Rightarrow \quad f(f(f(0)))=f(8)=4$$

$$\Rightarrow \quad f(f(f(f(0))))=f(4)=0$$

135. (b) $\because$ $f(x_1) = f(x_2)$ $\Rightarrow$ $\dfrac{2x_1}{1+2x_1} = \dfrac{2x_2}{1+2x_2}$

$\Rightarrow$ $x_1 + 2x_1x_2 = x_2 + 2x_1x_2$ $\Rightarrow x_1 = x_2$

$\therefore$ One-one

and $1 \notin f(x)$

$\therefore$ Into function.

136. (d) For $\alpha > 0$, $f(x)$ will be into if.

$$2\cdot 2+\alpha^2 > \alpha\cdot\frac{2}{2}+10 \quad \Rightarrow \quad \alpha^2-\alpha-6>0$$

$$\Rightarrow \quad (\alpha-3)(\alpha+2)>0 \quad \Rightarrow \quad \alpha>3$$

$$\therefore \quad \alpha_{\min} = 4$$

137. (d) $\because$ $y = h(x) = 3g(x)+7$ $\Rightarrow$ $g(x) = \dfrac{y-7}{3}$ $\Rightarrow$ $x = g^{-1}\left(\dfrac{y-7}{3}\right) = h^{-1}(y)$

$$\therefore \quad h^{-1}(x) = g^{-1}\left(\frac{x-7}{3}\right)$$

138. (c) $\because$ $P(1)=4,\quad P(2)=5,\quad P(3)=6$

$\because$ $P(x) = (x-1)(x-2)(x-3)$; $Q(x) = ax^2+bx+C$

Solving we get, $a = 0, b = 1, c = 2$

$$\therefore \quad 3a+2b+c=5$$

139. (a)

$$T_p = \frac{1}{q} = a+(p-1)d$$

$$T_q = \frac{1}{p} = a+(q-1)d$$

$$\therefore \quad a = \frac{1}{pq} = d$$

$$\therefore \quad T_{pq} = a+(pq-1)d = 1$$

Hence, T_{pq} is a root of given equation as 1 is one of the roots of given equation.

140. (c)

$$y = \log_2 x\left(1+\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\ldots\ldots\right) = 2\log_2 x \qquad \ldots(1)$$

$$4\log_4 x = \frac{5+9+13+\ldots\ldots+(4y+1)}{1+3+5+\ldots\ldots+(2y-1)}$$

$$2\log_2 x = \frac{2y^2+3y}{y^2} = y \quad \Rightarrow \quad y^2 = 2y+3$$

$\therefore$ $y = 3$ ($y = -1$, rejected)

and $x = 2^{3/2}$

$\therefore$ $x^2 y = 24$

141. (a) Domain $x \neq I$

$\therefore \quad \{x\}+\{-x\}=1$

Hence $f(x)=\dfrac{1}{2(1-\{x\})}-\{x\}=\dfrac{1}{2(1-\{x\})}+(1-\{x\})-1 \geq \sqrt{2}-1$ (by A.M. – G.M.)

Hence $a=\sqrt{2}-1=\tan\dfrac{\pi}{8}$

142. (a) $x=\sin^{-1}(\sin 10)=3\pi-10$

$y=\cos^{-1}(\cos 10)=4\pi-10$

$\therefore \quad y-x=\pi$

143. (b) $\because \quad x^2+(3-\lambda)x+2-\lambda=0$

$\therefore \quad \alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=(3-\lambda)^2-2(2-\lambda)=\lambda^2-4\lambda+5=(\lambda-2)^2+1$

$\therefore \quad \alpha^2+\beta^2$ will be minimum for $\lambda=2$

144. (c) Do yourself.

145. (b) Do yourself.

146. (c) $$\lim_{x\to\infty}\left(\frac{x^3+1}{x^2+1}-(ax+b)\right)=\lim_{x\to\infty}\left(\frac{x^3+1-(ax+b)(x^2+1)}{x^2+1}\right)$$

$$=\lim_{x\to\infty}\left(\frac{(1-a)x^3-bx^2-ax+(1-b)}{x^2+1}\right)=2$$

Therefore, $\quad 1-a=0 \quad \Rightarrow \quad a=1 \quad$ and $\quad -b=2 \;\Rightarrow\; b=-2$

147. (a) $\because$ Roots of 1st equation are imaginary.

$\therefore$ Both roots are in common.

$$\therefore \quad \frac{a}{1}=\frac{b}{2\lambda}=\frac{c}{\lambda^2+1}=k$$

$\because \quad a+b>c \quad \Rightarrow \quad 1+2\lambda>\lambda^2+1 \quad \Rightarrow \quad \lambda\in(0,2)$

148. (c) $$\lim_{x\to 2}\left(\frac{1}{x-2}-\frac{2(2x-3)}{x(x-1)(x-2)}\right)=\lim_{x\to 2}\frac{x^2-5x+6}{x(x-1)(x-2)}=\lim_{x\to 2}\frac{(x-3)}{x(x-1)}=\frac{-1}{2}$$

149. (c) Domain of $f(x):(2,\infty)$ and domain of $g(x):(-\infty,1)\cup(2,\infty)$

$\therefore$ Function will be identical when $x\in(2,\infty)$

150. (a) Do yourself.

151. (c) Do yourself.

152. (b) Do yourself.

153. (a) $$\because \quad (f(x)-3)\left(f\left(\frac{1}{x}\right)-3\right)=4$$

$\therefore \quad f(x)-3=\pm 2x^n \quad \Rightarrow \quad f(x)=3\pm 2x^n$

$\because \quad f(2)=3+2\cdot 2^n=11 \quad \Rightarrow \quad 2^n=4 \quad \Rightarrow \quad n=2$

$\therefore \quad f(x)=3+2x^2 \quad \Rightarrow \quad f(3)=21$

154. (b) For onto range : $\left[0, \frac{2\pi}{3}\right]$

$$\frac{-1}{\sqrt{3}} \le (x^2 - 4x + \alpha) < \infty \quad \Rightarrow \quad (x^2 - 4x + \alpha) > \frac{-1}{\sqrt{3}} \text{ for into function}$$

$$\therefore \quad x^2 - 4x + \left(\alpha + \frac{1}{\sqrt{3}}\right) > 0$$

$$\therefore \quad D = 16 - 4\left(\alpha + \frac{1}{\sqrt{3}}\right) < 0$$

$$\Rightarrow \quad \left(\alpha + \frac{1}{\sqrt{3}}\right) > 4 \quad \rightarrow \quad \alpha > 4 - \frac{1}{\sqrt{3}}$$

$\therefore$ Least integral value of $\alpha = 4$

155. (b) Do yourself.

156. (d) Do yourself.

157. (a) $\because \quad |x^2 + 6x + 6 = |x^2 + 4x + 9| + |2x - 3|$

$$\therefore \quad (x^2 + 4x + 9) + (2x - 3) \ge 0 \quad \rightarrow \quad x \ge \frac{3}{2}$$

158. (c) $(\sin^{-1} x + \sin^{-1} y)^2 = \pi^2 \quad \Rightarrow \quad \sin^{-1} x + \sin^{-1} y = \pm\pi$

$\therefore \quad x = y = \pm 1$

$\therefore \quad x^2 + y^2 = 2$

159. (b) $\tan^{-1}(\tan(f(-5) + f(20) + \cos^{-1}(f(\ 10) + f(17))))$

$$= \tan^{-1}(\tan(2 + 3 + \cos^{-1}(1 - 2))) = \tan^{-1}(\tan(5 + \pi)) = \tan^{-1}(\tan 5) = 5 - 2\pi$$

160. (d) $\cos^{-1}\left(\frac{2}{3x}\right) + \cos^{-1}\left(\frac{3}{4x}\right) = \frac{\pi}{2}$

$$\Rightarrow \quad \cos^{-1}\left(\frac{2}{3x}\right) = \sin^{-1}\left(\frac{3}{4x}\right) \quad \Rightarrow \quad \frac{2}{2x} = \sqrt{1 - \frac{9}{16x^2}}$$

$$\Rightarrow \quad \frac{4}{9x^2} = \frac{16x^2 - 9}{16x^2}$$

$$\Rightarrow \quad 64 = 144x^2 - 81 \quad \Rightarrow \quad x^2 = \frac{145}{144} \quad \Rightarrow \quad x = \frac{\sqrt{145}}{12}$$

161. (b) Do yourself.

162. (a) Let common difference be d and number of terms be n.

$$\therefore \quad m = l + 4d \quad \Rightarrow \quad d = \frac{m - l}{4}$$

$$p = 1 + (n - 1)d \quad \Rightarrow \quad n - 1 = \frac{4(p - l)}{(m - l)} \quad \Rightarrow \quad n = \frac{4p + m - 5l}{m - l}$$

$\therefore \quad \text{Sum} = \frac{n}{2}(l+p) = \frac{(l+p)(4p+m-5l)}{2(m-l)}$

$\Rightarrow \quad k = 2$

163. (c) $\because \quad (a-2)^2+(c-2)^2+(a-b)^2+(b-c)^2=0$

$\Rightarrow \quad a=b=c=2$

$\because$ Roots of $2x^2+2x+2=0$ are imaginary.

164. (d) Let roots be $a-3d, a-d, a+d, a+3d$

$\therefore$ Their sum $=4a=0 \quad \Rightarrow \quad a=0$

product $=9d^4=225 \quad \rightarrow \quad d^2=5$

and $q=$ sum of product taken two at a time $=-10d^2=-50$

165. (a) $\text{Sum} = \frac{1}{3}\left(\frac{1}{3}+\frac{1}{6}+\frac{1}{10}+\frac{1}{15}+\frac{1}{21}+\ldots\ldots\right)$

$=\frac{2}{3}\left(\frac{1}{2\cdot3}+\frac{1}{3\cdot4}+\frac{1}{4\cdot5}+\frac{1}{5\cdot6}+\ldots\ldots\right)$

$=\frac{2}{3}\left(\frac{1}{2}-\frac{1}{3}+\frac{1}{3}-\frac{1}{4}+\ldots\ldots\right)=\frac{1}{3}$

166. (b) Let $\frac{a+b}{c}=t$

$\because \quad (a+b)^2=(a+c)(b+c) \quad$ and $\quad c=\frac{2ab}{a+b}$

$=ab+(a+b)c+c^2$

$\Rightarrow \quad (a+b)^2=\frac{(a+b)c}{2}+(a+b)c+c^2$

Dividing by c^2, we get

$$\left(\frac{a+b}{c}\right)^2=\frac{3}{2}\left(\frac{a+b}{c}\right)+1$$

$\therefore \quad 2t^2-3t-2=0 \quad \Rightarrow \quad (t-2)(2t+1)=0$

$\Rightarrow \quad t=2$

167. (b) **Case-I :** Both equations have both the roots in common.

i.e., $\quad \frac{1}{1}=\frac{2k-6}{2k-2}=\frac{7-3k}{3k-5} \quad \Rightarrow \quad$ no value of k

Case-II : Equation $x^2+(2k-6)x+7-3k=0$ has equal roots and equation $x^2+(2k-2)x+(3k-5)=0$ has equal roots

$\therefore \quad (2k-6)^2-4(7-3k)=0 \Rightarrow 4k^2-12k+8=0 \Rightarrow k^2-3k+2=0 \Rightarrow k=1,2$

$(2k-2)^2-4(3k-5)=0 \Rightarrow 4k^2-20k+24=0 \Rightarrow (k-2)(k-3)=0 \Rightarrow k=2$

$\therefore \quad k=2$

168. (c) $x^2-4x+5=x-1$

$x^2-5x+6=0$

$(x-2)(x-3)=0$

$x=2 \qquad x=3$

$x_1=3 \qquad x_2=2$

$=\tan(x_1)\pi+\sec(x_2)\pi$

$=\tan 3\pi+\sec 2\pi$

$=0+1$

169. (a) $\sin\alpha+\sin\beta+\sin\gamma=-3$

Only possible when $\alpha=\beta=\gamma=\dfrac{3\pi}{2}$

$\therefore \quad \cos 2\alpha+\cos 4\beta+\cos 6\gamma=\cos 3\pi+\cos 6\pi+\cos 9\pi=-1+1-1=-1$

170. (b) $|x-3|^{3x^2-10x+3}=1$

Taking log on both the sides

$$(3x^2-10x+3)\log(|x-3|)=0$$

$\Rightarrow \quad \log(|x-3|)=0 \quad$ or $\quad 3x^2-10x+3=0$

$\Rightarrow \quad x=4,\ 2 \quad$ or $\quad x=3$ (rejected) or 1/3

171. (a) $$\underbrace{\frac{\sin x}{\cos 3x}}_{t_1}+\underbrace{\frac{\sin 3x}{\cos 9x}}_{t_2}+\underbrace{\frac{\cos 9x}{\cos 27x}}_{t_3}=0$$

Now, $$t_1=\frac{\sin x}{\cos 3x}\cdot\frac{2\cos x}{2\cos x}=\frac{\sin 2x}{2\cos x\cos 3x}=\frac{\sin(3x-x)}{2\cos x\cos 3x}$$

$$=\frac{\sin 3x\cdot\cos x-\sin x\cos 3x}{2\cos x\cdot\cos 3x}$$

$$t_1=\frac{1}{2}[\tan 3x-\tan x]$$

$$|||^{ly}\ t_2=\frac{1}{2}[\tan 9x-\tan 3x]$$

$$t_3=\frac{1}{2}[\tan 27x-\tan 9x]$$

$\Rightarrow \quad t_1+t_2+t_3=0 \qquad \Rightarrow \qquad \dfrac{1}{2}(\tan 27x-\tan x)=0$

$\Rightarrow \quad \tan 27x-\tan x=0 \qquad \Rightarrow \qquad \sin 27x\cos x-\cos 27x\sin x=0$

$\Rightarrow \quad \sin 26x=0 \quad \Rightarrow \quad 26x=n\pi, n\in I \quad \Rightarrow x=\dfrac{n\pi}{26}$

At $\quad n=1,\ x=\dfrac{\pi}{26}$

172. (d) $$\cos\left[\log_5\left(\frac{1+\tan^2 A-\sec^2 A\cdot\sin^2 A}{\sec^2 A\cdot\cos^2 A}\right)\right]=\cos\left[\log_5\left(\frac{\sec^2 A(1-\sin^2 A)}{1}\right)\right]=\cos 0=1$$

173. (b) $\cos x + \cos^2 x = 1 \quad \Rightarrow \quad \cos x = \sin^2 x$

Now, $E = \sin^{12} x + 3\sin^{10} x + 3\sin^8 x + \sin^6 x + 2$

$= \cos^6 x + 3\cos^5 x + 3\cos^4 x + \cos^3 x + 2 = (\cos^2 x + \cos x)^3 + 2 = 1 + 2 = 3$

$\log_{\tan\frac{\pi}{3}} E = \log_{\sqrt{3}} 3 = 2$

174. (c) $|2^x - 1|^3 + |2^x - 4| = 3$

Case-I: $x \in (-\infty, 0)$

$-2^x + 1 - 2^x + 4 = 3 \Rightarrow -2\cdot 2^x = -2 \Rightarrow 2^x = 1 \Rightarrow x = 0$ (rejected)

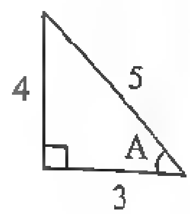
Case-II: $x \in [0, 2]$

$2^x - 1 - 2^x + 4 = 3 \Rightarrow 3 = 3$

Case-III: $x \in (2, \infty)$

$2^x - 1 + 2^x - 4 = 3 \Rightarrow 2\cdot 2^x = 8 \Rightarrow 2^x = 4 \Rightarrow x = 2$ (rejected)

175. (d) $\tan A = \frac{-4}{3} (A \in \text{IV})$



$\sin A = \frac{-4}{5}, \qquad \cos A = \frac{3}{5}$

$5\sin 2A + 2\sin A + 4\cos A = 5\left(\frac{2\tan A}{1+\tan^2 A}\right) + 2\sin A + 4\cos A$

$$= 5\left(\frac{2\left(\frac{-4}{3}\right)}{1+\frac{16}{9}}\right) + 2\left(\frac{-4}{5}\right) + 4\left(\frac{3}{5}\right) - 5\left(\frac{-8}{3}\right)\left(\frac{9}{25}\right) \; \frac{8}{5} + \frac{12}{5} = -\frac{24}{5} - \frac{8}{5} + \frac{12}{5} = \frac{-20}{5} = -4$$

176. (d) $\theta \in \left[99\pi + \frac{\pi}{2}, 100\pi\right]$

$\theta \in \left[\frac{3\pi}{2}, 2\pi\right]$

$\therefore \quad \alpha = -\sin^2\theta, \ \beta = \cos^2\theta$

$\alpha - \beta = -\sin^2\theta - \cos^2\theta = -1$

177. (b) $P(4) = Q(4)$

$\Rightarrow \quad k(64) + 3(16) - 3 = 2(64) - 20 + k$

$\Rightarrow \quad 63k = 128 - 20 - 48 + 3$

$\Rightarrow \quad 63k = 63 \Rightarrow k = 1$

178. (c) $a = \log 25, \ b = a + \log 9$

$\therefore \quad \log 5 = \frac{a}{2} \ ; \quad \log 9 = b - a \quad ; \quad \log 3 = \frac{b-a}{2}$

$\log\left(\frac{1}{81}\right) + \log\left(\frac{1}{2250}\right) = -2\log 9 - \log(2250) = -2(b-a) - (\log 225 + \log 10)$

$= -2(b-a) - (b+1) = -2b + 2a - b - 1 = 2a - 3b - 1$

179. (b) $$\frac{1}{3}\log_3 a+\frac{1}{2}\log_3 b=\frac{7}{2}$$
$$\frac{1}{2}\log_3 a+\frac{1}{3}\log_3 b=\frac{2}{3}$$
$$+$$
$$\frac{5}{6}(\log_3 a+\log_3 b)=\frac{21+4}{6}$$
$$\log_3(ab)=5$$
$$ab=243$$

180. (a) $$\sqrt{\left(\frac{1}{\sqrt{27}}\right)^{2-\log_5 13+(2\log_5 9)}}=\sqrt{\frac{(\sqrt{27})^{\log_{81}13}}{27}}=\left(\frac{\sqrt[8]{13}}{3}\right)^{\frac{3}{2}}$$
$$\therefore \quad a=8,\ b=13,\ c=3$$

181. (c) $\sin x\cdot\tan 4x=\cos x$
$$\sin x\cdot\sin 4x=\cos x\cdot\cos 4x$$
$$\cos 4x\cdot\cos x-\sin x\cdot\sin 4x=0$$
$$\cos(4x+x)=0$$
$$\cos 5x=0$$
$$5x=(2n-1)\frac{\pi}{2} \qquad (n\in I)$$
$$x=(2n-1)\frac{\pi}{10}$$
$$\therefore \quad x=\frac{\pi}{10},\frac{3\pi}{10},\frac{5\pi}{10},\frac{7\pi}{10},\frac{9\pi}{10}$$
Number of solutions = 5

182. (b) $\log_b a\cdot\log_c a+\log_a b\cdot\log_c b+\log_a c\cdot\log_b c=3$

Let $\log_a a=1$, $\quad \log_a b=B$, $\quad \log_a c=C$
$$\Rightarrow \quad \frac{1}{BC}+\frac{B^2}{C}+\frac{C^2}{B}=3$$
$$1+B^3+C^3=3BC$$
$$\therefore \quad 1+B+C=0$$
$$1+\log_a b+\log_a c=0 \quad\Rightarrow\quad \log_a(abc)=0$$
$$abc=1$$

183. (b) $$E=\sin\frac{\pi}{n}+\sin\frac{3\pi}{n}+\sin\frac{5\pi}{n}+\ldots\ldots n\text{ terms}$$
$$=\frac{\sin\left(n\frac{\pi}{n}\right)}{\sin\left(\frac{\pi}{n}\right)}\sin\left(\frac{\text{I}^{\text{st}}\ \text{Angle}+\text{Last Angle}}{2}\right)=0$$

184. (b) $S=\sum_{\alpha=1}^{17}\sin^2(5\alpha)^\circ=\frac{1}{2}\sum_{\alpha=1}^{17}(1-\cos 10\alpha)=\frac{1}{2}[17-\{\cos 10^\circ+\cos 20^\circ+\ldots\ldots+\cos 170^\circ\}]$

$=\frac{1}{2}[17-\{(\cos 10^\circ+\cos 170^\circ)+(\cos 20^\circ+\cos 160^\circ)+\ldots\ldots+(\cos 80^\circ+\cos 100^\circ)+\cos 90^\circ\}]$

$=\frac{1}{2}[17-0]=\frac{17}{2}$ $\quad\left\{\begin{matrix}\text{If } & A+B=180^\circ\\ \therefore & \cos A+\cos B=0\end{matrix}\right\}$

$[S]=[8.5]=8$

185. (b) $2\cos\theta-\sin\theta+2=0 \quad\Rightarrow\quad 2(1+\cos\theta)=\sin\theta$

$\Rightarrow\quad 2\cdot 2\cos^2\left(\frac{\theta}{2}\right)=2\sin\frac{\theta}{2}\cos\frac{\theta}{2} \quad\Rightarrow\quad \tan\frac{\theta}{2}=2$

Now, $\cos\theta=\dfrac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}=\dfrac{1-4}{1+4}=\dfrac{-3}{5}$; $\sin\theta=\dfrac{2\tan\theta}{1+\tan^2\theta}=\dfrac{4}{5}$

$$E=10\cos\theta+15\sin\theta=10\left(\frac{-3}{5}\right)+15\left(\frac{4}{5}\right)=-6+12=6$$

186. (c) $N^2=9\cos^2\theta+16\sin^2\theta+16\cos^2\theta+9\sin^2\theta$

$+2(\sqrt{(9\cos^2\theta+16\sin^2\theta)\cdot(16\cos^2\theta+9\sin^2\theta)}$

$=25+2\sqrt{(9+7\sin^2\theta)(9+7\cos^2\theta)}$

$=25+2\sqrt{81+63\cos^2\theta+63\sin^2\theta+49\cos^2\theta\sin^2\theta}$

$=25+2\sqrt{81+63+\frac{49\sin^2 2\theta}{4}}$

$N^2=25+2\sqrt{144+\frac{49\sin^2 2\theta}{4}}$

$N^2_{max}=25+2\sqrt{144+\frac{49}{4}}=25+2\sqrt{\frac{625}{4}}=25+25=50$

$N^2_{min}=25+2\sqrt{144}=25+24=49$

Sum = 50 + 49 = 99

187. (a) Do yourself.

188. (a) Multiply both sides of the equation by e^{-x}, we get

$$e^{-x}f(x)=\int_0^x e^{-y}f'(y)dy-(x^2-x+1)$$

Differentiate both sides with respect to x,

$$e^{-x} f'(x) - e^{-x} f(x) = e^{-x} f'(x) - (2x-1)$$

Now, solve for $f(x)$, we get

$$f(x) = (2x-1)e^{x}$$

$$\therefore \quad f(x) = 0 \quad \text{at} \quad x = \frac{1}{2}$$

189. (d) Given expression $= 1 + 2^2 + 1 + 3^2 + \operatorname{cosec}\left(\tan^{-1}\frac{4}{3} + \tan^{-1}\frac{4}{3}\right)$

$$= 15 + \operatorname{cosec}\left(\pi + \tan^{-1}\left(\frac{-24}{7}\right)\right) = 15 + \operatorname{cosec}\left(\operatorname{cosec}^{-1}\frac{25}{24}\right) = 15 + \frac{25}{24}$$

190. (c) $f(g(x)) = \ln\left(\frac{(1+x)^3}{(1-x)^3}\right) = 3f(x)$

191. (b) Do yourself.

192. (c) Given limit $= \lim\limits_{x \to \frac{\pi}{4}} \frac{f(\sec^2 x) \cdot 2\sec^2 x \tan x}{2x} - \frac{8}{\pi} f(2)$

$$\therefore \quad k^a = 8^2 = 64$$

193. (b) Since irrational roots occur in pairs, hence both root common

$$\therefore \quad \frac{51}{3} = 17 = \frac{m}{b} = \frac{c}{a} \quad \Rightarrow \quad \frac{c}{a} = 17$$

194. (a) $y = 2^{x(x-1)} \Rightarrow x^2 - x = \log_2 y \Rightarrow x^2 - x - \log_2 y = 0$

$$\Rightarrow \quad x = \frac{1 + \sqrt{1 + 4\log_2 y}}{2} = f^{-1}(y) \qquad \left(\because \text{ as } x \geq \frac{1}{2}\right)$$

$$\therefore \quad f^{-1}(x) = \frac{1 + \sqrt{1 + 4\log_2 x}}{2}$$

195. (a) $D : [-2, -1] \cup [1, 2]$

$$\because \quad -1 \leq \log_2\left(\frac{x^2}{2}\right) \leq 1 \quad ; \quad \therefore \text{ Range} = \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$$

196. (c) Add the two given equation, we get

$$f'(x) + g'(x) = (f(x) + g(x))^2 + 1$$

$$\Rightarrow \quad \int \frac{f'(x) + g'(x)}{(f(x) + g(x))^2 + 1} dx = \int 1 \cdot dx \quad \Rightarrow \quad \tan^{-1}(f(x) + g(x)) = x + C$$

Putting $x=0, c=\frac{\pi}{4}$

$\therefore$ Putting $x=\frac{\pi}{12}, f\left(\frac{\pi}{12}\right)+g\left(\frac{\pi}{12}\right)=\sqrt{3}$

197. (a) Let $f^{-1}(x)=h(x)$

$$\therefore \quad g'(x)=\frac{-1}{h^2(x)}\cdot h'(x)$$

$$\Rightarrow \quad g'(4)=\frac{-1}{h^2(4)}\cdot h'(4), \text{ where } h'(f(x))=\frac{1}{f'(x)}$$

$$\Rightarrow \quad h'(4)=\frac{1}{f'(3)}=\frac{3}{4}=\frac{-1}{9}\cdot\frac{3}{4}=\frac{-1}{12}$$

198. (b)

$$y=\sqrt{\frac{x}{y}} \Rightarrow y^3=x$$

$$\Rightarrow \quad 3y^2y'=1 \quad \Rightarrow \quad y'=\frac{1}{3y^2}=\frac{1}{12}$$

199. (a) $\because \quad f(f(x))=x$

$$\Rightarrow \quad f'(f(x))=\frac{1}{f'(x)}$$

$\therefore \quad f(x)=2 \Rightarrow (1025-x^{10})^{1/10}=2 \Rightarrow x=1$

$\therefore \quad a=1$

200. (b) Let $x=\tan\theta$

$$\therefore \quad y=2\tan^{-1}\left(\frac{\sec\theta-1}{\tan\theta}\right)-2\tan^{-1}\left(\tan\frac{\theta}{2}\right)=\theta=\tan^{-1}x$$

$$\frac{dy}{dx}=\frac{1}{1+x^2} \quad \Rightarrow \quad \frac{d^2y}{dx^2}=\frac{-2x}{(1+x)^2}$$

$$\therefore \quad \left.\frac{dy}{dx}\right|_{x=2}=\frac{-4}{25}$$

201. (d) $\because \quad \lim_{x\to y}\left|\frac{f(x)-f(y)}{x-y}\right|\le \lim_{x\to y} 6|x-y|$

$\Rightarrow \quad |f'(y)|\le 0 \quad \Rightarrow \quad f'(y)=0$

$\Rightarrow \quad f(y)=\text{constant}$

$\therefore \quad f(x)=6 \quad \Rightarrow \quad f(6)=6$

202. (d)

$$y=\frac{x}{x-c_1}+\frac{c_2x}{(x-c_1)(x-c_2)}+\frac{c_3x^2}{(x-c_1)(x-c_2)(x-c_3)}$$

$$=\frac{x^2}{(x-c_1)(x-c_2)}+\frac{c_3x^2}{(x-c_1)(x-c_2)(x-c_3)}=\frac{x^3}{(x-c_1)(x-c_2)(x-c_3)}$$

$\therefore \quad \ln y = 3\ln x - \ln(x-c_1) - \ln(x-c_2) - \ln(x-c_3)$

$$\Rightarrow \quad \frac{y'}{y} = \frac{3}{x} - \frac{1}{x-c_1} - \frac{1}{x-c_2} - \frac{1}{x-c_3}$$

$$\Rightarrow \quad y' = \frac{y}{x}\left(3 - \frac{x}{x-c_1} - \frac{x}{x-c_2} - \frac{x}{x-c_3}\right) = \frac{y}{x}\left(\frac{c_1}{c_1-x} + \frac{c_2}{c_2-x} + \frac{c_3}{c_3-x}\right)$$

203. (c) $\int_{-1}^{0} (f(x))^2\, dx = 10$

Integrating by parts

$$x(f(x))^2 |_{-1}^{0} - 2\int_{-1}^{0} xf'(x)f(x)dx = 10$$

$$(0)(f(0))^2 - (-1)(f(-1))^2 - 2\int_{-1}^{0} xf'(x)f(x)dx = 10$$

$$0 + 4 - 2\int_{-1}^{0} xf'(x)f(x)dx = 10$$

$$\Rightarrow \quad \int_{-1}^{0} xf'(x)f(x)dx = -3$$

204. (a) Do yourself.

205. (a) $|r| < 1$ and $S_\infty = \dfrac{a_1}{1-r} < 0 \quad \Rightarrow \quad a_1 < 0$

206. (a) $\quad m_1 = \dfrac{x^2 - y^2}{2xy} \quad$ and $\quad m_2 = \dfrac{-2xy}{x^2 - y^2}$

$\therefore \quad m_1 m_2 = -1 \quad \Rightarrow \quad \theta = \dfrac{\pi}{2}$

207. (a) $y = f(x) = \begin{cases} 0 & \text{if } x < 0 \\ 2x^2 & \text{if } x \geq 0 \end{cases}$

$\therefore \quad f(x)$ is continuous and derivable.

208. (d) $\because \quad$ function is odd.

$\therefore \quad$ D.I. = 0

209. (b) $\because \quad \displaystyle\int_0^1 \left(xf(x) - \frac{\sqrt{x}}{2}\right)^2 dx = 0$

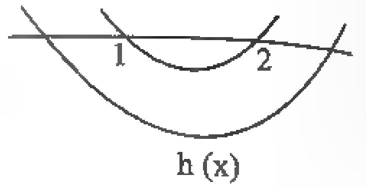
$\therefore \quad f(x) = \dfrac{1}{2\sqrt{x}}$

$\therefore \quad$ number of solution is 1.

210. (d) Limit $= \lim_{x\to 0^+} \dfrac{\int_0^{\tan^{-1}x} (\sin t^2)\,dt}{\frac{-x^3}{2}} = \lim_{x\to 0^+} \dfrac{\sin(\tan^{-1}x)^2}{\frac{-3}{2}x^2} = \dfrac{-2}{3}$

211. (d) Take $\cos^4 x$ out from denominator and put $\tan^4 x = t$

212. (d) $g(x_2) \le f(x_1)\ \forall\, x \in [0, 1]$



Hence $g(x_2)$ is less than minimum value of $f(x)$ in [0, 1].

Now $f(x)$ is an increasing function in [0, 1],

hence minimum value of $f(x)$ is $f(0) = -1$

$\therefore \quad g(x) \le -1$ for atleast one $x \in [1, 2]$

$h(x) = x^2 - 2ax + 5 \le 0$ fot atleast one $x \in [1, 2]$

Case-1 : $h(1) \le 0 \Rightarrow 6 - 2a \le 0 \Rightarrow a \ge 3$

and $h(2) \le 0 \Rightarrow 9 - 4a \le 0 \Rightarrow a \ge \dfrac{9}{4}$

Hence $a \ge 3$

Case-2 : $h(1) \ge 0$ and $h(2) \le 0$

Hence $a \in \left[\dfrac{9}{4}, 3\right]$

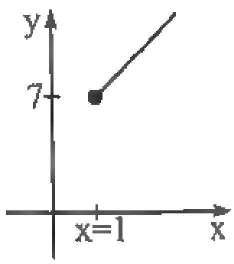
Case-3: $h(1) \le 0$ and $h(2) \ge 0$ (rejected) because product of root is 5.

$\therefore$ Case-I $\cup$ Case-2 $\cup$ Case-2 $\Rightarrow a \in \left[\dfrac{9}{4}, \infty\right)$

Hence $a_{\min} = \dfrac{9}{4}$

213. (c) $a - 1 \le 7$

$\Rightarrow a \le 8$



214. (c) $f(x) = (x^2 + ax + 2a)e^x$

$f'(x) = (x^2 + (a+2)x + 3a)e^x \ge 0\ \forall\, x$

$\Rightarrow D \le 0$

$a^2 - 8a + 4 \le 0$

$\Rightarrow 4 - 2\sqrt{3} \le a \le 4 + 2\sqrt{3}$

Hence, 7 intergral values *i.e.*, 1, 2, 3, 4, 5, 6, 7.

215. (c) $\sum_{\text{cyc}} a\cos A = \sum_{\text{cyc}} 2R\sin A\cos A = R\sum_{\text{cyc}} \sin 2A = 4R\prod_{\text{cyc}} \sin A \quad \ldots(1)$

$\dfrac{[ABC]}{R} = 4 \Rightarrow \dfrac{abc}{4R^2} = 4 \Rightarrow 8R^3\prod_{\text{cyc}} \sin A = 16R^2 \Rightarrow \prod_{\text{cyc}} \sin A = \dfrac{2}{R} \quad \ldots(2)$

On substituting eqn. (2) in eqn. (1), we get

$$\sum_{\text{cyc}} a\cos A = 4R\times\frac{2}{R}=8$$

216. (c) Do yourself.

217. (d) Since, A, B, C are in A.P. $\Rightarrow$ $2B = A+C$

and since sum of angles of a triangle is π.

$$\Rightarrow \quad A+B+C=\pi \quad \Rightarrow \quad 3B=\pi \quad\Rightarrow\quad B=\frac{\pi}{3}$$

Now, using the cosine formula

$$\cos B=\frac{a^2+c^2-b^2}{2ac} \quad\Rightarrow\quad \frac{1}{2}=\frac{a^2+c^2-b^2}{2ac}$$

$$\Rightarrow \quad a^2+c^2-b^2=ac \quad\Rightarrow\quad a^2+c^2-ac=b^2$$

$\therefore$ Required value $=\dfrac{a+c}{\sqrt{b^2}}=\dfrac{a+c}{b}$

Now, $a=2R\sin A$, $b=2R\sin B$, $c=2R\sin C$

where R is the circumradius.

$$\Rightarrow \quad \frac{a+c}{b}=\frac{2R(\sin A+\sin C)}{2R\sin(\pi/3)}=\frac{2\sin\frac{A+C}{2}\cos\frac{A-C}{2}}{\sin(\pi/3)}$$

$$=\frac{2\sin(\pi/3)\cos\frac{A-C}{2}}{\sin(\pi/3)} \qquad (\because\ A+C=2B \text{ and } B=\pi/3)$$

$$=2\cos\frac{A-C}{2}$$

218. (b) Consider the numbers $a/2$, $a/2$, $b/3$, $b/3$, $b/3$, $c/4$, $c/4$, $c/4$, $c/4$

using A.M. $\geq$ G.M. we get

$$\frac{a+b+c}{9}\geq\left(\frac{a^2b^3c^4}{2^{10}3^3}\right)^{1/9}$$

$\Rightarrow$ maximum value of $a^2b^3c^4$ is $2^{10}\times3^3$

Hence $x=10$ and $y=3$

$$\therefore \quad \log_{10}(x^y)=\log_{10}(10^3)=3$$

219. (d)
$$q=\left(\alpha+\frac{1}{\beta}\right)\left(\beta+\frac{1}{\alpha}\right)=\alpha\beta+\frac{1}{\alpha\beta}+2=2+\frac{1}{2}+2=\frac{9}{2}=p \ (\text{given } \alpha\beta=2)$$

$\Rightarrow$ $2q=9$

220. (b) $f(x)=\ln(x^2+ax+1)$ given $f(x)$ is defined $\forall\, x\in R$

$$\therefore \quad x^2+ax+1>0\ \forall\, x\in R$$

Hence, $a^2-4<0$ $\Rightarrow$ $a\in(-2,2)$

$\therefore$ Number of integers in the range of 'a' is 3.

221. (d) Consider

$$x^3+(a+b+c)x^2+(\Sigma ab)x+abc\equiv(x+a)(x+b)(x+c)$$

put $x=2$

$$8+\underbrace{4\Sigma a+2\Sigma ab+abc}_{202}=(2+a)(2+b)(2+c)$$

$$\therefore \quad (2+a)(2+b)(2+c)=210=2\cdot 3\cdot 5\cdot 7$$

$$=3\times 7\times 10$$

$$=3\times 5\times 14$$

$$=5\times 6\times 7$$

$$\text{required}\begin{cases}2\times 3\times 35\\ 2\times 5\times 21\end{cases}$$

$$a+b+c=14,\ 16,\ 12 \quad ; \quad a=0$$

Hence, number of possible values of $a+b+c$ is 3.

222. (b) If $\sin\theta$ and $\cos\theta$ are roots of $3x^2-x+k=0$, then

$$\sin\theta+\cos\theta=\frac{1}{3} \qquad \ldots(1)$$

$$\sin\theta+\cos\theta=\frac{k}{3} \qquad \ldots(2)$$

$$(\sin\theta+\sin\theta)^2=\sin^2\theta+2\sin\theta\cos\theta+\cos^2\theta$$

$$\frac{1}{9}=1+\frac{2k}{3} \quad\Rightarrow\quad k=\frac{-4}{3}$$

Also $\quad \sin^3\theta+\cos^3\theta=(\sin\theta+\cos\theta)(\sin^2\theta+\cos^2\theta-\sin\theta\cos\theta)$

$$=\frac{1}{3}\left(1+\frac{4}{9}\right)=\frac{13}{27}$$

$$\Rightarrow \quad 54(\sin^3\theta+\cos^3\theta)=26$$

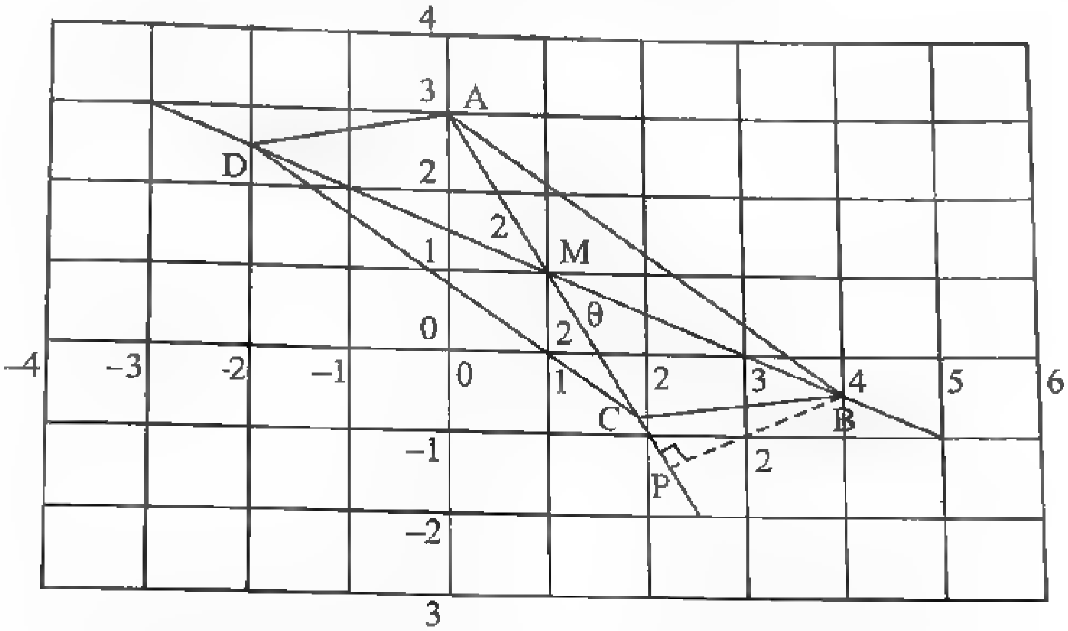
223. (d) $$\frac{b^2+c^2-a^2}{2abc}=\frac{b^2+c^2-a^2}{8}$$

$$\Rightarrow \quad k_1=\frac{a^2+b^2+c^2}{8}-\frac{b^2+c^2-a^2}{8}=\frac{a^2}{4}$$

Similarly we obtain $k_2=\dfrac{b^2}{4},\ k_3=\dfrac{c^2}{4}$

$$\Rightarrow \quad k_1k_2k_3=\frac{a^2b^2c^2}{64}=\frac{16}{64}=\frac{1}{4}$$

224. (a)



Let the diagonals AC and BD intersect at M and $\angle BMC = \theta$. We note that

$$\begin{cases} AC: x+2y-3=0 \Rightarrow y=-\frac{1}{2}x+\frac{3}{2} & \ldots(1) \\ BD: 2x+y-3=0 \Rightarrow y=-2x+3 & \ldots(2) \end{cases}$$

Therefore the gradients of diagonals AC and BD are $\tan^{-1}\left(\frac{-1}{2}\right)$ and $\tan^{-1}(-2)$ respectively and that

$$\theta = \tan^{-1}\left(\frac{-1}{2}\right) - \tan^{-1}(-2)$$

$$\tan\theta = \frac{\left(-\frac{1}{2}\right)-(-2)}{1+\left(-\frac{1}{2}\right)(-2)} = \frac{3}{4}$$

Now, let the perpendicular from point B to AC extension be BP.

Given that $AC = 4$ and $\{ABCD\} = 8$, $BP = 2$

Since $\frac{BP}{BM} = \sin\theta = \frac{3}{5} \Rightarrow BM = \frac{5}{3}BP = \frac{10}{3}$ and $BD = 2BM = \frac{20}{3}$

225. (c) For $\beta = \frac{\pi}{3}$, then $\tan\left(\alpha - \frac{\pi}{3}\right) = \dfrac{\sin\frac{2\pi}{3}}{3-\cos\frac{2\pi}{3}}$

$$\frac{\tan\alpha - \tan\frac{\pi}{3}}{1+\tan\alpha\tan\frac{\pi}{3}} = \frac{\frac{\sqrt{3}}{2}}{3-\left(-\frac{1}{2}\right)}$$

$$\frac{\tan\alpha-\sqrt{3}}{1+\sqrt{3}\tan\alpha}=\frac{\sqrt{3}}{7}$$
$$7\tan\alpha-7\sqrt{3}=\sqrt{3}+3\tan\alpha$$
$$4\tan\alpha=8\sqrt{3}$$
$$\tan\alpha=f\left(\frac{\pi}{3}\right)=2\sqrt{3}$$

226. (d) Note that $$b_n=\sum_{r=0}^{n}\frac{r}{{}^nC_r}=\sum_{r=0}^{n}\frac{n-r}{{}^nC_r}=na_n-b_n \qquad (n\geq 0)$$

So that $na_n=2b_n$ for $n\geq 0$ and hence it follows that $c_n=\dfrac{2}{n}$ for all $n\geq 1$

Thus we want to find ordered pairs (p, q) of positive integers such that $2p^{-1}+2q^{-1}=1$

or $\quad 2(p+q)=pq \quad$ or $\quad (p-2)(q-2)=4$

Thus $p-2$ can take the values 1, 2, 4 only and there are exactly 3 ordered pairs with the desired property, (3, 6), (4, 4) and (6, 3).

227. (c) If $x+1$ is factor of $f(x)=x^3+kx^2-3x+k+2$, then

$$f(-1)=0$$
$$\Rightarrow \quad -1+k+3+k+2=0$$
$$\Rightarrow \quad 2k+4=0 \quad \Rightarrow \quad k=-2$$

228. (b) Let $x=\sqrt{2-\sqrt{3}}+\sqrt{2+\sqrt{3}} \quad \Rightarrow \quad x^2=6$

$$\Rightarrow \quad \log_6(\sqrt{2-\sqrt{3}}+\sqrt{2+\sqrt{3}})=\log_6\sqrt{6}=\frac{1}{2}$$

229. (c) $\log_3(x)^2=2 \quad \Rightarrow \quad x^2=9 \quad \Rightarrow \quad x=\pm 3$

230. (b) $$a=7^{\frac{1}{\log_8\sqrt{343}}}=7^{2\log_7 2}=4$$
$$b=11^{\frac{1}{\log_5\sqrt{11}}}=25$$

231. (a) If $a+b+c=0$, then $a^3+b^3+c^3=3abc$

$$(x-1)^3+(2x-1)^3+(2-3x)^3=3(x-1)(2x-1)(2-3x)$$
$$=-3(x-1)(2x-1)(3x-2)$$

232. (d) $\quad \forall\, x\in(-\infty,-1]$

$$f(x)=\sin^{-1}\left(\frac{2x}{1+x^2}\right)-2\tan^{-1}x$$
$$=-\pi-2\tan^{-1}x-2\tan^{-1}x$$
$$=-\pi-4\tan^{-1}x$$
$$g(x)=\sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)+4\tan^{-1}x$$

$$= \frac{\pi}{2} - \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) + 4\tan^{-1} x$$

$$= \frac{\pi}{2} - (-2\tan^{-1} x) + 4\tan^{-1} x$$

$$= \frac{\pi}{2} + 6\tan^{-1} x$$

Now, $f(x) - g(x) = \dfrac{-3\pi}{2} - 10\tan^{-1} x$

$$\forall\, x \in (-\infty, -1], \quad \tan^{-1} x \in \left(\frac{-\pi}{2}, \frac{-\pi}{4}\right]$$

So, $(f(x) - g(x)) \in \left[\pi, \dfrac{7\pi}{2}\right)$

233. (b) $A = \{1, 2, 3, 4, 5, 6, 7\}$

Case-I : When exactly 4 values follows $f(i) = i$

$${}^7C_4 \times 3!\left(\frac{1}{2!} - \frac{1}{3!}\right) = 70$$

Case-II : When exactly 5 values follows $f(i) = i$

$${}^7C_5 \times 1 = 21$$

Case-III : When all 7 values follows $f(i) = i$

number of function = 1

Total functions = 70 + 21 + 1 = 92

234. (a) $\alpha^2 - \alpha + 2 = 0$ ...(1)

So, $$\frac{6(-\alpha^3 + 2\alpha^2 - \alpha)}{\alpha^5 - 3\alpha^4 + 3\alpha^3 - \alpha^2} = \frac{-6\alpha(\alpha^2 - 2\alpha + 1)}{\alpha^2(\alpha^3 - 3\alpha^2 + 3\alpha - 1)}$$

$$= \frac{-6\alpha(\alpha-1)^2}{\alpha^2(\alpha-1)^3} = \frac{-6}{\alpha(\alpha-1)} = \frac{-6}{\alpha^2 - \alpha} = \frac{-6}{-2} = 3$$ From equation (1)

235. (b) Term independent of x in expansion of $\left(3x - \dfrac{1}{x}\right)^{20}$

$$T_{r+1} = {}^{20}C_r (3x)^{20-r}\left(\frac{-1}{x}\right)^r$$

When $r = 10$

$$A = T_{11} = {}^{20}C_{10}\ 3^{10}$$ ...(1)

Term independent of x in expansion of $\left(x + \dfrac{\sqrt[9]{3^{10}}}{x}\right)^{18}$

$$T_{r+1} = {}^{18}C_r\,(x)^{18-r}\left(\frac{(3^{10})^{1/9}}{x}\right)^r$$

When $r = 9$

$$B = T_{10} = {}^{18}C_9\, 3^{10} \qquad \ldots(2)$$

So, $\left(\frac{9}{38}A + B\right) = \left(\frac{9}{38}\times {}^{20}C_{10}\times 3^{10} + {}^{18}C_9 \times 3^{10}\right)$ [From eqns. (1) and (2)]

$$= \left(\frac{9}{38}\times\frac{20}{10}\times\frac{19}{9}\times {}^{18}C_8\times 3^{10} + {}^{18}C_9\times 3^{10}\right) = 3^{10}\times {}^{19}C_9$$

236. (b) For $f(x) - f^{-1}(x)$

only 3 solutions

(0, 2), (2, 0), (1, 1)

237. (d) $\lim_{x\to 0} |\cos x + \sin 2x + \sin 3x|^{\cot x} = e^m$

$$= e^{\lim_{x\to 0}(|\cos x + \sin 2x + \sin 3x| - 1)\cot x} = e^m$$

$$\Rightarrow \quad m = \lim_{x\to 0}(|\cos x + \sin 2x + \sin 3x| - 1)\cot x$$

$$= \lim_{x\to 0}(\cos x + \sin 2x + \sin 3x - 1)\cot x$$

$$= \lim_{x\to 0}\frac{\cos x - 1 + \sin 2x + \sin 3x}{x\cdot\frac{\tan x}{x}}$$

$$= \lim_{x\to 0}\left(\frac{\cos x - 1}{x} + \frac{\sin 2x}{x} + \frac{\sin 3x}{x}\right) = 2 + 3 = 5$$

238. (b) Suppose number of red, white, blue and green balls are a, b, c and d respectively.

Then $a + b + c + d = 18$ but $a, b, c, d \geq 2$

$a + b + c + d = 10$

Total number of ways $= {}^{13}C_3 = \frac{13\times 12\times 11}{6} = 286$

239. (d) $\tan^{-1}\left(\frac{\tan\alpha}{3 + 2\tan^2\alpha}\right) = \alpha - \tan^{-1}\left(\frac{2\tan\alpha}{3}\right)$

$$\Rightarrow \tan^{-1}\left(\frac{\tan\alpha}{3 + 2\tan^2\alpha}\right) + \tan^{-1}\left(\frac{2\tan\alpha}{3}\right) = \alpha = \frac{\pi}{12}$$

240. (a) $\sin^2\theta + 2\cos\theta + 1 = \frac{P-1}{2P+3} = \frac{1}{2} - \frac{\frac{S}{2}}{(2P+3)}$

$$\Rightarrow \quad 3 - (1 - \cos\theta)^2 = \frac{1}{2} - \frac{\frac{S}{2}}{2P+3}$$

$$\Rightarrow \quad -1 \le \frac{1}{2} - \frac{\frac{S}{2}}{2P+3} \le 3$$

$$\frac{-3}{2} \le \frac{\frac{-S}{2}}{2P+3} \le \frac{5}{2} \quad \Rightarrow \quad -1 \le \frac{1}{2P+3} \le \frac{3}{5}$$

$$\Rightarrow \quad 2P+3 \le -1 \cup 2P+3 \ge \frac{5}{3}$$

$$\Rightarrow \quad P \le -2 \cup \frac{-2}{3} \le P$$

241. (c) Let $x+y+z=\theta$ and $k=2$

$$\therefore \quad \cos x+\cos y+\cos z = k\cos\theta \quad \text{and} \quad \sin x+\sin y+\sin z = k\sin\theta$$

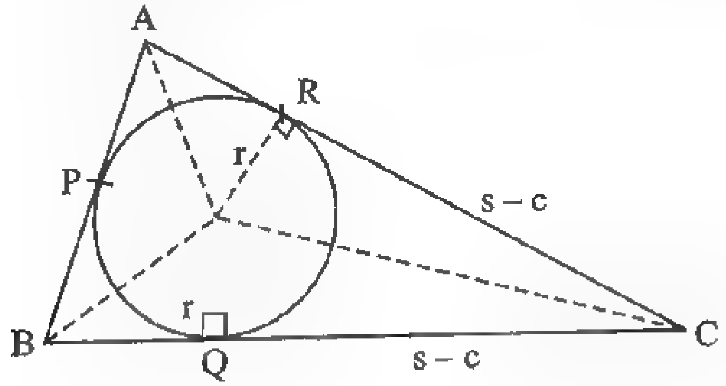
$$\cos(x+y)+\cos(y+z)+\cos(z+x) = \cos(\theta-z)+\cos(\theta-x)+\cos(\theta-y)$$

$$= \cos\theta\,(k\cos\theta)+\sin\theta\,(k\sin\theta) = k = 2$$

242. (d) $\alpha = \frac{\pi}{3}$

$$\therefore \quad \frac{\cos 2\alpha+\sec\alpha+3\sqrt{3}}{\tan\alpha} = \frac{-\frac{1}{2}+2+3\sqrt{3}}{\sqrt{3}} = \frac{\frac{3}{2}+3\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{2}+3$$

243. (d) Area of quadratic $QCRI$ is



$$2\times\frac{1}{2}r(s-c) = r(s-c)$$

$$s-a+s-b+s-c = 15$$

$$\Rightarrow \quad s = 15$$

$$\Delta = \sqrt{15\cdot 3\cdot 5\cdot 7} = 15\sqrt{7}$$

$$\Rightarrow \quad r = \frac{\Delta}{s} = \sqrt{7}$$

Required area $= 7\sqrt{7}$

244. (c) $f(\ln(1+|x|)) = (1-\ln^7(1+|x|))^{\frac{1}{7}}$

$\Rightarrow \quad f(f(x)) = x$

$\therefore \quad f(f(\cos x)) = \cos x$

245. (d) $5(x-\alpha)(x-\beta)(x-\gamma)+8-x^3 = (x-a)(x-b)(x-c)+(x-b)(x-c)(x-d)$

$+(x-c)(x-d)(x-a)+(x-d)(x-a)(x-b)$

$= 4x^3-3x^2(a+b+c+d)+2x(ab+bc+ca+bd+cd+ad)-\lambda = 0$

$\therefore$ Sum of the roots of the equation is $= \dfrac{3(a+b+c+d)}{4}$

246. (a) $\lim\limits_{x\to1}\dfrac{f(1+x^3-x)-f(x)}{\sin(x-1)} = \lim\limits_{x\to0}\dfrac{f(1-x)-f(1)}{x}+10$

$\lim\limits_{x\to1}\dfrac{f'(1+x^3-x)(3x^2-1)-f'(x)}{1} = \lim\limits_{x\to0}\dfrac{f(1-x)-f(1)}{x}+10$

$f'(1) = -f'(1)+10 \quad \Rightarrow \quad f'(1) = 5$

247. (d) $\log_p 5^{42} = 42\log_p 5 = (2\times3\times7)\log_p 5$

For above number to be an integer, p must be the from 5^m where m is divisor of 42.

$\therefore$ Number of divisors of 42 are (1 + 1) (1 + 1) (1 + 1) = 8

Now, the product of all the integral values of p is

$$5^{(\text{sum of all the divisors of }42)} = 5^{(1+2)(1+3)(1+7)} = 5^{96}$$

248. (a) Line $PQ: \dfrac{x+1}{4} = \dfrac{y-2}{-2} = \dfrac{z+3}{6} \quad \rightarrow \quad \dfrac{x+1}{2} = \dfrac{y-2}{-1} = \dfrac{z+3}{3} = r$

For $|PQ-QR|$ to maximum P, Q and R must be collinear.

$\therefore \quad x_0 = 2r-1, \quad y_0 = -r+2, \quad z_0 = 3r-3$

Putting values of x_0, y_0, z_0 in the given relation

$2r-1-2(-r+2)+3(3r-3)+1=0 \quad \Rightarrow \quad r=1$

$\therefore \quad x_0 = 1,\ y_0 = 1,\ z_0 = 0 \ \Rightarrow\ x_0+y_0+z_0 = 2$

249. (a) I.F. $= e^{\int 2\cot x\,dx} = \sin^2 x$

$\therefore \quad y\sin^2 x = \int 8\operatorname{cosec} x\sin^2 x\,dx + C = -8\cos x + C$

$(\pi/2, 8) \Rightarrow C = 8$

$\therefore \quad y = \dfrac{8(1-\cos x)}{\sin^2 x} = \dfrac{8}{1+\cos x}$

$y|_{\min} = 4$

250. (d) $x^2 - x\sin 2\theta + 2\cos^2\theta = (x-\alpha)(x-\beta)$

Putting $x = 2$

$(2-\alpha)(2-\beta) = 4-2\sin 2\theta + 1+\cos 2\theta = 5+\cos 2\theta - 2\sin 2\theta$

$(2-\alpha)(2-\beta)|_{\max} = 5+\sqrt{5} \equiv a+\sqrt{a}$

$\therefore \quad a = 5$

251. (b) $2x-y+1=0$; $3x-y=0$; $2x+y-5=0$

$\Rightarrow$ $x=1, y=3$

$\therefore$ Least distance $=\left|\dfrac{3-12+19}{5}\right|=2$

252. (c) Doubtful points : $-1, \dfrac{-2}{3}, \dfrac{-1}{2}, \dfrac{-1}{3}, 0, \dfrac{1}{3}, \dfrac{1}{2}, \dfrac{2}{3}, 1$

Continuous at $-1, 0$

Discontinuous at $\dfrac{-2}{3}, \dfrac{-1}{2}, \dfrac{-1}{3}, \dfrac{1}{3}, \dfrac{1}{2}, \dfrac{2}{3}, 1 \Rightarrow 7$

253. (c) $\vec{\mathbf{a}}=\vec{\mathbf{b}}\times\vec{\mathbf{c}}+2\vec{\mathbf{b}}$

Taking dot product with $\vec{\mathbf{b}}$

$$\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}=2|\vec{\mathbf{b}}|^2 \Rightarrow |\vec{\mathbf{a}}||\vec{\mathbf{b}}|\cos\theta=2|\vec{\mathbf{b}}|^2$$

$$\Rightarrow \cos\theta=\frac{4}{|\vec{\mathbf{a}}|} \Rightarrow |\vec{\mathbf{a}}|=4 \Rightarrow \theta=0^\circ$$

$\Rightarrow$ $\vec{\mathbf{a}}=2\vec{\mathbf{b}}$

Now, $\vec{\mathbf{b}}\times\vec{\mathbf{c}}=0 \Rightarrow \vec{\mathbf{b}}=\vec{\mathbf{c}}$ or $\vec{\mathbf{b}}=-\vec{\mathbf{c}}$

$$|2\vec{\mathbf{a}}+\vec{\mathbf{b}}+\vec{\mathbf{c}}| \begin{cases} |3\vec{\mathbf{a}}|=12 \\ |2\vec{\mathbf{a}}|=8 \end{cases}$$

$\therefore$ Required sum $=12+8=20$

254. (d) (NN) (RR) (AAA) E, D, B, H, I

Required number of words $=\dfrac{7!}{2!\cdot 2!\cdot 3!}\times {}^8C_5\times 5!=\dfrac{7!}{4\times 6}\times\dfrac{8!}{5!\cdot 3!}\times 5!$

$$=\frac{7\cdot 6\cdot 5!\, 8\cdot 7\cdot 6\cdot 5!}{4\times 6\times 6}=98(5!)^2$$

255. (b) Matrices value of whose determinant is zero.

$$\underbrace{\begin{bmatrix}2&3\\2&3\end{bmatrix}}_{2}\ \underbrace{\begin{bmatrix}2&2\\3&3\end{bmatrix}}_{2}\begin{bmatrix}2&2\\2&2\end{bmatrix}\begin{bmatrix}3&3\\3&3\end{bmatrix}\to 6$$

From the remaining 10 matrices, 5 have negative values of their determinant and 5 have positive values of their determinant.

$\therefore$ Required probability $=\dfrac{5}{11}$

256. (b) Given integral $= \int_0^1 0\,dx + \int_1^2 1\{x\}\,dx + \int_2^3 2^3\{x\}\,dx + \ldots\ldots + \int_9^{10} 9^3\{x\}\,dx$

$$= (1^3 + 2^3 + \ldots\ldots 9^3)\int_0^1 \{x\}\,dx = \left(\frac{9\times 10}{2}\right)^2 \times \int_0^1 x\,dx = \frac{2025}{2}$$

257. (c) $\because$ Tangents at end of focal chord meet at directrix at 90°.

$\therefore$ PQ is focal chord and its mid-point will be circumcentre.

Let $P = (at^2, 2at)$ $\quad\therefore\quad Q = \left(\frac{a}{t^2}, \frac{-2a}{t}\right)$ and circumcentre $= (h, k)$

$\therefore \quad 2h = a\left(t^2 + \frac{1}{t^2}\right)$ and $2k = 2a\left(t - \frac{1}{t}\right)$

$\therefore \quad 2h = a\left\{\left(t - \frac{1}{t}\right)^2 + 2\right\}$

$\therefore \quad 2h = a\left\{\frac{k^2}{a^2} + 2\right\} \Rightarrow 2ha = k^2 + 2a^2$

$\therefore$ Locus is, $y^2 = 2a(x - a)$

$\therefore$ Focus $= \left(\frac{3a}{2}, 0\right)$

258. (a) $\because \quad T_r(A) = a + b + c = 10 \quad ; \quad (a \neq b \neq c)$

Therefore it may be (1, 3, 6), (1, 4, 5), (2, 3, 5)

$\therefore$ number of matrices $= \underbrace{3!\times 3}_{\text{arranging diagonal elements}} \times \underbrace{3!}_{\text{arranging non-diagonal elements}}$

$= 3(3!)^2$

259. (a) Let vectors be $\vec{a}_1, \vec{a}_2, \ldots\ldots, \vec{a}_{2019}$

$\therefore \quad \vec{a}_1 + \vec{a}_2 + \ldots\ldots + \vec{a}_{2017} + \vec{a}_{2018} = \lambda\,\vec{a}_{2019}$

and $\quad \vec{a}_1 + \vec{a}_2 + \ldots\ldots + \vec{a}_{2017} + \vec{a}_{2019} = \mu\,\vec{a}_{2018}$

$\Rightarrow \quad \lambda\,\vec{a}_{2019} - \vec{a}_{2018} + \vec{a}_{2019} = \mu\,\vec{a}_{2018}$

$\therefore \quad \lambda + 1 = \mu + 1 = 0 \Rightarrow \lambda = -1, \mu = -1$

$\therefore \quad \vec{a}_1 + \vec{a}_2 + \ldots\ldots + \vec{a}_{2017} = \vec{0}$

260. (d) Favourable cases are (2, 3), (2, 6), (4, 3), (4, 6), (6, 3), (6, 6) and (3, 2), (6, 2), (3, 4), (6, 4), (3,6).

$\therefore \quad$ Probability $= \frac{11}{36}$

261. (a) $\because \quad |z+16|^2 = 16|z+1|^2$

$\Rightarrow \quad z\bar{z} + 16z + 16\bar{z} + 256 = 16(z\bar{z} + z + \bar{z} + 1)$

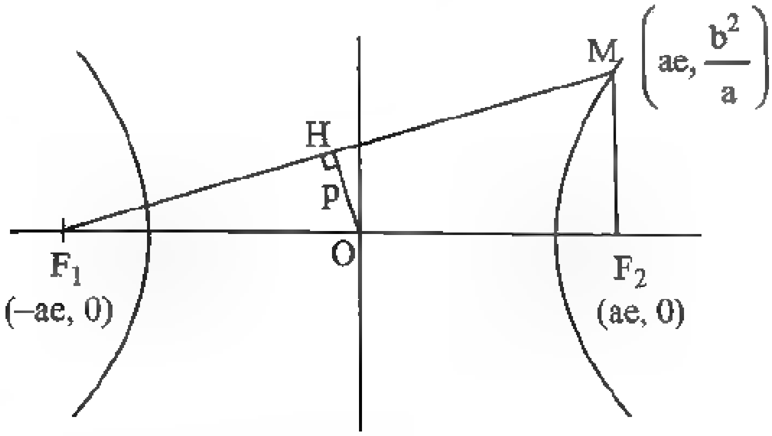
$\Rightarrow \quad 15z\bar{z} = 240 \quad \Rightarrow \quad z\bar{z} = 16 \quad \Rightarrow \quad |z| = 4$

262. (a) Clearly $|x| \geq 1$ and $x^2 + y^2 \leq 1$

$\therefore$ No value of (x, y) satisfies in equation

$\therefore \quad \lambda \in R.$

263. (c)



$$\frac{OH}{OF_2} = \lambda \in \left(\frac{1}{3}, \frac{1}{2}\right) e \in ?$$

Equation of HF_1 : $\quad y = \dfrac{b^2}{a(2ac)}(x + ac)$

$$y2a^2e = b^2x + aeb^2$$

$$b^2x - 2a^2ey + aeb^2 = 0$$

$$OH = p = \frac{aeb^2}{\sqrt{b^4 + 4a^4e^2}}$$

$$OF_2 = ae$$

$$\frac{OH}{OF_2} = \lambda = \frac{b^2}{\sqrt{b^4 + 4a^4e^2}} = \frac{b^2}{\sqrt{b^4 + \dfrac{4b^4}{(e^2-1)^2}e^2}} = \frac{1}{\sqrt{1 + \dfrac{4e^2}{(e^2-1)^2}}}$$

$$\lambda = \frac{e^2 - 1}{e^2 + 1}$$

$$\underbrace{\frac{1}{3} < \frac{e^2-1}{e^2+1}} < \frac{1}{2}$$

LHS : $\quad e^2 + 1 < 3e^2 - 3$

$$24 < 2e^2$$
$$e^2 > 2$$
RHS : $$2e^2 - 2 < e^2 + 1$$
$$e^2 < 3$$
$$\sqrt{2} < e < \sqrt{3}$$

264. (c) Let $\frac{m}{s} = \sin^2\theta$; $\frac{n}{t} = \cos^2\theta$

$$s = \frac{m}{\sin^2\theta} = m\operatorname{cosec}^2\theta$$
$$t = n\sec^2\theta$$
$$s + t = m\operatorname{cosec}^2\theta + n\sec^2\theta$$
$$m(1+\cot^2\theta) + n(1+\tan^2\theta)$$
$$\underbrace{m\cot^2\theta + n\tan^2\theta}_{\text{A.M} \geq \text{G.M.}} + 3$$
$$m\cot^2\theta + n\tan^2\theta \geq 2\sqrt{mn}$$
$$s + t_{mn} = 2\sqrt{mn} + 3 = 2\sqrt{2} + 3$$
$$mn = 2$$

Given $m + n = 3$

$\Rightarrow \quad m = 1, \quad n = 2$

$\therefore \quad m < n$

Now use $T = S$

to get $2x + y - 4 = 0$

265. (c) We have N = $\boxed{a}\boxed{b}\boxed{c}\boxed{d}$

First place a can be filled in 2 ways *i.e.*, 4, 5 ($4000 \leq N < 6000$)

For b and c, total possibilities are '6' ($3 \leq b \leq 6$)

i.e., 34, 35, 36, 45, 46, 56

Last place d can be filled in 2 ways *i.e.*, 0, 5 (N is a multiple of 5)

Hence total numbers $= 2 \times 6 \times 2 = 24$

266. (c) $$f(x) = \sec^2 x + 4\operatorname{cosec}^2 x = 1 + \tan^2 x + 4(1 + \cot^2 x) = 5 + \tan^2 x + 4\cot^2 x$$

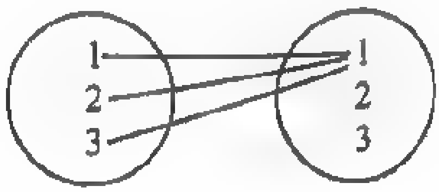
Hence $$f(x)_{\min} = 5 + 4 = 9$$

267. (a) Do yourself

268. (b) $$f[f(x)] = f(x) \; \forall \; x \in S = \{1, 2, 3\}$$

I. When range contains 1 element

$${}^3C_1 \times 1 = 3$$



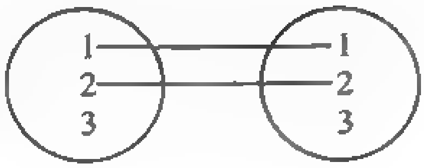
II. When range contains 2 elements

e.g., let $f(1)=1$ and $f(2)=2$

Let $f(3)=1$ OR Let $f(3)=2$

(i) If $x=1$, LHS $= f[f(x)] = f(1)$

RHS $= 1$



In this case also LHS = RHS $\forall\, x \in S$

(ii) If $x=2$, LHS = RHS

(iii) If $x=3$, LHS = RHS

$\therefore$ ${}^3C_2 \times 2 = 6$

Remaining element can be mapped 2 ways.

III. When range contains 3 elements

$f(1)=1$ $f(2)=2$ $f(3)=3$

269. (a) Do yourself.

270. (d)
$$\sum_{\omega=1}^{\infty} \sin^{-1}\left[\frac{2\omega+1}{\omega(\omega+1)(\sqrt{\omega^2+2\omega}+\sqrt{\omega^2-1})}\right]$$

$$\sum \sin^{-1}\left[\frac{(2\omega+1)(\sqrt{\omega^2+2\omega}-\sqrt{\omega^2-1})}{\omega(\omega+1)}\right]$$

$$\sum \sin^{-1}\left[\frac{\sqrt{(\omega+1)^2-1}-\sqrt{\omega^2-1}}{\omega(\omega+1)}\right]$$

$$\sum_{\omega=1}^{\infty} \sin^{-1}\left[\frac{1}{\omega}\sqrt{1-\frac{1}{(\omega+1)^2}}-\frac{1}{\omega+1}\sqrt{1-\frac{1}{\omega^2}}\right]$$

$$\sum_{\omega=1}^{\infty}\left(\sin^{-1}\frac{1}{\omega}-\sin^{-1}\frac{1}{\omega+1}\right)$$

$$S_n = \sin^{-1} - \sin^{-1}\frac{1}{2}$$

$$\sin^{-1}\frac{1}{2}-\sin^{-1}\frac{1}{3}$$

$$\sin^{-1}\frac{1}{n}-\sin^{-1}\frac{1}{n+1}$$

$$S_n = \sin^{-1} - \sin^{-1}\frac{1}{n+1}$$

$$S_\infty = \frac{\pi}{2}$$

271. (c) $\dfrac{\alpha^2+\alpha+1}{\alpha^2-\alpha+1}=\dfrac{\alpha^3-1}{(\alpha-1)(\alpha^2-\alpha+1)}$. Since α is a root of the equation $x^3-2x^2+6x-1=0$,

therefore $\alpha^3-1=2\alpha(\alpha-3)$ and $(\alpha-1)(\alpha^2-\alpha+1)=-4\alpha$.

So, $$a\left(\frac{\alpha^2+\alpha+1}{\alpha^2-\alpha+1}\right)=\frac{3\alpha-\alpha^2}{2}$$

Similar for β and γ. Therefore the given expression equals

$\dfrac{3(\alpha+\beta+\gamma)-(\alpha^2+\beta^2+\gamma^2)}{2}$. Now $\alpha+\beta+\gamma=2$, $\alpha\beta+\beta\gamma+\alpha\gamma=6$.

So, $\alpha^2+\beta^2+\gamma^2=-8$, and the value of the given expression is $\dfrac{3\times 2+8}{2}=7$.

272. (d) $\dfrac{A_1}{B_1}=\dfrac{a_1}{b_1}=\dfrac{1-1}{2}=0 \quad\Rightarrow\quad a_1=0$

Let the common difference of $\{A_n\}$ and $\{B_n\}$ be d_a and d_b respectively. Then

$$\begin{cases}\dfrac{A_2}{B_2}=\dfrac{d_a}{2b_1+d_b}=\dfrac{1}{4} \quad\Rightarrow\quad 4d_a=2b_1+d_b & \ldots(1)\\[2mm] \dfrac{A_3}{B_3}=\dfrac{2d_a}{2b_1+2d_b}=\dfrac{1}{3} \quad\Rightarrow\quad 3d_a=b_1+d_b & \ldots(2)\end{cases}$$

From eqn. (1) – eqn. (2) : $d_a=d_1$ and from eqn. (1) : $d_b=2b_a$.

Therefore $a_n=(n-1)b_1$ and $b_n=b_1+2b_1(n-1)=(2n-1)b_1$ and

$$\frac{a_3+a_5+a_7}{3(b_3+b_9)}+\frac{a_4+a_{10}}{2(b_2+b_{10})}=\frac{2b_1+4b_1+6b_1}{3(5b_1+17b_1)}+\frac{3b_1+9b_1}{2(3b_1+19b_1)}=\frac{12}{3(22)}+\frac{12}{2(22)}=\frac{5}{11}$$

273. (d) Rewrite the integral as

$$I_2=\int_0^1\left(\frac{x}{5+x}\right)^{7/2}\left(\frac{1-x}{5+x}\right)^{9/2}\frac{dx}{(5+x)^2}$$

and do the substitution $\dfrac{x}{5+x}=t$, so that $\dfrac{dx}{(5+x)^2}=\dfrac{dt}{5}$ and the integral becomes

$\dfrac{1}{(5)^{11/2}}\displaystyle\int_0^{1/6}(t)^{7/2}(1-6t)^{9/2}dt$ and now from here do the substitution $6t=u$ and we simply

obtain $I_2=\dfrac{1}{5^{9/2}\times 6^{7/2}}I_1$ and we conclude $a=30$.

274. (d) $$y=\cos^{-1}\cos\left(\log_2 2^{\ln e^{\sin^{-1}\sin x}}\right)\text{ for }\frac{-\pi}{2}\le x\le\frac{\pi}{2}$$

$$=\cos^{-1}\cos(\log_2 2^{\ln e^x})$$

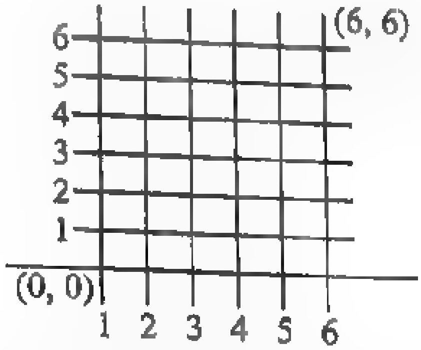
$$=\cos^{-1}\cos(\log_2 2^x)$$

$$=\cos^{-1}\cos x \text{ for } 0\le x\le\pi$$

$$= x$$

$$\Rightarrow \quad \frac{dy}{dx} = 1$$

275. (c)



Number of ways to select horizontal path :

$$(2, 2, 2), (2, 2, 1, 1), (2, 1, 1, 1, 1), (1, 1, 1, 1, 1, 1)$$

Similarly, number of ways of select vertical paths (13 ways)

$$13 \times 13 \times 2 = 338$$

276. (c) Let $\theta = \sin^{-1}\left(\frac{3\sin 2\alpha}{5+4\cos 2\alpha}\right)$. Then $\tan^{-1} x = \frac{\theta}{2}$

$$\Rightarrow \quad x = \frac{\theta}{2}$$

Now, $\quad \sin\theta = \frac{3\sin 2\alpha}{5+4\cos 2\alpha}$

$$\frac{2\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}} = \frac{6\sin\alpha\cos\alpha}{5+8\cos^2\alpha-4} \quad ; \quad \text{note that } x = \tan\frac{\theta}{2}$$

$$\frac{2x}{1+x^2} = \frac{6\tan\alpha}{\sec^2\alpha+8} \qquad \text{Divide up and down by } \cos^2\alpha$$

$$= \frac{6\tan\alpha}{9+\tan^2\alpha} \qquad \text{Divide up and down by 9}$$

$$= \frac{\frac{2}{3}\tan\alpha}{1+\frac{1}{9}\tan^2\alpha}$$

$$\Rightarrow \quad x = \frac{1}{3}\tan\alpha$$

277. (b) Let the coordinate of A be (h, k). Then those of B are $(-h, -k)$.

Eccentricity of the ellipse is $e = \sqrt{1-\left(\frac{b}{a}\right)^2}$.

From these and the condition of perpendicular of $\overline{AF}$ and BF, we get

$$h = \frac{a}{e}\sqrt{2e^2 - 1},\ k = \frac{a(1-e^2)}{e}$$

In the lower limit of the given range, thus, $e = \frac{2}{\sqrt{3}+1} = \sqrt{3} - 1$. In the upper limit,

we get $e = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$. Hence, e has the range $\left[\frac{\sqrt{2}}{2}, \sqrt{3}-1\right]$.

278. (b) By the properties of an ellipse, $|PF_1| + |PF_2| = 2a$, and by the properties of a hyperbola $|PF_1| - |PF_2| = 2a_2$. These equations combine to $|PF_1| - a_1 + a_2$ and $|PF_2| = a_1 - a_2$. Also by the properties of an ellipse and hyperbola, $|F_1F_2| = 2a_1e_1 = 2a_2e_2$. Since $|F_1F_2| = 2|PF_2|, 2a_1e_1 = 2(a_1 - a_2)$. Substituting $a_2 = \frac{a_1e_1}{e_2}$ and rearranging eliminates a_1 and gives $e_2 = \frac{e_1}{1\ e_1}$.

For a hyperbola, $e_2 > 1$, which means $\frac{e_1}{1-e_1} > 1$, which solves to $e_1 > \frac{1}{2}$.

For an ellipse, $e_1 < 1$ Therefore $\frac{1}{2} < e_1 < 1$

Therefore, $D = e_2 - e_1 = \frac{e_1}{1-e_1} - e_1 = \frac{e_1^2}{1-e_1}$. For $\frac{1}{2} < e_1 < 1$, the range of D is $\left(\frac{1}{2}, \infty\right)$

279. (a) Do yourself

280. (a) Now, the integrand can be factorised using these trigonometric identities

$$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

So, ……

$$\int_{1/3}^{1} \frac{\pi\cos\left[2\cos\left(\frac{\pi}{2}x\right)\cos\left(\frac{\pi}{6}x\right)\right]}{2\sin\left(\frac{\pi}{2}x\right)\left[2\sin\left(\frac{\pi}{2}x\right)\cos\left(\frac{\pi}{6}x\right)\right]}dx$$

After simplification :

$$\int_{1/3}^{1} \frac{\pi\cos\left(\frac{\pi}{2}x\right)}{2\sin^2\left(\frac{\pi}{2}x\right)}dx = \int_{1/3}^{1} \pi\cot\left(\frac{\pi}{2}x\right)\text{cosec}\left(\frac{\pi}{2}x\right)dx$$

u-substitution

$$u = \frac{\pi}{2}x$$

$$du = \frac{\pi}{2}dx$$

$$\int_{\pi/6x}^{\pi/2} \cot(u)\operatorname{cosec}(u)\,du = -\cos\left(\frac{\pi}{2}\right) - \left[-\cos\left(\frac{\pi}{6}\right)\right] = 1.$$

281. (a) Number of terms in the first series is $2m+1$. So $S_A = 3m(2m+1)$.

Number of terms in the second series in m. $S_B = 3m^2$.

Therefore, $\frac{S_A}{S_B} = \frac{2m+1}{m} = 2 + \frac{1}{m}$

So, $k = 2, l = m, k + l = 2 + m$

282. (d) For all y in the range of f, we have $yf(y) = 1$, so $f(y) - \frac{1}{y}$. Since 999 is in the range of f, we have $f(999) = \frac{1}{999}$, so $\frac{1}{999}$ is in the range of f, then the range of f contains $\left[\frac{1}{999}, 999\right]$ by the intermediate value theorem, so for all y in $\left[\frac{1}{999}, 999\right]$, $f(y) = \frac{1}{y}$. Hence statements 1, 2, 4 are all true.

On the other hand, let $y = g(x)$ be the equation of the line through the points $\left(999, \frac{1}{999}\right)$ and (1000, 999) and considerd the function.

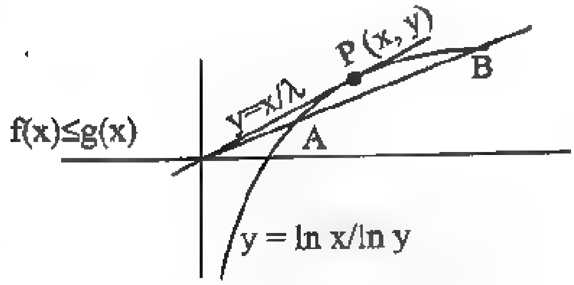
$$f(x) = \begin{cases} 999, & \text{if } x \le \frac{1}{999} \\ \frac{1}{x}, & \text{if } \frac{1}{999} \le x \le 999 \\ g(x), & \text{if } 999 \le x \le 1000 \\ 999, & \text{if } x \ge 1000 \end{cases}$$

Then the range of f equals $\left[\frac{1}{999}, 999\right]$, and f is continuous, so it satisfies the conditions of the problem. This shows that statements 3, 5, 6, 7 are not necessarily true. Hence, the answer is 8.

283. (d) $$\left(\frac{1}{x}\right)^{\lambda} \le \left(\frac{1}{9}\right)^{r}$$

$$x \ln 9 \le \lambda \ln x$$

$$\frac{x}{\lambda} \le \frac{\ln x}{\ln 9}$$



when $f(x)$ and $g(x)$ are tangent to each other at $P(x_1, y_1)$ when $f'(x)=g'(x)$ at $P(x_1, y_1)$

$$\frac{1}{\lambda}=\frac{1}{x\ln 9}$$

$$x_1=\frac{\lambda}{\ln 9}$$

$$y_1=\frac{1}{\ln 9}$$

$$y_1=\frac{\ln x_1}{\ln 9}$$

$$\frac{1}{\ln 9}=\frac{\ln x_1}{\ln 9} \quad \Rightarrow \quad x_1=e$$

$\therefore$ $\lambda = e\ln 9$

for $f(x)<g(x)$

$$\lambda > e\ln 9$$

$$\lambda > 2.71\times 2.197 = 5.95$$

If $\lambda = 6$ then $\left(\frac{1}{x}\right)^{6/x} \le \frac{1}{9}$ true for $x = 3, 4$

284. (a)

$$L=\lim_{x\to\infty}\sqrt{n}\int_0^1\frac{dx}{(1+x^2)^n}$$

$$(1+x^2)^n = 1+nx^2+\ldots\ldots$$

$\therefore$ $(1+x^2)^n > 1+nx^2$

$$\frac{1}{(1+x^2)^n}<\frac{1}{1+nx^2}$$

$\therefore$ $\int_0^1\frac{dx}{(1+x^2)^n}<\int_0^1\frac{dx}{(1+x^2)}$

$$=\frac{1}{n}\int_0^1\frac{dx}{\frac{1}{n}+x^2}=\frac{1}{n}\sqrt{n}\,(\tan^{-1}x\sqrt{n})_0^1$$

$$=\frac{1}{\sqrt{n}}\tan^{-1}\sqrt{n}$$

$$\therefore \qquad \lim_{x\to\infty}\sqrt{n}\int_0^1 \frac{dx}{(1+x^2)^n} < \lim_{x\to\infty}\tan^{-1}\sqrt{n}$$

$$\therefore \qquad L<\frac{\pi}{2}$$

$$\therefore \qquad L<2$$

285. (a) $$I=\int_{-20}^{20}\frac{f(x)}{g(x)}dx$$

Using King's rule, we have

$$I=\int_{-20}^{20}\frac{f(-x)}{g(-x)}dx=\int_{-20}^{20}\frac{f(-x)\cdot f(x)}{g(x)}dx$$

Addition the two, we get :

$$2I=\int_{-20}^{20}\frac{f(x)\cdot(1+f(-x))}{g(x)}dx=\int_{-20}^{20}\frac{f(x)\cdot(1+f(-x))}{g(x)}dx=\int_{20}^{20}f(x)dx=2020$$

$$\Rightarrow \qquad I=1010$$

286. (c) $$S=\frac{\sin^2\frac{2\pi}{7}}{\sin^2\frac{\pi}{7}}+\frac{\sin^2\frac{4\pi}{7}}{\sin^2\frac{2\pi}{7}}+\frac{\sin^2\frac{\pi}{7}}{\sin^2\frac{4\pi}{7}}$$

$$S=4\left(\cos^2\frac{\pi}{7}+\cos^2\frac{2\pi}{7}+\cos^2\frac{4\pi}{7}\right)=4\left(1-2\cos\frac{\pi}{7}\times\cos\frac{2\pi}{7}\times\cos\frac{4\pi}{7}\right)$$

$$=4\left(1+2\times\frac{1}{8}\right)=5$$

287. (c) $x^n=x^2+x+1$

$n\ln x=\ln(x^2+x+1)$

$$\therefore \qquad n=\frac{\ln(x^2+x+1)}{\ln x} \qquad (\text{as } n\to\infty, x\to 1)$$

$$\therefore \qquad e^{\left(\lim\limits_{x\to1}\frac{\ln(x^2+x+1)\times(x-1)}{\ln x}\right)}=e^{\ln 3}=3$$

288. (c) Let $\log_2 n=m \Rightarrow n=2^m$

$$\therefore \qquad \prod_{k=1}^{m}(x^{2^{m-k}}+1)$$

$$=(x^{2^{m-1}}+1)(x^{2^{m-2}}+1)(x^{2^{m-3}}+1)\ldots\ldots(x+1)$$

$$=(x+1)(x^2+1)(x^4+1)\ldots\ldots(x^{2^{m-1}}+1)$$

$$=\frac{x^{2^m}-1}{x-1}=\frac{x^n-1}{x-1}=\frac{x^A-B}{x-C}$$

$\therefore \qquad A = n = 2^{92}, \quad B = C = 1$

$\Rightarrow \qquad B + C + \log_2 A = 1 + 1 + 92 = 94$

289. (b) Let $0 < a < b < \frac{\pi}{2}$. If $f(x) = \begin{vmatrix} \sin x & \sin a & \sin b \\ \cos x & \cos a & \cos b \\ \tan x & \tan a & \tan b \end{vmatrix}$, then minimum possible number of roots of $f'(x) = 0$ lying in (a, b).

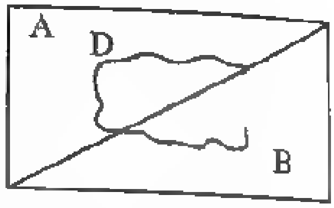
290. (a) Draw the graph of $\ln x$ and $\cos x - \sin x$ and then interpret.

291. (d) $$e^{\lim\limits_{x\to 0} \frac{2}{x}\left(\frac{a^x-1+b^x-1}{2}\right)} = e^{\ln ab} = ab = 6$$

$(a, b) = (1, 6), (6, 1)\ (2, 3)\ (3, 2)$

$\Rightarrow \qquad P(E) = \frac{4}{36} - \frac{1}{9}$

292. (c) A = even that the item came from lot A ; $P(A) = \frac{3}{7}$



B = item came from B ; $P(B) = \frac{4}{7}$

D = item from mixed lot 'C' is defective

$$P(D) = P(D \cap A) + P(D \cap B)$$

$$= P(A) \cdot P(D/A) + P(B) \cdot P(D/A) = \frac{3}{7} \times \frac{3}{8} \times \frac{4}{7} \times \frac{5}{8} = \frac{29}{56}$$

293. (a) $$a^2 + 4b^2 + 4c^2 - 2ab - 4bc - 2ac = 0$$

$$(a - 2b)^2 + (2b - 2c)^2 + (2c - a)^2 = 0$$

$\Rightarrow \qquad a = 2b = 2c$

$\therefore$ Number of ordered triplets satisfying are 3 *i.e.*, (2, 1 1), (4, 2, 2), (6, 3, 3).

Two points (2, 1, 1) and (4, 2, 2) lying inside the given tetrahedron.

$\therefore$ Required probability is $\frac{2}{3} = \frac{5}{\lambda} \quad \Rightarrow \lambda = 9$

294. (c) The point $(a_1, b_1), (a_2, b_2), \ldots\ldots, (a_n, b_n), (a_{n+1}, b_{n+1})$

given $\qquad (a_{n+1} + ib_{n+1}) =$ say z_{n+1}

$$z_{n+1} = (\sqrt{3}\, a_n - b_n) + i(\sqrt{3}\, b_n + a_n)$$

$$= \sqrt{3}\,(a_n - ib_n) + i(a_n + ib_n)$$

$$\sqrt{3}(a_n + ib_n) + i(a_n + ib_n) = \sqrt{3}\, z_n + iz_n$$

$$z_n(\sqrt{3} + i) \text{ where } z_n = a_n + ib_n$$

$$= 2z_n\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$$

Hence, $$z_{n+1} = 2z_n e^{i\frac{\pi}{6}}$$

$$\frac{z_{n+1}}{z_n} = 2e^{i\frac{\pi}{6}} \text{ say } \alpha$$

Put $n = 1, 2, 3, \ldots\ldots, 99$

$$\frac{z_2}{z_1} = \alpha \quad ; \quad \frac{z_3}{z_2} = \alpha, \ldots\ldots\ldots, \frac{z_{100}}{z_{99}} = \alpha$$

Multiplying $$\frac{z_{100}}{z_1} - \alpha^{99} = 2^{99} e^{i\frac{33\pi}{2}} = 2^{99} \cdot i$$

$$z_{100} = 2^{99}(z_1)i$$

$$z_1 = \frac{z_{100}}{2^{99} \cdot i} = \frac{(2+4i)}{2^{99} i}$$

$$z_1 = 2^{-97} - 2^{-98} i$$

$$a_1 + ib_1 = 2^{-97} - 2^{-98} i$$

$\therefore$ $$a_1 = 2^{-97} \quad ; \quad b_1 = -2^{-98}$$

$$a_1 + b_1 = 2^{-98}(2-1) = 2^{-98}$$

295. (b) $M = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$, $M^2 = MM = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$, $M^3 = \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix}$, $M^4 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$,

$M^{4k} = I \; \forall \; k \in N$.

So, $I + M + M^2 + M^3 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\therefore$ $I + M + M^2 + M^3 + M^4 + M^5 + \ldots\ldots + M^{2010}$

$= (I + M + M^2 + M^3) + M^4(I + M + M^2 + M^3) + \ldots\ldots + M^{2008}(I + M + M^2)$

$= I + M + M^2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} + \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} = M$ (itself)

296. (d) Given expression $= \dfrac{a\omega^3 + b\omega + c\omega^2}{b + c\omega + a\omega^2} + \dfrac{a + b\omega + c\omega^2}{c\omega^3 + a\omega + b\omega^2}$

$$= \omega + \frac{1}{\omega} = \omega + \omega^2 = -1$$

297. (d) Equation of line through O (0, 0, 0) and perpendicular to the plane $2x - y - z = 4$, is

$$\frac{x-0}{2} = \frac{y-0}{-1} = \frac{z-0}{-1} = t \text{ (let)}$$

$\therefore$ Any point on it is $(2t, -t, -t)$

As above point lies on the plane $3x - 5y + 2z = 6$, so

$\Rightarrow \quad 6t + 5t - 2t = 6 \quad \Rightarrow \quad 9t = 6 \;\Rightarrow\; t = \dfrac{2}{3}$.

$\therefore$ Co-ordinates of point of intersection are $\left(\frac{4}{3}, \frac{-2}{3}, \frac{-2}{3}\right) \equiv (x_0, y_0, z_0)$ [Given]

Hence, $(2x_0 - 3y_0 + z_0) = 4$

298. (c) Normal vector of the plane $\vec{\mathbf{n}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 3 & 0 & -1 \\ 1 & 2 & -1 \end{vmatrix}$

$$\vec{\mathbf{n}} = 2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 6\hat{\mathbf{k}} = 2(\hat{\mathbf{i}} + \hat{\mathbf{j}} + 3\hat{\mathbf{k}})$$

$\therefore$ Equation of plane $1(x+1) + 1(y-2) + 3(z-0) = 0$

P : $\boxed{x + y + 3z = 1}$

Hence, $(a+b+c) = 1+1+3 = 5$

299. (c) $|[\vec{\mathbf{a}}\ \vec{\mathbf{b}}\ \vec{\mathbf{c}}]| = 30$

$|abc \sin\theta \cos\phi| = 30 \Rightarrow \theta = \frac{\pi}{2}, \quad \phi = 0 \Rightarrow \vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ are mutually perpendicular

$$(2\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}}) \cdot [(\vec{\mathbf{a}} \times \vec{\mathbf{c}}) \times (\vec{\mathbf{a}} - \vec{\mathbf{c}}) + \vec{\mathbf{b}}]$$

$$= (2\vec{\mathbf{a}} + \vec{\mathbf{b}} + \vec{\mathbf{c}}) \cdot [(\vec{\mathbf{a}} \cdot \vec{\mathbf{a}})\vec{\mathbf{c}} + c^2 \vec{\mathbf{a}} + \vec{\mathbf{b}})$$

$$= 2a^2c^2 + b^2 + a^2c^2 = 3a^2c^2 + b^2 = 300 + 9 - 309$$

$$\therefore \quad \frac{k}{103} = \frac{309}{103} = 3$$

300. (c) $f(x) = \begin{cases} \frac{1}{x}; & \text{if } x^2 > 1 \quad \Rightarrow x < -1 \quad \text{or} \quad x > 1 \\ ax^3 + bx^2; & \text{if } 0 < x^2 < 1 \quad \Rightarrow -1 < x < 1 \\ \frac{1/x + ax^3 + bx^2}{2}; & \text{if } x^2 = 1 \end{cases}$

$\because$ f is continuous

$\therefore$ at $x = 1$ $\quad 1 = a + b$ ...(1)

and at $x = -1$ $\quad -1 = -a + b$ ...(2)

$\therefore$ $b = 0$ and $a = 1$

$\therefore$ point A and B are = $(-1, 3)$ and $(1, -1)$.

$\therefore$ $g'(x) = \lambda(x-1)(x+1)$

$$g(x) = \lambda\left(\frac{x^3}{3} - x\right) + c$$

$$g(-1) = \frac{2\lambda}{3} + c = 3 \quad \text{...(3)}$$

$$g(1) = -\frac{2\lambda}{3} + c = -1$$

$$c = 1 \quad \text{and} \quad \lambda = 3$$

$\therefore$ $g(x) = x^3 - 3x + 1$

$\therefore$ $g(2) = 3$

More Than One Correct Type Questions

301. (a,c,d) $f'(x)=e^{\frac{-1}{x^2}}\left(\frac{2}{x^3}\right)+\sqrt{1+\sin\frac{\pi x}{2}}\left(\frac{\pi}{2}\right) \Rightarrow (A)$

Also $f'(x)$ is bounded but $f(x)$ is unbound because $\lim\limits_{x\to\infty} f(x)\to\infty$

Also $f''(x)$ does not exist at 3, 7, 11 ....

$\Rightarrow$ (d) is obvious.

302. (a,c) Curve is $y=\tan^{-1}(\sqrt{x^2-1})-2-\frac{\pi}{3}$

Now, verify options.

303. (c,d) Solve $f'(x)\geq 2\,\forall\, x\in(0,\infty)$ $\Rightarrow$ Range of a is $[1,\infty)$.

304. (a,b,c) Since, f attains its maximum value at some $c\in(0,4)$

$\therefore$ $f'(c)=0$

Now, in $y=f'(x)$ using LMVT in $[0, c]$ and $[c, 4]$

$$|f''(d)|=\left|\frac{f'(c)-f'(0)}{c}\right|\leq 5$$

$$\Rightarrow \quad |f'(0)|\leq 5c \qquad \dots(1)$$

Again, $$|f''(d)|=\left|\frac{f'(4)-f'(c)}{4-c}\right|\leq 5$$

$$\Rightarrow \quad |f'(4)|\leq 5(4-c) \qquad \dots(2)$$

Eqn. (1) + eqn. (2) $\Rightarrow$ $|f'(0)|+|f'(4)|\leq 20.$

305. (a,d) $a\cdot c=b\cdot d$ [power of point]

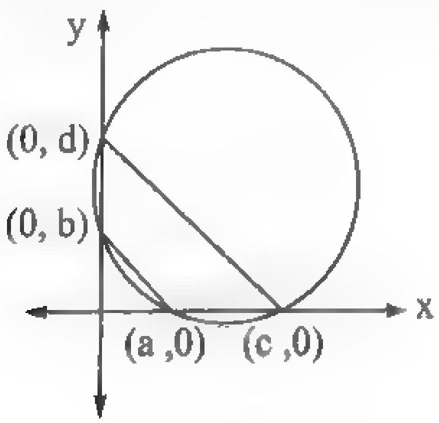
$a^2+c^2=b^2+d^2$

$(a-c)^2=(b-d)^2$ and $(a+c)^2=(b+d)^2$

$\therefore$ $a+c=b+d$ and $a-c=b-d$

$\Rightarrow$ $a=b$ and $c=d$

$\Rightarrow$ Both line have slope $=-1$ $\Rightarrow$ Parallel lines never meet



$\therefore \quad a-c=a-b$ ...(i)

$a=d$ and $b=c$

$$\frac{x}{d}+\frac{y}{c}=1;\quad \frac{x}{c}+\frac{y}{d}=1$$

$\Rightarrow \quad cx+dy=dx+cy \quad \Rightarrow c(x-y)=d(x-y)$

$\Rightarrow \quad y=x$

2 lines meet $y=x$.

306. (a,c) Let $3^k=x$

$2^k=y$

$$\therefore \quad \frac{6^k}{(3^k-2^k)(3^{k+1}-2^{k+1})}=\frac{xy}{(x-y)(3x-2y)}$$

$$=\frac{y(3x-2y)-2y(x-y)}{(x-y)(3x-2y)}=\frac{y}{x-y}-\frac{2y}{3x-2y}$$

$$=\frac{2^k}{3^k-2^k}-\frac{2^{k+1}}{3^{k+1}-2^{k+1}}$$

$$\therefore \quad \sum_{k=1}^{\infty}\frac{6^k}{(3^k-2^k)(3^{k+1}-2^{k+1})}=\sum_{k=1}^{\infty}\frac{2^k}{3^k-2^k}-\frac{2^{k+1}}{3^{k+1}-2^{k+1}}$$

$$T_1=\frac{2}{3-2}-\frac{2^2}{3^2-2^2}$$

$$T_2=\frac{2^2}{3^2-2^2}-\frac{2^3}{3^3-2^3}$$

$\vdots$

$$T_k=\frac{2^k}{3^k-2^k}-\frac{2^{k+1}}{3^{k+1}-2^{k+1}}$$

$$\sum_{k=1}^{\infty}T_k=\lim_{k\to\infty}\left(2-\frac{2\cdot 2^k}{3\cdot 3^k-2\cdot 2^k}\right)=2$$

307. (a,c) Do yourself

308. (a,b,c,d) Using A.M. $\geq$ G.M.

$$\frac{\Sigma a_i}{16}\geq (a_1a_2\ldots\ldots a_{16})^{1/16}$$

$$\therefore \quad a_1a_2\ldots\ldots a_{16}\leq\left(\frac{a_1+a_2+\ldots\ldots+a_{16}}{16}\right)^{16}\leq\left(\frac{392}{16}\right)^{16}\leq\left(\frac{49}{2}\right)^{16}$$

$\Rightarrow \quad S=49$ and $W=2$

$$\frac{6}{2}(2a+5\cdot 3d)=147$$

$2a+15d=49$ ...(i)

$$2a+15d = M = 49$$

Also, $\frac{4}{2}(2a+3\cdot 5d)=N$; $(2a+15d)2=N$ $\Rightarrow$ $N=98$

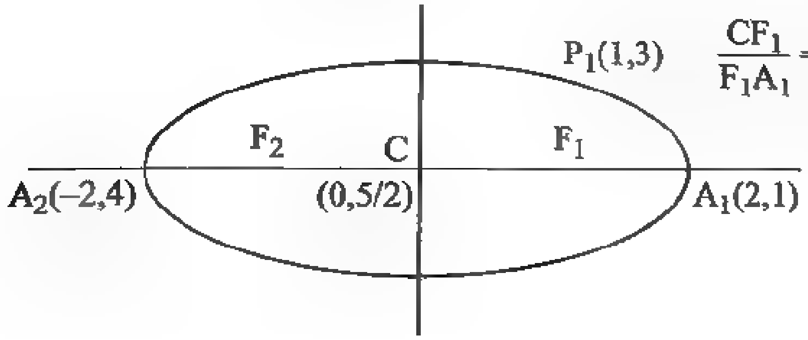
309. (a,b,c,d) $\theta=\frac{\pi}{2}$

310. (b,c,d) $2a=\sqrt{16+9}=5$

$$C\equiv\left(0,\frac{5}{2}\right)$$

$$F_1\equiv\left[2e,\frac{5}{2}(5-3e)\right]$$

$$F_2\equiv\left[-2e,\frac{1}{2}(5+3e)\right]$$



$$\frac{CF_1}{F_1A_1}=\frac{ae}{a-ae}=\frac{e}{1-e}$$

$$PF_1+PF_2=2a$$

$$e^2=\frac{5}{6} \quad\Rightarrow\quad e=\sqrt{\frac{5}{6}}$$

$\therefore$ $b^2=\frac{25}{4}$, now verify all the options.

311. (a,b,c,d) Do yourself.

312. (a,b,c) (a) $a, a+d, a+2d$

$$a^2+(a+d)^2=(a+2d)^2$$

$$2a^2+d^2+2ad=a^2+4d^2+4ad$$

$$a^2-2ad=3d^2$$

$$\left(\frac{a}{d}\right)^2-\frac{2a}{3}=3$$

$$t^2-2t-3=0 \quad\Rightarrow\quad (t-3)(t+1)=0 \Rightarrow t=3$$

$a=3d$ [possible]

(b) $a=a$; $b=ar$; $c=ar^2$

$$c^2=a^2+b^2$$

$$a^2r^4=a^2(1+r^2)$$

$$r^2t^2 - t - 1 = 0$$

$$r^2 = \frac{1+\sqrt{5}}{2} > 1 \qquad \text{[possible]}$$

$a, b, c \to$ A.P.

$$b = \frac{2ac}{a+c}$$

$$c^2 = a^2 + \frac{4a^2c^2}{(a+c)^2}$$

$$a^4 + 2a^3c + 4a^2c^2 - 2ac^3 - c^4 = 0$$

Divide by a^2c^2

$$\left(\frac{a^2}{c^2} - \frac{c^2}{a^2}\right) + 2\left(\frac{a}{c} - \frac{c}{a}\right) + 4 = 0$$

$$\frac{a}{c} - \frac{c}{a} - t; \; t < 0$$

$$t\left(\frac{a}{c} - \frac{c}{a}\right) + 2t + 4 = 0 \quad \Rightarrow \quad t\sqrt{t^2+4} + 2t + 4 - 0, \quad t < 0$$

$$\Rightarrow \quad t^2(t^2+4) = 4(t+2)^2$$

$$\Rightarrow \quad f(t) = t^4 - 16t - 16 = 0 \quad \Rightarrow \quad f(0) = -16$$

$$\Rightarrow \quad f' = 4t^3 - 4 \quad \Rightarrow \quad f'' = 12t^2 > 0$$

One negative, one positive root.

$\therefore$ $\frac{a}{c} - \frac{c}{a}$ has positive roots.

313. (a,b,c) Subtracting the given two equations.

$$5x^2 - 15x + b + a = 0 \begin{matrix} \nearrow \alpha \\ \searrow \beta \end{matrix}$$

$$\alpha + \beta = 3$$

$$\alpha + \beta + x_1 = 5$$

$$\alpha + \beta + x_2 = 0$$

$$x_1 = 2; \quad x_2 = -3$$

Put x_1 and x_2

$\therefore$ $8 - 20 + 14 - a = 0 \quad \Rightarrow \quad a = 2$

$\therefore$ $b = 3.$

314. (a,b,c) Differentiate w.r.t. y keeping x constant

$$0 = xf'(xy) - \frac{x}{y^2} f'\left(\frac{x}{y}\right)$$

$$\frac{1}{y^2} f'\left(\frac{x}{y}\right) = f'(xy)$$

Put $y = x$

$$f'(1) = x^2 f'(x^2) = 1$$

$$f'(t) = \frac{1}{t} \quad \forall\, t > 0$$

$$f(x) = \ln x$$

315. (a,c,d) $\quad f(xy) = e^{xy-x} \cdot f(x) + e^{xy-y} f(y)$

Differentiating w.r.t. x

$$yf'(xy) = e^{xy-x} f'(x) + f(x)e^{xy-x}(y-1) + f(y)e^{xy-y} \cdot y$$

Put $x = 1$

$$yf'(y) = e^{y-1} f'(1) + f(1)e^{y-1}(y-1) + yf(y)$$

Now put $x = y = 1$; $\quad f(1) = 0$

$\therefore \quad yf'(y) = e^y + yf(y) \quad \Rightarrow \quad x(f'(x) - f(x)) = e^x$

$$e^{-x}(f'(x) - f(x)) = \frac{1}{x}$$

Integrate both sides

$$e^{-x} f(x) = \ln x + C$$

$f(1) = 0 \quad \therefore \quad C = 0$

$\Rightarrow \quad f(x) = e^x \ln x$. Now verify.

316. (a,c,d) $\quad$ Let degree of $f(x)$ be n

$\therefore \quad n^2 = n + 2 \quad \Rightarrow \quad n = 2$

$\therefore \quad f(x) = ax^2 + bx$

On comparing the coefficient

$$b = 0 \quad ; \quad a = \frac{1}{\sqrt{3}}$$

$$\therefore \quad f(x) = \frac{x^2}{\sqrt{3}}$$

317. (a,d) $\quad$ Consider $F(x) = f(x) - g(x)$

$$F'(x) = f'(x) - g'(x) = \ln(x + \sqrt{x^2 + 1}) = 0 \quad \text{at} \quad x = 0$$

$F'(x) > 0\, \forall\, x > 0 \quad \Rightarrow \quad f(x)$ is increasing

and $\quad F'(x) < 0\ \forall x < 0 \quad \Rightarrow \quad f(x)$ is decreasing

Hence, minimum value of $F(x)$ occur at $x = 0$ but $F(0) = 0 \ \Rightarrow \ F(x) > 0\, \forall\, x \in R - \{0\}$

$\Rightarrow \quad$ (a) and (d) are correct.

318. (a,b,d) Put $a = g'(1)$, $b = g''(2)$

$$f(x) = x^2 + ax + b \quad \cdots(1)$$

$$g(x) = cx^2 + x(2x+a) + 2$$

$$\therefore \quad g(x) = (c+2)x^2 + ax + 2 \quad \cdots(2)$$

$$g'(x) = 2(c+2)x + a$$

$$\therefore \quad g'(1) = 2(c+2) + a = a$$

$$\therefore \quad c = -2$$

$$g''(x) = 2(c+2)$$

$$\therefore \quad g''(2) = 2(c+2) = b$$

$$\therefore \quad b = 0$$

$$\Rightarrow \quad f(x) = x^2 + ax$$

$$g(x) = ax + 2$$

Now, $c = f(1)$

$$\therefore \quad f(1) = 1 + a = c = -2 \quad \Rightarrow \quad a = -3$$

$$\Rightarrow \quad f(x) = x^2 - 3x, \qquad g(x) = 2 - 3x$$

Now, verify.

319. (a,b) $$b = \lim_{x\to1} \frac{x^a - 2x + 1 - x + 1}{(x-1)(x^a - 2x + 1)} = \lim_{x\to1} \frac{x^a - 3x + 2}{(x-1)(x^a + 1 - 2x)}; \qquad x = 1 + h$$

$$b = \lim_{h\to0} \frac{(1+h)^a - 3(1+h) + 2}{h[(1+h)^a + 1 - 2(1+h)]} = \lim_{h\to0} \frac{(1 + ah + \ldots\ldots) - 3h - 1}{h[1 + ah \ldots\ldots - 1 - 2h]}$$

$$= \lim_{h\to0} \frac{(a-3)h + \frac{a(a-1)}{2}h^2}{(a-2)h^2 + \ldots\ldots}$$

$$a = 3 \quad \text{and} \quad b = 3 \quad \Rightarrow \quad a + b = 6$$

320. (a,c,d) $y = \sqrt{x+y} \quad \Rightarrow \quad y^2 = x + y$

$$\therefore \quad y^2 - y - x = 0 \quad \ldots(1)$$

$$(2y-1)y' = 1$$

$$\Rightarrow \quad y' = \frac{1}{2y-1} \quad \Rightarrow \quad \text{(a)}$$

From equation (1), $2y = 1 + \sqrt{1+4x}$

Hence, $2y - 1 = \sqrt{1+4x}$

$$\Rightarrow \quad y' = \frac{1}{\sqrt{1+4x}} \quad \Rightarrow \quad \text{(c)}$$

On dividing by y, equation (1) gives

$$y-1=\frac{x}{y} \quad \Rightarrow \quad 2y-2=\frac{2x}{y}$$

(d) : $\dfrac{y}{2x+y}=\dfrac{1}{\frac{2x}{y}+1}=\dfrac{1}{(2y-2)+1}=\dfrac{1}{2y-1}$ which is same as (a).

321. (a,b,c) $f(x)=\begin{cases} x, & x<0 \\ \sin x, & 0\le x\le \pi/2 \\ 1, & x>\pi/2 \end{cases} = a\displaystyle\int_0^{\pi/2}|x-t|\sin t\,dt+bx+c$

If $x<0$, $\quad x=a\displaystyle\int_0^{\pi/2} t\sin t dt-ax\int_0^{\pi/2}\sin t\,dt+bx+c \;\Rightarrow\; a+c=0$ and $b-a=1$

If $x>\dfrac{\pi}{2}$, $\quad 1=ax\displaystyle\int_0^{\pi/2}\sin t\,dt-a\int_0^{\pi/2}t\sin t\,dt+bx+c \;\Rightarrow\; c-a=1$ and $a+b=0$

$\Rightarrow \quad a=\dfrac{-1}{2};\; b-c=\dfrac{1}{2}$

322. (a,c) $\quad f(x)=ax^2+bx+c$

Given, $\quad \dfrac{-b}{2a}=0 \quad\Rightarrow\quad b=0$

Now, let maximum value of $g(x)$ occur at $x=p$

$\Rightarrow \quad g'(p)=0$

$\quad [(f'(p))^2+f''(p)]e^{f(p)}=0$

$\Rightarrow \quad [(f'(p))^2+f''(p)]=0 \quad \ldots(1)$

Also, $\quad g(x)=4\sqrt{e}$ has rational roots (given)

Therefore, $\quad |f'(p)|e^{f(p)}=4\sqrt{e}$ has rational root (p must be rational)

On comparing $\quad |f'(p)|=4$ and $f(p)=\dfrac{1}{2} \quad \ldots(2)$

From equations (1) and (2),

$\quad 16+2a=0 \quad\Rightarrow\quad a=-8$

Hence, $\quad f(x)=-8x^2+c$

Now, $\quad f(p)=\dfrac{1}{2} \;\Rightarrow\; -8p^2+c=\dfrac{1}{2}$ and $f'(p)=-16p=\pm 4 \;\Rightarrow\; p=\pm\dfrac{1}{4}$

Hence, $\quad c=1$

Therefore, $\quad f(x)=-8x^2+1$

$\quad g(x)=|16x|e^{1-8x^2}$

Now, verify the options.

323. (c,d) (a) $f(x) = -(x-2)^{1/3} \quad \Rightarrow \quad \lim_{h\to 0} \frac{f(2+h)-f(2)}{h}$ does not exist.

(b) $f(x) = e^{-x}$

solution for $f(x) = 0$ does not exist.

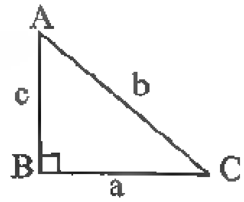
(c) $f(x)$ is monotonously decreasing and $f(-x+1)$ monotonously increases.

And since its clear that $y = f(x)$ and $y = f(-x+1)$ meets at point $\left(\frac{1}{2}, f\left(\frac{1}{2}\right)\right)$,

they must meet at only one point.

Thus, the solution for $f(x) - f(-x+1)$ is only one, $x = \frac{1}{2}$.

324. (a,b,c) b is maximum where $B = 90°$



$$\frac{b}{\sin B} = \frac{a}{\sin A} = 2R$$

$$b = \frac{11}{3/7} = 2R \quad , \quad b = \frac{77}{3}$$

$$\Delta = \frac{1}{2} \times 11 \times c \qquad \left[c = \sqrt{b^2 - a^2} = \frac{22\sqrt{10}}{3}\right]$$

(D) $\frac{24\sqrt{10}}{49}$

325. (a,d) Do yourself.

326. (a,b,d) $f(0^-) = f(0) = f(0^+)$

$f(0^-) = 3$

$$\lim_{h\to 0} \frac{a(1 - h\sin h) + b\cos h + 5}{h^2} = 3$$

$$\lim_{h\to 0} \frac{a\left[1 - h\left(h - \frac{h^3}{6} \ldots\right)\right] + b\left(1 - \frac{h^2}{2!} + \frac{h^4}{4!} + \ldots\ldots\right) + 5}{h^2} = 3$$

$$\lim_{h\to 0} \frac{(a+b+5) + h^2\left(-a - \frac{b}{2}\right) + \ldots\ldots}{h^2} = 3$$

$a + b + 5 = 0$ and $a + \frac{1}{2} = -3$

$-3 + \frac{b}{2} = -5 \quad ; \quad \frac{b}{2} = -2$

$\therefore \quad b = -4 \quad ; \quad a = -1$

Now, $f(0^+) = 3$

$$\lim_{h\to 0}\left(1+\frac{ch+dh^3}{h^2}\right)^{1/h}=3$$

$$C=0$$

$$\lim_{h\to 0}(1+dh)^{1/h}=3$$

$$e^d=3$$

$\therefore \quad d=\ln 3$

$\therefore \quad 2e^d+7c-3a-5b=2(3)+7(0)-3(-1)-5(-4)=6+3+20=29$

327. (b,c) $\quad x^2-ax-(a^2+1)x+a(a^2+1)=0$

$\Rightarrow \quad (x-a)(x-(a^2+1))=0$

Clearly greater root is $a^2+1=\alpha$ (let). Let $f(x)=x^2 \quad a^2x-2(a^2-2)=0$

Then the condition that α lies between the roots of $f(x)$ is $f(\alpha)<0$. Solving we get $a^2>5$ from where we get least positive integral values of a as 3.

(a) $$\sqrt{\frac{27}{\sqrt{\frac{27}{\sqrt{\frac{27}{\sqrt{\dots}}}}}}}=27^{1/2\ 1/4+1/8}=3$$

(b) $$\sqrt{4\sqrt{4\sqrt{4\sqrt{\dots\dots}}}}=4^{1/2+1/4+1/8+\dots}=4$$

(c) $$\sqrt{5}\sqrt{\sqrt{5}\sqrt{\sqrt{5}\sqrt{\sqrt{5}\sqrt{\dots}}}}=\sqrt{5}^{\,1+1/2+1/4+\dots\dots}=5$$

(d) $$\sqrt{2+\sqrt{2+\sqrt{2+\dots}}}=2$$

328. (b,c,d) If graph of $|f(x)|$ and $f(|x|)$ is same then $f(x)>0\ \forall\ x>0$

Given $f(2)=0 \quad \rightarrow \quad f'(2)-0$ because $f(x)$ cannot be –ve for $x>0$.

By symmetry $f(\ 2)=f'(-2)=0$

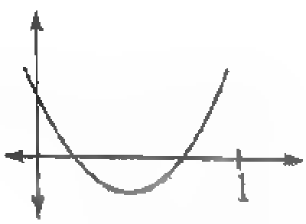
$\therefore \quad f(x)=x(x-2)^2(x+2)^2$ **(Note :** Function must be odd)

$$f(x)=x(x^2-4)^2$$

329. (a,b,c) $\quad f(x)=ax^2-bx+c$

S.R. $=\dfrac{b}{a}<2 \quad \Rightarrow \quad b<2a \quad \dots(1)$

P.R. $=\dfrac{c}{a}<1 \quad \Rightarrow \quad c<a \quad \dots(2)$



and $\quad D=b^2-4ac>0 \quad \Rightarrow \quad b^2>4ac \quad \dots(3)$

To find minimum value of abc

$f(0)=c>0 \quad ; \quad C_{\min.}=1$

$\therefore \quad (3) \quad \Rightarrow \quad b^2 > 4a$

$\qquad (1) \quad \Rightarrow \quad b < 2a$

$\Rightarrow \quad b = 5; \quad a = 5; \quad c = 1$

Also, $\quad f(1) > 0 \quad \Rightarrow \quad a - b + c > 0; \quad a + 1 > b$

$\lambda \le \log_5 25$

$\lambda \le 2$

330. (a,c,d)

(1) Reciprocal roots $\Rightarrow \quad c/a = 1 \quad \Rightarrow \quad c = a$

(2) Distinct roots $\Rightarrow \quad b^2 - 4ac = b^2 - 4a^2 > 0$

$$\Rightarrow \quad \frac{b^2}{a^2} - 4 > 0 \qquad (\because\ a \ne 0)$$

(3) Positive roots $\Rightarrow \quad \frac{b}{a} < 0$

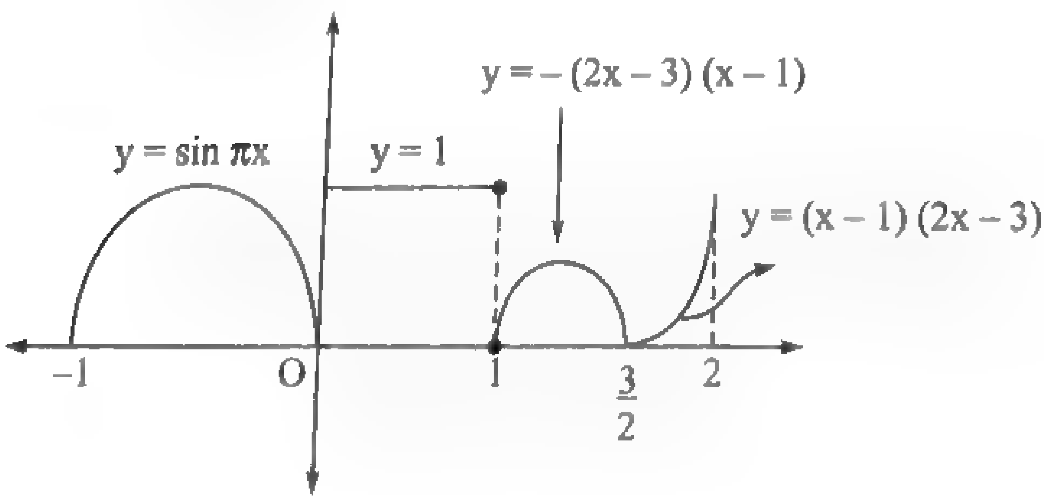
Substituting $b/a = t$, from (2) and (3). It can be easily established that $t < -2$

$$a\,f'(1) = 2(2a+b) = a^2\left(2 + \frac{b}{a}\right) = a^2(2+t) < 0 \qquad (\because\ t < -2 \text{ and } a^2 > 0)$$

331. (a,b) $\quad f(x) = \frac{x^2-1}{2};\ g(x) = \frac{-1}{2}\ln x;\ k = \frac{1}{2}$

332. (a,b) $\quad f(x) = \begin{cases} \{x\}\sqrt{4x^2 - 12x + 9}, & 1 \le x \le 2 \\ \cos\left(\frac{\pi}{2}(|x| - \{x\})\right), & -1 \le x < 1 \end{cases}; \quad f(x) = \begin{cases} |2x-3| \cdot \{x\} & 1 \le x \le 2 \\ \cos\frac{\pi}{2}(|x| - \{x\}), & -1 \le x < 1 \end{cases}$

$$= \begin{cases} -(2x-3)(x-1), & 1 \le x \le \frac{3}{2} \\ (2x-3)(x-1), & \frac{3}{2} \le x < 2 \\ \cos\frac{\pi}{2}(-x-(x+1)), & -1 \le x < 0 \\ \cos\frac{\pi}{2}(x-(x)), & 0 \le x \le 1 \end{cases}$$



$$= \begin{cases} \cos(2x+1)\frac{\pi}{2} = -\sin \pi x, & -1 \le x < 0 \\ 1, & 0 \le x < 1 \\ -(2x+3)(x-1), & 1 \le x < \frac{3}{2} \\ (2x+3)(x-1), & \frac{3}{2} \le x < 2 \\ 0 & x = 2 \end{cases}$$

Hence, $f(x)$ is discontinuous at 0, 1 and 2.

333. (a,d) $\int \underset{\text{I}}{x} \cdot \underbrace{e^{x^2+x} \cdot (2x+1)}_{\text{II}} dx + \int e^{x^2+x} dx$

$x \cdot e^{x^2+x} + \int e^{x^2+x} dx$

$\therefore \quad f(x) = xe^x$

$f'(x) = e^x(1+x) = 0 \quad \Rightarrow \quad x = -1$

$f''(x) = e^x(2+x)$

$f''(-1) = 0 \quad \Rightarrow \quad$ Minima

$\therefore \quad f(\ 1) = \frac{-1}{e} = m$

$\left[\frac{-1}{m}\right] - [e] = 2$

334. (a,b,c) $q \in \left[\frac{1}{3}, 3\right]$

335. (a,c,d) $Y = \int_0^1 \frac{2x^2+3x+3}{(x+1)(x^2+2x+2)}$ By partial fraction decomposition

$$= \int_0^1 \left(\frac{2}{x+1} - \frac{1}{x^2+2x+2}\right) dx = \int_0^1 \left(\frac{2}{x+1} \quad \frac{1}{(x+1)^2+1}\right) dx$$

$$= 2\ln(x+1) - arc\tan(x+1)\Big|_0^1$$

$$= 2\ln 2 - arc\tan 2 + \frac{\pi}{4} \quad \Rightarrow \quad \text{(a)}$$

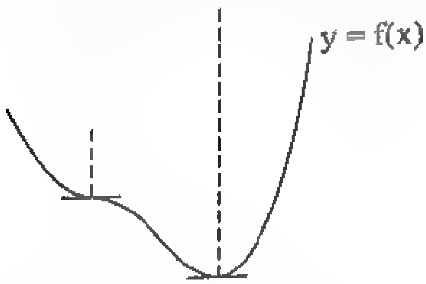
From (a)

$$A = 2\ln 2 + \frac{\pi}{4} - arc\tan 2 = 2\ln 2 + arc\tan\left(\frac{1-2}{1+2}\right) = 2\ln 2 - arc\cot 3 \quad \Rightarrow \quad \text{(c)}$$

From (a)

$$A = 2\ln 2 + \frac{\pi}{4} - arc\tan 2 = -\frac{\pi}{4} + \ln 4 + \frac{\pi}{2} - arc\tan 2 = -\frac{\pi}{4} + 2\ln 2 + arc\cot 2 \quad \Rightarrow \quad \text{(d)}$$

336. (a,c,d) Since, $f'(0) = 0$ and $f(x)$ decreases in the intervals $(-\infty, 0)$ and $(0, k)$, we infer that it contacts the line $y = f(0)$ as it passes through



Then, $f(x)$ would look like the picture shown on the right.

Since $f'(2)>0$, we can notice that $k<2$, and therefore $f'(x)=0$ has one root in the open interval (0, 2), **(*a*, true)** and obviously $f(x)$ does not have a local maximum **(*b*, false)**.

If $f(0)=0$, then $f(x)=x^3(x-p)$.

This leads to $f'(x)=3x^2(x-p)+x^3=x^2(4x-3p)$ and from $f'(2)=16$, we get $p=\frac{4}{3}$.

$f'(x)=4x^2(x-1)$, and therefore the local minimum (and thus the actual minimum of this function) occurs at $x=1$, which yields $f(1)=-\frac{1}{3}$, **(*c*, true).**

337. (a,d) Given $x-g'(x)\geq 0\,\forall x \Rightarrow g'(x)\leq x\,\forall x$

$$\therefore \int_0^1 g'(x)dx\leq\int_0^1 x\,dx \Rightarrow g(1)-g(0)\leq\frac{1}{2} \Rightarrow \text{(a)}$$

Again $f'(x)+f(x)g'(x)\geq 0\,\forall\, x$

$$\therefore \frac{d}{dx}(f(x)e^{g(x)})\geq 0\,\forall\, x$$

Hence, $f(x)\cdot e^{g(x)}$ is an increasing function $\forall\, x$

$$\Rightarrow f(0)\cdot e^{g(0)}\leq f(1)e^{g(1)}$$

$$\frac{f(0)}{f(1)}\leq e^{g(1)-g(0)}\leq e^{1/2} \Rightarrow \text{(d)}$$

338. (b,c) Range of $f(x)$ must be all real numbers.

Hence, $f(x)=1-\dfrac{2x+c}{x^2+x+2c}$ where $P(x)=x^2+x+2c$

For range to be R, $P\left(\dfrac{-c}{2}\right)<0$

$\Rightarrow c\in(-6,0)$.

339. (b,d) $I=\displaystyle\int\frac{x^2}{(x\sin x+\cos x)^2}dx$

Put $x^2=x\sec x\cdot x\cos x$

$$I=\int\frac{x\sec x\cdot x\cos x}{(x\sin x+\cos x)^2}dx$$

Integrating by parts taking $x\sec x$ as first function and $\frac{x\cos x}{(x\sin x+\cos x)^2}$ as second function, we get

$$I = x\sec x\int\frac{x\cos x}{(x\sin x+\cos x)^2}\,dx+\int(x\sec x\tan x+\sec x)\left(\int\frac{x\cos x}{(x\sin x+\cos x)^2}\,dx\right)dx \quad ...(1)$$

$$I_1=\int\frac{x\cos x}{(x\sin x+\cos x)^2}\,dx$$

Put $x\sin x+\cos x=t \Rightarrow \quad x\cos x\,dx=dt$

$\Rightarrow \quad I_1=\int\frac{1}{t^2}\,dt \quad \Rightarrow \quad I_1=\frac{-1}{t}$

(Ignoring arbitrary constant as per the question)

$\Rightarrow \quad I_1=\frac{-1}{x\cos x+\sin x}$

Putting value of I_1 in (1), we get

$$I=\frac{-x\sec x}{x\sin x+\cos x}+\int\frac{\sec x\,(1+\tan x)}{x\sin x+\cos x}\,dx$$

which is the option d.

Simplifying this further by taking $\cos x$ common in denominator in the second term we get

$$I=\frac{-x\sec x}{x\sin x+\cos x}+\int\frac{\sec x\,(1+x\tan x)}{\cos x\,(1+x\tan x)}\,dx$$

340. (b,c,d) First observe that g is the inverse of p.

Hence $p(a)=0$, $p(b)=1$ Also f' is a continuous function which maps from $[0,1]\to[0,1]$

by Intermediate value theorem we can say that there is $k\in[0,1]$ such that $f'(k)=k\Rightarrow$ (d)

Now, $\quad f'(x)\le 1\le(p(b)-p(a))$

Hence, $\quad \int_0^1 f'(x)\,dx\le(p(b)-p(a))$

Divide both sides by $b-a=\int_0^1 g'(x)\,dx$

$$\frac{\int_0^1 f'(x)\,dx}{\int_0^1 g'(x)\,dx}\le\frac{(p(b)-p(a))}{b-a}=p'(c),\ c\in(a,b) \qquad \text{[By LMVT]}\Rightarrow\text{(c)}$$

Also, use LMVT for $y=f(x)$ in [0, 1] ⇒ $f'(c)=\dfrac{f(1)-f(0)}{1-0}\le 1$

$f(1)-f(0)\le 1$

$f(1)\le 1+f(0) \Rightarrow$ (b)

341. (a,b,c) $I_n = n\pi \; \forall \; n=0, 1, 2, 3, \ldots\ldots$

Now, proceed.

342. (a,c) Given, $P(a)=0$

$Q(b)=0$

Consider, $R(x)=P(x)-Q(x)$

$R(a)=-Q(a)$

$R(b)=P(b)$

$R(a)R(b)=-Q(a)P(b)<0$

∴ using I.V.T.

$R(c)=0$ for some c

⇒ $P(c)-Q(c)=0$ for some c

|||ly consider $R(x)=P(x)-2Q(x)$

$R(a)=-2Q(a)$; $R(b)=P(b)$

∴ $R(a)R(b)=-2Q(a)\,P(b)<0$

∴ $R(c)=0$ for some c (using I.V.T.)

⇒ $P(c)-2Q(c)=0$ for some c ⇒ (c)

343. (b,c,d) $$T=3\sum_{n=1}^{\infty}\left(\underbrace{\frac{1}{\pi}\sum_{k=1}^{\infty}\cot^{-1}\left(1+2\sqrt{\sum_{r=1}^{k} r^3}\right)}_{J}\right)^n$$

(1) $\displaystyle\sum_{r=1}^{k} r^3=\left(\frac{k(k+1)}{2}\right)^2$

(2) $\cot^{-1}x=\tan^{-1}\left(\dfrac{1}{x}\right)$

$$J=\frac{1}{\pi}\left(\sum_{k-1}^{\infty}\tan^{-1}\left(\frac{1}{1+k(k+1)}\right)\right)=\frac{1}{\pi}\left(\sum_{k-1}^{\infty}\tan^{-1}(k+1)-\tan^{-1}k\right)$$

$$\frac{1}{\pi}\left(\frac{\pi}{2}-\frac{\pi}{2}\right)=\frac{1}{4}$$

∴ $$T=3\sum_{n=1}^{\infty}\left(\frac{1}{4}\right)^n$$

Infinite G.P. $=3\left(\dfrac{1/4}{1-(1/4)}\right)=1$

344. (b,c,d) $F(x)$ is an increasing function and $G(x)$ is a decreasing function

(a) $F(F(x)-G(x)) = \underbrace{F'(f(x)-G(x))}_{+\text{ve}}\cdot\left(\underbrace{F'(x)}_{+\text{ve}}-\underbrace{G'(x)}_{-\text{ve}}\right) > 0$

(b) $G(F(x)-G(x)) = \underbrace{G'(F(x)-G(x))}_{-\text{ve}}\cdot\left(\underbrace{F'(x)}_{+\text{ve}}-\underbrace{G'(x)}_{-\text{ve}}\right) < 0$

(c) $G'(x)-F'(x)<0$

(d) $G'(F(x))\cdot F'(x)<0$

345. (a,c,d) Do yourself.

346. (a,b,c) $(3\sec\theta+5\operatorname{cosec}\theta)x+(7\sec\theta-3\operatorname{cosec}\theta)y+11(\sec\theta-\operatorname{cosec}\theta)-0$

$\sec\theta(3x+7y+11)+\operatorname{cosec}\theta(5x-3y-11)=0$

Hence, family of lines are concurrent at the point of intersection of

$$3x+7y+11=0 \qquad \text{and} \qquad 5x-3y-11=0$$

Hence point B is $(1, -2)$.

Now proceed.

347. (a,c) $$\sum_{m=1}^{6}\csc\left(\alpha+(m-1)\frac{\pi}{4}\right)\csc\left(\alpha+\frac{m\pi}{4}\right)=4\sqrt{2}$$

On solving, we get $\sin 2\alpha=\dfrac{1}{2}$

Hence, $2\alpha=\dfrac{\pi}{6}$ or $\dfrac{5\pi}{6}$ $\Rightarrow$ $\alpha=\dfrac{\pi}{12}$ or $\dfrac{5\pi}{12}$

348. (a,b,d) Given that both the matrices

$A-\dfrac{I}{2}$ and $A+\dfrac{I}{2}$ are orthogonal that means

$$\left(A-\frac{I}{2}\right)\left(A'-\frac{I}{2}\right)=I \qquad (\text{as } I'-I)$$

$$AA'-\frac{AI}{2}-\frac{A'I}{2}+\frac{I}{4}=I \qquad \ldots(1) \qquad (\text{as } I^2=I)$$

Also, $$\left(A+\frac{I}{2}\right)\left(A+\frac{I}{2}\right)'=I$$

$$\left(A+\frac{I}{2}\right)\left(A'+\frac{I}{2}\right)=I$$

$$AA'+\frac{AI}{2}+\frac{A'I}{2}+\frac{I}{4}=I \qquad \ldots(2)$$

Subtracting eqn. (1) from eqn. (2), we get

$$AI+A'I=0 \quad\Rightarrow\quad A=-A'$$

$\Rightarrow$ Hence, A is an skew-symmetric matrix.

Now, for order of matrix add eqn. (1) and eqn. (2), we get

$$AA' = \frac{3I}{4}$$

Hence, $|A|^2 \neq 0$ have this so even order.

349. (a,b,c) Consider a differentiable function $f : R \to R$ such that $f(0) = 0$ and $f'(0) = 1$

(b) $f(x) = x - 2x^2 \sin\left(\frac{1}{x}\right)$, if x is distinct to 0 and $f(0) = 0$.

(d) $h(x) = \int_0^x f(t)\, dt$ fulfills $h'(x) = f(x)$.

$g(x) = \int_0^x h(t)\, dt \quad \Rightarrow \quad g'(x) = h(x)$

$\Rightarrow \quad g''(x) = h'(x) = f(x)$

350. (a,b,c,d) Note that C_k is binomial coefficient ${}^nC_k$.

$$\left(\sum_{i=0}^{n} C_i\right)^2 = \sum_{i=0}^{n} C_i^2 + 2\sum_{i=0}^{n}\sum_{j=i+1}^{n} C_i C_j$$

$$\Rightarrow \quad \sum_{i=0}^{n}\sum_{j=i+1}^{n} C_i C_j = \frac{1}{2}\left[\left(\sum_{i=0}^{n} C_i\right)^2 - \sum_{i=0}^{n} C_i^2\right]$$

$$= \frac{1}{2}\left[\left(\sum_{i=0}^{n} {}^nC_i\right)^2 - \sum_{i=0}^{n} ({}^nC_i)^2\right] = \frac{1}{2}[2^{2n} - {}^{2n}C_n] = 2^{2n-1} - \frac{(2n)!}{2(n!)^2}$$

Therefore, $a = 2n-1,\ b = 2n,\ c = 2,\ d = n$ all are true.

351. (a,b,d)

Firstly, note that to calculate the total number of matrices in the sample space, we may place the three 1's in any of the 9 entries of M and the remaining 6 entries would be all 0. Hence, total number of matrices M in the sample space is ${}^9C_3 = 84$

For M to be non-singular, all rows must be linearly independent so that M has full rank. Hence, each row must have exactly one 1 and no two 1's must be present on the same column. This can be done in 6 ways. Hence, probability is $\frac{6}{84} = \frac{1}{14}$

Prob $(M = I_3) = \frac{1}{84}$ because all 1's need to be present on the principal diagonal and hence there is only one such M.

For trace $(M) = 0$, 0's are present on the principal diagonal. Hence, 1's can be placed on any of the 6 remaining entries. Hence, probability is $\frac{5}{21}$.

352. (b,d) Shift LHS integral to RHS to get

$$\int_2^{150}(f^2(x)-(x-1)\ln(x-1)(2f(x)-(x-1)\ln(x-1)))\,dx=0$$

$$\Rightarrow \int_2^{150}(f^2(x)-2f(x)(x-1)\ln(x-1)+((x-1)\ln(x-1))^2)\,dx=0$$

$$\Rightarrow \int_2^{150}(f(x)-(x-1)\ln(x-1))^2\,dx=0$$

But, $(f(x)-(x-1)\ln(x-1))^2\geq 0$

So, $(f(x)-(x-1)\ln(x-1))^2=0$

$\Rightarrow$ $f(x)-(x-1)\ln(x-1)=0$

$\Rightarrow$ $f(x)=(x-1)\ln(x-1)$

(a) is incorrect because area is equal to 1/4

353. (c,d) Do yourself.

354. (b,c,d) $2\sin 2A+\sin(2B+C)=\sin C$

$2\sin 2A+\sin(2(\pi-A-C)+C)=\sin C$ Given that $\angle C=\frac{\pi}{3}$

$$2\sin 2A+\sin\left(\frac{5\pi}{3}-2A\right)=\frac{\sqrt{3}}{2}$$

$$2\sin 2A-\sin\left(2A+\frac{\pi}{3}\right)=\frac{\sqrt{3}}{2}$$

$$2\sin 2A-\frac{1}{2}\sin 2A-\frac{\sqrt{3}}{2}\cos 2A=\frac{\sqrt{3}}{2}$$

$$\frac{3}{2}\sin 2A-\frac{\sqrt{3}}{2}\cos 2A=\frac{\sqrt{3}}{2}$$

Divide by $\sqrt{3}$

$$\frac{\sqrt{3}}{2}\sin 2A-\frac{1}{2}\cos 2A=\frac{1}{2}$$

$$\sin\left(2A-\frac{\pi}{6}\right)=\frac{1}{2}$$

$$\Rightarrow\quad 2A-\frac{\pi}{6}=\frac{\pi}{6}\quad\Rightarrow\quad A=\frac{\pi}{6}$$

$$\Rightarrow\quad B=\frac{\pi}{2},\quad a=\frac{2}{\sqrt{3}}$$

Now, verify.

355. (a,b,d) $m_1^2=\dfrac{2b^2+2c^2-a^2}{4}$; $m_2^2=\dfrac{2c^2+2a^2-b^2}{4}$; $m_3^2=\dfrac{2a^2+2b^2-c^2}{4}$

$$\therefore \quad \begin{bmatrix} m_1^2 \\ m_2^2 \\ m_3^2 \end{bmatrix}=\frac{1}{4}\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix}\begin{bmatrix} a^2 \\ b^2 \\ c^2 \end{bmatrix}$$

$$\therefore \quad \begin{bmatrix} m_1^2 \\ m_2^2 \\ m_3^2 \end{bmatrix}=\frac{A}{4}\begin{bmatrix} a^2 \\ b^2 \\ c^2 \end{bmatrix} \Rightarrow 4A^{-1}\begin{bmatrix} m_1^2 \\ m_2^2 \\ m_3^2 \end{bmatrix}=\begin{bmatrix} a^2 \\ b^2 \\ c^2 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} a^2 \\ b^2 \\ c^2 \end{bmatrix}=\frac{4}{9}\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix}\begin{bmatrix} m_1^2 \\ m_2^2 \\ m_3^2 \end{bmatrix}$$

$$\therefore \quad M=\frac{4}{9}\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix}$$

Now, verify.

356. (a,b,c) $f'(x)=3ax^2+2bx+c \quad \Rightarrow \quad f'(0)=c=3$

$$f''(x)=6ax+2b \quad \Rightarrow f''\left(\frac{-2}{3}\right)=-4a+2b=0 \Rightarrow b=2a$$

$$f'\left(\frac{-2}{3}\right)=3a\times\frac{4}{9}+4a\cdot\left(\frac{-2}{3}\right)+3=\frac{5}{3}$$

$$\Rightarrow \quad \frac{4a}{3}=\frac{4}{3} \quad \Rightarrow \quad a=1,\ b=2$$

$$\therefore \quad f(x)=x^3+2x^2+3x+4$$

$$\frac{d}{dx}(g(x)\cdot f(g(x)))=\frac{d}{dx}(xg(x))=xg'(x)+g(x)$$

$$\because \quad f(0)=4 \quad \Rightarrow \quad g(4)=0$$

and $g'(f(x))=\dfrac{1}{f'(x)} \quad \Rightarrow \quad g'(4)=\dfrac{1}{f'(0)}=\dfrac{1}{3}$

$$\therefore \quad (g(x)\cdot f(g(x)))'|_{x=4}=\frac{4}{3}$$

357. (a,b,d)

$\because \quad f(-1-x)=f(-1+x)$

$\therefore \quad f(x)$ is symmetrical w.r.t. line $x=-1$

$\therefore \quad f(x)=a(x+1)^2+b$

$f'(-1)=0$

$\therefore \quad f(1)=5$ and $f(-1)=1$

$\therefore \quad b=1, \quad a=1$

$\therefore \quad f(x)=(x+1)^2+1=x^2+2x+2$

358. (a,b,c) Differentiating w.r.t. x, we get

$$(x^2-1)e^x(x^2+4x+1)=e^x(f(x)+f'(x))$$

$$\Rightarrow \quad g(x)=f(x)+f'(x)=(x^2-1)(x^2+4x+1)$$

$$g(x)=0 \quad \Rightarrow \quad x=\pm 1 \quad \text{or} \quad x^2+4x+1=0$$

$$\therefore \quad \alpha^2+4\alpha+1=0$$

$\therefore$ Given expression $=(\alpha^2+4\alpha+1)(\alpha^2+1)+1=1$

359. (a,b,c,d) Check at $x=k \quad k\in I$

(a) $\left.\begin{array}{l} f(k^+)-k \\ f(k^-)=k-1+1=k \end{array}\right]$ continuous

(b) $f(k^+)=k^2+0+2k\cdot 0=k^2$

$f(k^-)=(k-1)^2+1+2+2(k-1)(1)=k^2$

(c) for $x>1$ $\quad \{x\}$ is less than e^x and both function are positive.

$\therefore \quad \forall\, x>1 \qquad 0<\frac{\{x\}}{e^x}<1$

$\therefore \quad [\;]=0 \qquad \rightarrow \qquad$ continuous

(d) $f(k^+)-0$

$f(k^-)=[\;]\times 0=0 \qquad \Rightarrow \qquad$ continuous

360. (a,b,c,d) Using, A.M. $\geq$ G.M.

$$\frac{\sum a_i}{16}\geq (a_1a_2\ldots\ldots a_{16})^{1/16}$$

$$\therefore \quad a_1a_2\ldots\ldots a_{16}\leq\left(\frac{a_1+a_2+\ldots\ldots a_{16}}{16}\right)^{16}\leq\left(\frac{392}{16}\right)^{16}\leq\left(\frac{49}{2}\right)^{.6}$$

$\Rightarrow \quad S=49$ and $W=2$

$$\frac{6}{2}(2a+5\cdot 3d)=147$$

$$2a+15d=49 \qquad \ldots(1)$$

$$2a+15d=M=49$$

Also, $\quad \frac{4}{2}(2a+3\cdot 5d)=N \quad ; \quad (2a+15d)2=N \quad \Rightarrow \quad N=98$

361. (b,c,d) $\quad \sum_{k-1}^{\infty}\left(\sin^{-1}\left(\frac{1}{\sqrt{k}}\right)-\sin^{-1}\left(\frac{1}{\sqrt{k+1}}\right)\right)=\theta \quad ; \quad \theta=\frac{\pi}{2}$

362. (a,d) Do yourself.

363. (a,b) (c) = 2; (d) = −2

364. (a,b,c) $a^2+ab+10=0$

$b^2+ab+10=0$

$\therefore \quad (a^2-b^2)=0$

$(a-b)(a+b)=0$

$a\neq b \quad \therefore \quad a+b=0$

But, $a(a+b)=-10$

$\therefore$ no such a and b exists. $\Rightarrow$ (d)

365. (a,b) Do yourself.

366. (a,b,d) Do yourself.

367. (a,b,d) Do yourself.

368. (b,c,d) Do yourself.

369. (b,d) $f(x)=(x-2)^2+6$

370. (a,d) $f(x)=3(\tan^{-1}\sqrt{x-2})^2+\sec^{-1}\sqrt{x}-\dfrac{\pi}{2}$

$\therefore \quad \text{Range}=\left[\dfrac{-\pi}{4},\dfrac{3\pi^2}{4}\right)$

$$\lim_{x\to 2^+}\frac{3(\tan^{-1}\sqrt{x-2})^2-\operatorname{cosec}^{-1}\sqrt{x}+\dfrac{\pi}{4}}{\sin(x-2)}=\lim_{t\to 0^+}\frac{3(\tan^{-1}\sqrt{t})^2-\operatorname{cosec}^{-1}\sqrt{t+2}-\dfrac{\pi}{4}}{t}$$

$$=\lim_{t\to 0^+}\left(\frac{3(\tan^{-1}\sqrt{t})^2}{(\sqrt{t})^2}-\frac{\cot^{-1}(\sqrt{t+1})-\tan^{-1}(1)}{t}\right)=\frac{1}{4}$$

371. (a,b,c) Put $\alpha=\beta=0$ we get $\lambda=2$

372. (a,d) $P(x)=x^3+ax^2+bx$

Note that $x=0$ is one of the root.

Therefore 3 roots in A.P. can be taken as $0, d, 2d$ (where $d>0$)

Now, sum of the root $=3d=-a$ ...(1)

sum taken 2 at a time $=2d^2=b$ ...(2)

Also given $1+a+b=10$ ...(3)

From eqns. (1), (2) and (3), we get

$$a+b=9$$

Hence, $2d^2-3d=9 \quad \Rightarrow \quad 2d^2-3d-9=0$

$\Rightarrow \quad 2d^2-6d+3d-9=0 \quad \Rightarrow \quad (d-3)(2d+3)=0$

$\therefore \quad d=3$

Hence, roots are 0, 3, 6.

Hence, $a=-9$ and $b=18$

$b-a=27$

Sum of the roots of $P(x)=-a=9$

373. (a,b) Range : $(-\infty, \infty)$

374. (a,b) $\vec{\mathbf{A}} \times \vec{\mathbf{B}} = -13\hat{\mathbf{i}} - 9\hat{\mathbf{j}} + 7\hat{\mathbf{k}}$

Take cross with $\vec{\mathbf{A}}$

$\therefore$ $\vec{\mathbf{A}} \times (\vec{\mathbf{A}} \times \vec{\mathbf{B}}) = \vec{\mathbf{A}} \times (-13\hat{\mathbf{i}} - 9\hat{\mathbf{j}} + 7\hat{\mathbf{k}})$

$$(\vec{\mathbf{A}} \cdot \vec{\mathbf{B}})\vec{\mathbf{A}} - |\vec{\mathbf{A}}|^2 \vec{\mathbf{B}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & 1 & 5 \\ -13 & -9 & 7 \end{vmatrix} = 52\hat{\mathbf{i}} - 79\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$$

$$11\vec{\mathbf{A}} - 30\vec{\mathbf{B}} = 52\hat{\mathbf{i}} - 79\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$$

$$(22 - 30x)\hat{\mathbf{i}} + (11 - 30y)\hat{\mathbf{j}} + (55 - 30z)\hat{\mathbf{k}} = 52\hat{\mathbf{i}} - 79\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$$

Now, comparing coefficients of $\hat{\mathbf{i}}, \hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$, we get

$$22 - 30x = 52 \quad \Rightarrow \quad x = -1$$
$$11 - 30y = -79 \quad \Rightarrow \quad y = 3$$
$$55 - 30z = -5 \quad \Rightarrow \quad z = 2$$

Now, verify.

375. (a,c,d) Let, $f(x) = x^3 + ax^2 + bx + c$, then

$$f(x) = x^3 + ax^2 + bx + c$$
$$f'(x) = 3x^2 + 2ax + b$$
$$f''(x) = 6x + 2$$
$$f'''(x) = 6$$

Equating the coefficient of $f(x) + f'(x) + f''(x) + f'''(x) = x^3$

We have $3 + a = 0 \quad \Rightarrow \quad a = -3$

$b + 2a + 6 = 0 \quad \Rightarrow \quad b = 0$

and $c + b + 2a + 6 = 0 \quad \Rightarrow \quad c = 0$

Therefore, $f(x) = x^3 - 3x^2$ and

$$g(x) = \int \frac{f(x)}{x^3}\, dx = \int \frac{x^3 - 3x^2}{x^3}\, dx = \int \left(1 - \frac{3}{x}\right) dx = x - 3\ln x + C,$$

Where, C is the constant of integration

$$g(1) = 1 - 3\ln 1 + C = 1 \quad \Rightarrow \quad C = 0$$

$\Rightarrow$ $g(x) = x - 3\ln x$

376. (a,c) Do yourself

377. (a,b,d) $\lim\limits_{x \to 0}\left(\dfrac{1}{x^2}\displaystyle\int_0^x \dfrac{t + t^2}{1 + \sin t}\right) = \dfrac{1}{2}$

378. (a,b,d)

Consider the function $h(x) = f(x) - k\,g(x)$ where $k \in \{1, 2, 3, 4\}$ on the interval $[0, 1]$

Using LMVT, we get

$$h'(c) = \frac{h(1) - h(0)}{1 - 0} = f'(c) - kg'(c)$$

$$(f(1) - k\,g(1)) - (f(0) - k\,g(0)) = f'(c) - k\,g'(c)$$

$$(6 - 2k) - (2 - 0) = f'(c) - k\,g'(c)$$

$$4 - 2k = f'(c) - k\,g'(c)$$

Now, if $k = 1$, we get

$$f'(c) - g'(c) = 2 = f(0) \quad \Rightarrow \quad \text{(a) is true.}$$

if $k = 2$

$$f'(c) - 2g'(c) = 0 = g(0) \quad \Rightarrow \quad \text{(b) is true.}$$

if $k = 3$

$$f'(c) - 3g'(c) = -2 = -g(1) \quad \Rightarrow \quad \text{(c) is false.}$$

if $k = 4$

$$f'(c) - 4g'(c) = -4 = -2g(1) \quad \Rightarrow \quad \text{(d) is true.}$$

379. (b,c,d) L.H.L. $= \lim\limits_{x \to 0^-} \dfrac{\sin[a(x+1)] + \sin x}{2x}$

for limit exist $a \in [0, 1)$

$$= \lim_{x \to 0^-} \frac{\sin x + 0}{2x} = \frac{1}{2}$$

$$\text{R.H.L.} = \lim_{x \to 0^+} \frac{(1+bx)^{1/2} - 1}{bx} = \lim_{x \to 0^+} \frac{\left(1 + \frac{bx}{2} + \ldots\right) - 1}{bx} = \frac{1}{2} \;\forall\; b \in R - \{0\}$$

$c = \dfrac{1}{2}$; $a \in [0, 1)$

So, $b \in R - \{0\}$

$$f(1) = \frac{\sqrt{b+1} - 1}{b} = \frac{1}{\sqrt{b+1} + 1} = \frac{1}{3}$$

$\therefore \quad b = 3$

380. (a,c,d) Do yourself

381. (a,b,c) $S_n = \sum\limits_{r=1}^{n} \tan^{-1}\left(\dfrac{1}{1 + (r+2)(r+1)}\right)$; $S_n = \sum\limits_{r=1}^{n} \tan^{-1}(r+2) - \tan^{-1}(r+1)$

$S_n = \tan^{-1}(n+2) - \tan^{-1}(2)$ now verify.

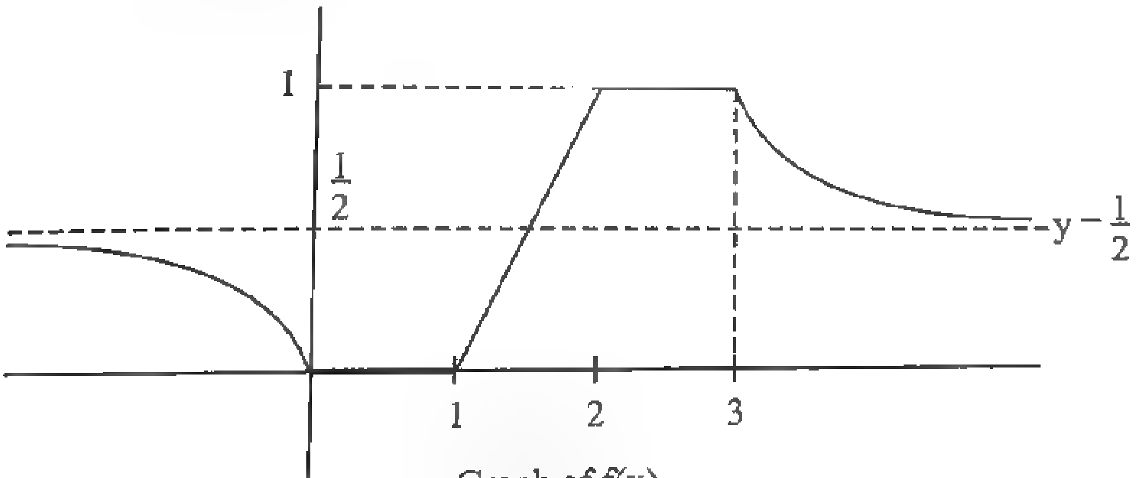
382. (a,b,d) $d(x, [a, b]) = \begin{cases} |x - a| & x < a \\ 0 & a \le x \le b \\ |x - b| & b < x \end{cases}$

$$\therefore \quad d(x,[0,1]) = \begin{cases} |x|, & x \le 0 \\ 0, & x \in [0,1] \\ |x-1|, & x \in [1,2] \\ |x-1|, & x \in [2,3] \\ |x-1|, & x \ge 3 \end{cases}$$

$$\Rightarrow \quad d(x,[2,3]) = \begin{cases} |x-2|, & x \le 0 \\ |x-2|, & x \in [0,1] \\ |x-2|, & x \in [1,2] \\ 0, & x \in [2,3] \\ |x-3|, & x \ge 3 \end{cases}$$

$$\Rightarrow \quad f(x) = \begin{cases} \dfrac{|x|}{|x|+|x-2|} & x \le 0 \\ 0 & 0 \le x \le 1 \\ \dfrac{|x-1|}{|x-1|+|x-2|} & 1 \le x \le 2 \\ \dfrac{|x-1|}{|x-1|+0} = 1 & 2 \le x \le 3 \\ \dfrac{|x-1|}{|x-1|+|x-3|} & x \ge 3 \end{cases} = \begin{cases} \dfrac{x}{x+x-2} = \dfrac{x}{2(x-1)}, & x \le 0 \\ 0, & x \in [0,1] \\ x-1|, & x \in [1,2] \\ 1, & x \in [2,3] \\ \dfrac{x-1}{2(x-2)}, & x \ge 3 \end{cases}$$

$$\Rightarrow \quad 0 \le f(x) \le 1$$



Graph of f(x)

383. (a,b,c) $\quad xf(y) < yf(x) \; \forall \; 0 < x < y < 1$

$$\Rightarrow \quad \frac{f(y)}{y} < \frac{f(x)}{x} \; \forall \; x < y$$

$$\Rightarrow \quad g(x) = \frac{f(x)}{x} \text{ is decreasing}$$

$$\Rightarrow \quad g'(x) < 0 \qquad \Rightarrow \qquad xf'(x) < f(x)$$

$$\Rightarrow \quad f'(x) < \frac{f(x)}{x} \; \forall \; x \; 0 < x < 1$$

Hence, $\qquad f'(x) < g(x) \; \forall \; x \; 0 < x < 1$

Hence, $f'(x)$ is less than minimum value of $g(x)$ which is $g(1)$.

$\therefore \quad f'(x) < f(1)$ ...(1)

$xf'(x) < f(x) \quad \Rightarrow \quad \int_0^1 xf'(x)dx < \int_0^1 f(x)\,dx$

$\Rightarrow \quad xf(x)|_0^1 - \int_0^1 f(x)\,dx < \int_0^1 f(x)\,dx$

$\Rightarrow \quad f(1) < 2\int_0^1 f(x)\,dx$ ...(2)

384. (c,d) Let A is $(1, 0)$, B is $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ and P is (x, y)

$\Rightarrow \quad \overrightarrow{\mathbf{OP}} = (2-t)\overrightarrow{\mathbf{OA}} + t\,\overrightarrow{\mathbf{OB}}, t \in R$

$\Rightarrow \quad x\hat{\mathbf{i}} + y\hat{\mathbf{j}} = (2-t)\hat{\mathbf{i}} + t\left(\frac{\hat{\mathbf{i}}}{2} + \frac{\sqrt{3}\,\hat{\mathbf{j}}}{2}\right)$

$\therefore \quad x = 2 - \frac{t}{2}; \quad y = \frac{\sqrt{3}t}{2}$

$|\overrightarrow{\mathbf{AP}}| = |(x-1)\hat{\mathbf{i}} + y\hat{\mathbf{j}}| = \sqrt{(x-1)^2 + y^2} = \sqrt{t^2 - t + 1} \geq \frac{\sqrt{3}}{2}$

385. (a,b) Two possible A.P. 1/2, 1/2, 3/2, ...

−3/2, 1/2, 5/2, ...

386. (b,c) $A(t) = \frac{4}{3}\sin t - \cos t - \frac{1}{3}$

387. (a,b,c) $f(x) = 0$, now verify.

388. (a,b,d) $f(x) = \frac{e^x}{x^2}$

389. (a,c) Equation of circle is $(x-4)^2 + y^2 = 8$

390. (a,b,d) Do yourself.

391. (a,b,c) $\frac{1-\cos x}{\cos x} = (\sqrt{2}-1)\frac{\sin x}{\cos x} \quad \Rightarrow \quad \frac{1-\cos x}{\sin x} = (\sqrt{2}-1)$

$$\frac{2\sin^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}} = \sqrt{2}-1$$

either $\sin\frac{x}{2} = 0$ *i.e.,* $x = 2n\pi$

$\therefore \quad x = 0, 2\pi$

or $\qquad \tan\frac{x}{2} = \sqrt{2}-1 \qquad \Rightarrow \qquad \frac{x}{2} = n\pi + \frac{\pi}{8}$

$\therefore \qquad x = 2n\pi + \frac{\pi}{4}$

$\therefore \quad$ only $x = \frac{\pi}{4}$

Hence, $d = 3$ *i.e.*, $0, 2\pi, \pi/4$

Now, $d = 3$ lies between the roots of the equation $x^2 + (k-1)x + k^2 + k - 11 = 0$

$$\therefore \qquad f(3) < 0$$
$$9 + 3k - 3 + k^2 + k - 11 < 0$$
$$k^2 + 4k - 5 < 0$$
$$(k+5)(k-1) < 0$$
$$k \in (-5, 1) \qquad \Rightarrow \qquad \text{a, b, c}$$

392. (a,b,d) $\quad a = 52, b = 51, c = 1$

P (2 aces are drawn in exactly n draws) = P (exactly 1 ace in $n-1$ draws)

P (second ace in nth draw)

$$= \frac{{}^{48}C_{n-2} \cdot {}^{4}C_1}{{}^{52}C_{n-1}} \times \frac{{}^{3}C_1}{53-n}$$

$$= \frac{48! \cdot (n-1)!\,(53-n)! \cdot 4}{(n-2)!\,(50-n)! \cdot 52!} \times \frac{3}{53-n}$$

$$= \frac{(n-1)(53-n)(52-n)(51-n) \cdot 12}{52 \cdot 51 \cdot 50 \cdot 49} \times \frac{1}{53-n}$$

$$= \frac{(n-52)(n-51)(n-1)}{13 \cdot 17 \cdot 50 \cdot 49} \equiv \frac{1}{k}(n-a)(n-b)(n-c)$$

$a = 52, b = 51, c = 1$ and $k = 13 \cdot 17 \cdot 50 \cdot 49$

$\Rightarrow \quad$ (a), (b), (d)

393. (a,b) $\quad f(x) = 0 \;\forall\; x \geq 1$

(a) is true because domain of $f(x)$ is $x \geq 1$.

(c) is false because domain of $f(x)$ is $x \geq 1$.

394. (a,d) $\quad f'(x) = \dfrac{2e^{-1/x^2}}{x^2} < \dfrac{2e^{-1/x^2}}{x^3} \;\forall\; x \in (0, 1/\sqrt{2})$

$$f(x) < e^{-1/x^2}$$

Let $\quad g(x) = e^{-1/x^2}$

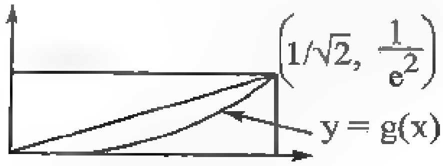
$$\int_0^{1/\sqrt{2}} f(x)\,dx < \int_0^{1/\sqrt{2}} g(x)\,dx$$

Now, $g'(x)=\dfrac{2e^{-1/x^2}}{x^3}$

$$g''(x)=e^{-1/x^2}\left(\frac{4}{x^6}-\frac{6}{x^4}\right)>0$$

$\Rightarrow$ $\dfrac{-\sqrt{2}}{\sqrt{3}}<x<\dfrac{\sqrt{2}}{\sqrt{3}}$

$\therefore$ $y=g(x)$ is concave up $\forall\, x\in(0,1/\sqrt{2})$



$\Rightarrow$ $\displaystyle\int_0^{1/\sqrt{2}} g(x)dx<\frac{1}{2\sqrt{2e^2}}$ $\Rightarrow$ $\displaystyle\int_0^{1/\sqrt{2}} f(x)dx<\frac{1}{2\sqrt{2e^2}}$ $\Rightarrow$ (a)

For point of inflection $f'(x)=\dfrac{2e^{-1/x^2}}{x^2}$

Now, $f''(x)=0$ gives quadratic in x, hence 2 point of inflection. $\Rightarrow$ (d)

395. (a,b,c) $x^2=2y$ and $\left(y+\dfrac{1}{2}\right)^2-4px$

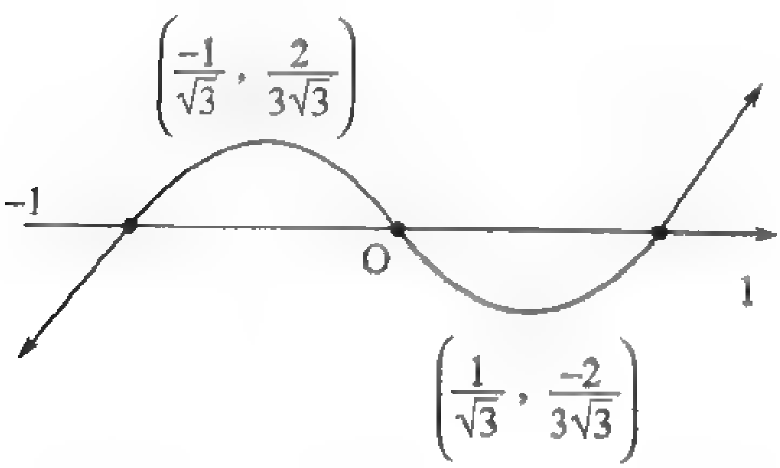
Suppose the common tangent be $y+\dfrac{1}{2}=mx+\dfrac{p}{m}$,

then, $x^2=2mx+\dfrac{2p}{m}-1$ has equal roots

$$x^2-2mx-\left(\frac{2p}{m}-1\right)=0 \quad\Rightarrow\quad D=0 \quad\Rightarrow\quad m^2+\frac{2p}{m}-1=0$$

$\Rightarrow$ $m^3-m=-2p$

Let $f(m)=m^3-m$ $\Rightarrow$ $f'(m)=0$ if $m=\dfrac{1}{\sqrt{3}},\dfrac{-1}{\sqrt{3}}$



; $f(p)=\begin{cases}1 & p<\dfrac{-1}{3\sqrt{3}}\\ 2 & p=\dfrac{-1}{3\sqrt{3}}\\ 3 & \dfrac{-1}{3\sqrt{3}}<p<\dfrac{1}{3\sqrt{3}}\\ 2 & p=\dfrac{-1}{3\sqrt{3}}\\ 1 & \dfrac{1}{3\sqrt{3}}<p\end{cases}$

196. (a,b,c)' $P(A\cap B)=P(A)P(B)$

$P(A\cap(B\cap C))=P(A)P(B\cap C)$

$P(A\cap(B\cup C))=P(A)P(B\cup C)$

$\Rightarrow P(A\cap B)+P(A\cap C)-P(A\cap B\cap C)=P(A)[P(B)+P(C)-P(B\cap C)]$

$\Rightarrow P(A)P(B)+P(A\cap C)-P(A)P(B\cap C)=P(A)P(B)+P(A)P(C)-P(A)P(B\cap C)$

$\Rightarrow P(A\cap C)=P(A)P(C) \quad \Rightarrow$ A and C are independent events.

$\Rightarrow P\left(\frac{A}{C}\right)=P(A)=\frac{1}{2}$

$$P\left(\frac{\overline{B}\cup\overline{C}}{A}\right)=P\left(\frac{\overline{B\cap C}}{A}\right)=P(\overline{B\cap C})=1-P(B)P(C)=1-\frac{1}{3}\cdot\frac{1}{4}=\frac{11}{12}$$

$$P\left(\frac{A}{\overline{B\cap C}}\right)=P(A)=\frac{1}{2}$$

197. (a,c,d) $S_n=\sum_{k=1}^{n}\tan^{-1}\frac{1}{k(k+1)+1}$

$$S_n=\sum_{k=1}^{n}\tan^{-1}\frac{\frac{1}{k(k+1)}}{1+\frac{1}{k(k+1)}}$$

$$S_n=\sum_{k=1}^{n}\tan^{-1}\frac{\frac{1}{k}-\frac{1}{k+1}}{1+\frac{1}{k(k+1)}}$$

$$S_n=\sum_{k=1}^{n}\left(\tan^{-1}\frac{1}{k}-\tan^{-1}\frac{1}{k+1}\right)$$

$$S_n=\tan^{-1}\frac{1}{1}-\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{2}-\tan^{-1}\frac{1}{3}+\ldots\ldots+\tan^{-1}\frac{1}{n-1}-\tan^{-1}\frac{1}{n}+\tan^{-1}\frac{1}{n}-\tan^{-1}\frac{1}{n+1}$$

$$S_n=\tan^{-1}1-\tan^{-1}\frac{1}{n+1}$$

$$\therefore\quad 5+\sum_{n=1}^{62}\frac{1+\tan S_n}{1-\tan S_n}=5+\sum_{n=1}^{62}\tan\left(\frac{\pi}{4}+S_n\right)=5+\sum_{n=1}^{62}\tan\left(\frac{\pi}{4}+\frac{\pi}{4}-\tan^{-1}\frac{1}{n+1}\right)$$

$$=5+\sum_{n=1}^{62}\tan\left(\frac{\pi}{2}-\tan^{-1}\frac{1}{n+1}\right)=5+\sum_{n=1}^{62}\cot\left(\tan^{-1}\frac{1}{n+1}\right)$$

$$=5+\sum_{n=1}^{62}\frac{1}{\tan\left(\tan^{-1}\frac{1}{n+1}\right)}=5+\sum_{n=1}^{62}\frac{1}{\frac{1}{n+1}}=5+\sum_{n=1}^{62}(n+1)$$

$$=4+(1+2+3+\ldots\ldots+63)=4+\frac{63\cdot 64}{2}=4+2016=2020$$

398. (a,b,d) Othrocentre is $(\alpha+4, \beta-3)$

Use O, G, C collinear and G divide O and C in 2 : 1, to get p and q

$$p=\frac{-1}{2};\quad q=\frac{-5}{2} \quad \Rightarrow \quad \text{(b) and (d)}$$

(a) $m_1 m_2 = -1$

$$\frac{\beta-2}{\alpha-1}\left(\frac{\beta+7}{\alpha+2}\right)=1$$

$$(\beta-2)(\beta+7)=(\alpha-1)(\alpha+2)$$

$$\beta^2+5\beta-14=\alpha^2+\alpha-2$$

$$\beta^2-\alpha^2+5\beta-\alpha=12$$

399. (b,c,d) Let $\vec{v}_1=\hat{i}+\hat{j}+\hat{k}$ and $\vec{v}_2=f(x)\hat{i}+g(x)\hat{j}+h(x)\hat{k}$

Now, $\vec{v}_1\cdot\vec{v}_2=2=|\vec{v}_1|\,|\vec{v}_2|\cos\theta=\sqrt{3}\sqrt{f^2(x)+g^2(x)+h^2(x)}\cos\theta$

Hence, $\frac{4}{3}\sec^2\theta=f^2(x)+g^2(x)+h^2(x)\geq\frac{4}{3}$

Hence, $I_{\min}=\int_0^{3/4}(f^2(x)+g^2(x)+h^2(x))\,dx=\int_0^{3/4}\frac{4}{3}\,dx=1$

400. (a,b) $8x^3+(\lambda+2)x^2-(2k+\lambda)x-27=0 \; \begin{cases} a \\ b \\ c \end{cases}$ ...(1)

$$\lambda^2+2\lambda(k+1)+4k=2^3\cdot3^5$$

$$\Rightarrow \quad (\lambda+2)(\lambda+2k)=2^3\cdot3^5 \quad \text{...(2)}$$

$$a+b+c=\frac{-(\lambda+2)}{8}$$

$$ab+bc+ca=\frac{-(2k+\lambda)}{8}$$

$$abc=\frac{27}{8}$$

$$\therefore \quad (a+b+c)(ab+bc+ca)=\left(\frac{-(\lambda+2)}{8}\right)\left(\frac{-(2k+\lambda)}{8}\right)=\frac{2^3\cdot3^5}{8\times8}=\frac{3^5}{8}$$

$$(abc)(a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)=\frac{3^5}{8}$$

$$\frac{27}{8}(a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)=\frac{3^5}{8}$$

Hence, $(a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)=9$

Now, using A.M. ≥ H.M., in a, b, c we get

$$(a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\geq\frac{1}{9}$$

Hence, $a=b=c=\frac{3}{2}$

∴ Triangle is equilateral.

Now, verify options.

401. (a,b,c) Let a and b be the number of shots A, B respectively takes until they hit the target.

$$S=\sum_{n-1}^{\infty}(P(a=n))\times(P(b>n))$$

Now, for $P(a=n)$, A must miss their first $n-1$ shots and hit on the nth shot.

Thus, probability of $A=n=\left(1-\frac{3}{5}\right)^{n-1}\times\frac{3}{5}=\frac{3}{2}\times\left(\frac{2}{5}\right)^{n}$

Next we have $P(b>n)-\sum_{k=n+1}^{\infty}P(b=k)=\sum_{k-n+1}^{\infty}\left(\frac{2}{7}\right)^{k-1}\times\frac{5}{7}=\left(\frac{2}{7}\right)^{n}$

Thus, $S=\frac{3}{2}\sum_{n=1}^{\infty}\left(\frac{2}{5}\times\frac{2}{7}\right)^{n}=\frac{3}{2}\times\frac{\frac{4}{35}}{1-\frac{4}{35}}=\frac{6}{31}$ → (a)

Now, probability that A, B require the same number of shots will be

$$\sum_{n=1}^{\infty}(P(A=n))\times(P(B=n))=\sum_{n-1}^{\infty}\left(\left(\frac{4}{35}\right)^{n-1}\times\frac{3}{5}\times\frac{5}{7}\right)-\frac{3}{7}\times\frac{1}{1-\frac{4}{35}}=\frac{15}{31}\quad\Rightarrow\quad(c)$$

Now, probability that B require less shots than A will be

$$1-\frac{16}{31}-\frac{15}{31}=\frac{10}{31}\qquad\Rightarrow\qquad(b)$$

Hence (d) is false.

402. (a,b,c,d)

(a) $\log_5(\sqrt{7\sqrt{7\sqrt{7\ldots\ldots}}})=\log_5 7>1$

(b) $\frac{1}{\sqrt{3}+\sqrt{2}}<\frac{1}{\sqrt{7}+\sqrt{6}}\quad\Rightarrow\quad\sqrt{3}-\sqrt{2}>\sqrt{7}-\sqrt{6}$

(c) $\log_3 10>2$ and $\log_{10}70<2$

(d) $\log_3(3+\sqrt{2})>1$ and $\log_2(2-\sqrt{2})<1$

403. (b,c,d)

(a) $7^{-\log_7 6}+81^{(1-\log_9 2)}=7^{-\log_7 6}+81\cdot 81^{-\log_9 2}=\frac{1}{6}+\frac{81}{4}$

(b) $(1-\log_6 2)(1+\log_6 2)+(\log_6 2)^2=1-(\log_6 2)^2+(\log_6 2)^2=1$

(c) $\log_3 5+\log_3 6-\log_3 10=\log_3 3=1$

(d) $\left(2^{\frac{1}{3}}+5^{\frac{1}{3}}\right)\left(2^{\frac{2}{3}}-2^{\frac{1}{3}}\cdot 5^{\frac{1}{3}}+5^{\frac{2}{3}}\right)=2+5=7$

404. (b,c,d) $(\log_2 x-4)\cdot\log_2 x=5$

Let $\log_2 x=a$

$\Rightarrow \quad a^2-4a-5=0 \quad\Rightarrow\quad (a-5)(a+1)=0$

$\Rightarrow \quad a=5$ or $a=-1 \quad\Rightarrow\quad x=32$ or $x=\frac{1}{2}$

405. (a,b) $y=\dfrac{\cos x-\cos 3x+\cos 3x-\cos 9x+\cos 9x-\cos 17x}{\sin 3x-\sin x+\sin 9x-\sin 3x+\sin 17x-\sin 9x}$

$$y=\frac{\cos x-\cos 17x}{\sin 17x-\sin x}=\frac{2\sin 9x\cdot\sin 8x}{2\sin 8x\cdot\cos 9x}=\tan 9x$$

$\therefore$ (a) and (b) are correct.

406. (b,c,d) $\log_3(2^x+1)=t \quad\Rightarrow\quad (2+t)\cdot t=3$

$\Rightarrow \quad t^2+2t-3=0 \quad\Rightarrow\quad t=-3,1$

$\therefore \quad \log_3(2^x+1)=-3,1 \quad\Rightarrow\quad 2^x+1=\frac{1}{27},3$

$\therefore \quad 2^x=\frac{-26}{27},2$

$\therefore \quad 2^x=2 \quad\Rightarrow\quad x=1$

407. (a,d) Let $\log_3 x=A$ and $\log_3 y=B$

$\therefore \quad \frac{A}{2}+\frac{B}{3}=\frac{7}{2} \quad\Rightarrow\quad 3A+2B=21$

and $\quad \frac{A}{3}+\frac{B}{2}=\frac{2}{3} \quad\Rightarrow\quad 2A+3B=4$

$\therefore \quad A=\log_3 x=11 \quad\Rightarrow\quad x=3^{11}$

and $\quad B=\log_3 y=-6 \quad\Rightarrow\quad y=3^{-6}$

408. (a,b,c,d) $g(x)=(x+2)(x-1)$

$\therefore \quad f(x)=Q(x)\cdot(x+2)(x-1)+(4x+3)$

$$\left.\begin{aligned}\therefore\quad & f(1)=a+b+6=7\\ \text{and}\quad & f(-2)=4a-2b+3=-5\end{aligned}\right\}\begin{aligned}a&=-1\\ b&=2\end{aligned}$$

409. (a,b,d) $x+y=\sin\theta\cdot\cos\theta$ (a) is correct.

$x-y=\sin\theta\cdot\cos\theta\cdot\cos 2\theta=\frac{\sin 4\theta}{4}$ (b) and (d) are correct.

410. (b,c) $\gamma=8\alpha^3$, $\gamma=5^6\beta^6$, $\gamma^3=\alpha^2\beta^2$

$\therefore$ $(2\alpha)^3=(25\beta^2)^3$

$\therefore$ $2\alpha=25\beta^2$

Also $\gamma^3=2^9\cdot\alpha^9=\alpha^2\cdot\beta^2$ $\Rightarrow$ $\beta^2=2^9\cdot\alpha^7=\frac{2\alpha}{25}$

$\therefore$ $\alpha^6=\frac{1}{2^8\ 25}=\frac{1}{2^8\cdot 5^2}$ $\Rightarrow$ $\alpha^3=\frac{1}{2^4\cdot 5}$ Option (b)

Also $\gamma=8\cdot\frac{1}{2^4\cdot 5}=\frac{1}{10}$ Option (c)

Also $\beta^6=\frac{\gamma}{5^6}=\frac{1}{10\cdot 5^6}$

411. (a,b,c)

Clearly, sum of roots $=\frac{-\beta}{\alpha}>0$ $\Rightarrow$ $\beta>0$ (As $\alpha<0$)

Also, product of roots $=\frac{\gamma}{\alpha}<0$ $\Rightarrow$ $\gamma>0$ (As $\alpha<0$)

$\therefore$ $\alpha<0,\ \beta>0$ and $\gamma>0$

Hence, $\alpha\beta<0,\quad \alpha^2+\gamma\beta>0$ $\Rightarrow$ $\beta+\gamma-\alpha>0$ and $\alpha\gamma\beta<0$.

Now, verify alternatives.

412. (c) $a>0$ and $D=0$ $\Rightarrow$ $k>3$ and $4k^2-4(3k-6)(k\quad 3)=0$

$\Rightarrow$ $k=6,\frac{3}{2}$ (rejected)

$\therefore$ $k=6$

413. (a,c) $f\left(\frac{\pi}{7}\right)=5\times5=25$ and $f\left(\frac{2\pi}{7}\right)=-3(-3)=9$

414. (a,c,d) $f(n)=\sum_{r=1}^{n}(\log_{10}(9r+1)-\log_{10}(9r-8))$

$\therefore$ $f(n)=\log_{10}(9n+1)$.

415. (a,c) $2t^2+2\sqrt{2}\,t=3;\ t=\sin^2\theta$

$\therefore$ $t=\sin^2\theta=\frac{1}{\sqrt{2}}$ $\Rightarrow$ $\sin\theta=\pm\frac{1}{\sqrt[4]{2}}$

416. (a,b,c,d) $f(x)=\lambda-(x-2)^2, x\in[0,5]$

$\therefore$ maximum $=\lambda$

and $g(x)$: $\min = \lambda^2 - 2\lambda^2 + 10 - 2\lambda = 10 - 2\lambda - \lambda^2$

$\therefore \quad \lambda < 10 - 2\lambda - \lambda^2$

$\Rightarrow \quad \lambda^2 + 3\lambda - 10 < 0$

$\therefore \quad \lambda \in (-5, 2)$

417. (a,b,d) $\quad f(x) = \dfrac{(\sin x - 10)(x^2 - 4x + 3)(x^2 + x + 1)}{x^2 - 16}$

for $\quad f(x) \geq 0 \Rightarrow \dfrac{(\sin x - 10)(x^2 - 4x + 3)(x^2 + x + 1)}{x^2 - 16} \geq 0$

$\Rightarrow \quad \dfrac{(x-1)(x-3)}{(x-4)(x+4)} \leq 0$

$x \in (-4, 1] \cup [3, 4)$

Integral values of x are $-3, -2, -1, 0, 1, 3$, *i.e.*, 6

and their sum $= -2$.

for $\quad f(x) \leq 0 \Rightarrow \dfrac{(\sin x - 10)(x^2 - 4x + 3)(x^2 + x + 1)}{x^2 - 16} \leq 0$

$\Rightarrow \quad \dfrac{(x-1)(x-3)}{(x-4)(x+4)} \geq 0$

$x \in (-\infty, -4) \cup [1, 3] \cup [4, \infty]$

Integral values of x are infinite and their sum $= 1 + 2 + 3 = 6$

418. (a,b,d) $\quad$ Clearly, $\quad \tan(A+B) = \dfrac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{6}} = 1 \Rightarrow A + B = \dfrac{\pi}{4} \Rightarrow \sin^2(A+B) = \dfrac{1}{2}$

419. (b,d)

For the sum of infinite G.P. to be a finite quantity, common ratio r must satisfy $0 < |r| < 1$

$\therefore$ Options (b) and (d) correct.

420. (b,c) $\quad \dfrac{a}{1} = \dfrac{p+2}{4} = \dfrac{10q}{5} = k$

$a = k,\ p = 4k - 2,\ q = \dfrac{k}{2}$

For a, p, q to be natural number $k = 2$

$\therefore \quad (a + p + q)|_{\text{least}} = 2 + 6 + 1 = 9$

$a + q + \dfrac{3}{p} + \ldots\ldots, \infty = 2 + 1 + \dfrac{1}{2} + \ldots\ldots, \infty = \dfrac{2}{1 - \frac{1}{2}} = 4$

421. (a,c,d) $\quad 3^x = 4^{x^2}$

$\Rightarrow \quad x \log_2 3 = x^2 \log_2 4$

$\Rightarrow \quad x = 0,\ x = \dfrac{1}{2} \log_2 3 = \log_4 3$

422. (a,c) $\sin x+1=\dfrac{1}{2+\sin x+1}$

$\Rightarrow \quad (\sin x+1)(\sin x+3)=1 \quad \Rightarrow \quad \sin^2 x+4\sin x+2=0$

$\Rightarrow \quad (\sin x+2)^2=2 \quad \Rightarrow \quad \sin x=-2\pm\sqrt{2}$

$\Rightarrow \quad \sin x=-2-\sqrt{2}$ (rejected)

$\Rightarrow \quad \sin x=-2+\sqrt{2}$ (negative) $\Rightarrow \quad \pi+\alpha, \quad 2\pi-\alpha$

423. (b,c,d) $S_n=6+17+34+57+\ldots\ldots+T_n$

$$\text{Sub}\ \frac{S_n = \quad 6+17+34+57+\ldots\ldots+T_{n-1}+T_n}{0=6+11+17+23+\ldots\ldots+(T_n-T_{n-1})-T_n}$$

$T_n=6+11+17+23+\ldots\ldots+(T_n-T_{n-1})=6+\dfrac{n-1}{2}(2\times 11+(n-1-1)6)$

$T_n=6+(n-1)(3n+5)$

$T_n=3x^2+2x+1\equiv(\log_2 a)n^2+(\log_3(b-a))n+\log_4 c$

$\therefore \quad \log_2 a=3 \Rightarrow a=8; \quad \log_3(b-a)=2 \rightarrow b-a=9 \Rightarrow b=17$

$\log_4 c=1 \Rightarrow c=4$

Now, verify the options.

424. (a,c) $px^2-3px+14\geq|3\sin\theta-4\cos\theta|\,\forall\, x,\ \theta\in R$

$\Rightarrow \quad px^2-3px+14\geq 5\,\forall\, x\in R$

$\Rightarrow \quad px^2-3px+9\geq 0$

$p>0, \quad D\leq 0 \Rightarrow 9p^2-4p\cdot 9\leq 0$

$p(p-4)\leq 0 \Rightarrow p\in[0,4]$

$\therefore \quad p\in[0,4]$

For $p=0, f(x)=14\geq 5$

Sum of integral values of $p=10$.

Now, $f(x)\leq 14+\sin^2\alpha\,\forall\, x, a\in R$

$p=0$ is only the possible value.

425. (a,b) Let $y^{\log_3(\sqrt{3y})}=t$

$t=t^2-6 \quad \Rightarrow \quad t^2-t-6=0 \quad \Rightarrow \quad (t-3)(t+2)=0$

$t=3$

$\therefore \quad y^{\log_3(\sqrt{3y})}=3$

$\dfrac{1}{2}\log_3 3y\cdot\log_e y=1$

$\frac{1}{2}(1+a)a=1 \quad \Rightarrow \quad a^2+a-2=0 \quad \Rightarrow \quad (a+2)(a-1)=0$

$a=1,-2$

$y=3, 1/9$

$y_1=1/9$ and $y_2=3$

426. (a,c) $P(x)=2-\sin 3x$

427. (b,c,d) $\alpha+\gamma=\pi$

$\beta+\delta=\pi$

428. (a,b,c) $D<0$

$p\in(2,5)$

$\therefore$ $a=3;\ b=4;\ c=5$

Triangle is right angled triangle. Now verify.

429. (a,b) $D=4(1+\log_2(\sin\theta))$ must be a perfect square.

$\therefore$ $\sin\theta=1/2$. Now verify.

430. (a,c) $C_3 \to C_3+C_2-C_1$

$$f(n)=\begin{vmatrix} 2 & 1 & -1 \\ \frac{1}{(n+3)^2} & \frac{1}{n+1} & 0 \\ \frac{1}{(n+2)^2} & \frac{1}{n+2} & 0 \end{vmatrix}$$

Expand by C_3

$$f(n)=\frac{1}{(n+1)(n+2)^2}-\frac{1}{(n+2)(n+3)^2}$$

$$\sum_{n=1}^{n} f(n)=\left(\frac{1}{2\cdot 3^2}-\frac{1}{3\cdot 4^2}\right)+\left(\frac{1}{3\cdot 4^2}-\frac{1}{4\cdot 5^2}\right)+\ldots\ldots+\left(\frac{1}{(n+1)(n+2)^2}-\frac{1}{(n+2)(n+3)^2}\right)$$

$$\sum_{n=1}^{n} f(n)=\frac{1}{2\cdot 3^2}-\frac{1}{(n+2)(n+3)^2}$$

$$n=7,\ \sum_{n=1}^{7} f(n)=\frac{1}{2\cdot 9}-\frac{1}{9\cdot 100}=\frac{49}{900}$$

$$n\to\infty,\ \sum_{n=1}^{\infty} f(n)=\frac{1}{2\cdot 9}-0=\frac{1}{18}$$

431. (a,b,c) Let $P(h,k)$

$A\,(5\cos\alpha, 5\sin\alpha)$ $\qquad P(h,k)$ $\qquad B(5\cos\beta, 5\sin\beta)$

$$h=\frac{15\cos\alpha+10\cos\beta}{5}, \quad k=\frac{15\sin\alpha+10\sin\beta}{5}$$

$$h = 3\cos\alpha + 2\cos\beta, \qquad k = 3\sin\alpha + 2\sin\beta$$

Square and add

$$h^2 + k^2 = 13 + 12\cos(\alpha - \beta)$$

$$x^2 + y^2 = 13 + 12\cos(\alpha - \beta)$$

Now, verify.

432. (a,c,d) $\dfrac{\sqrt{3}}{2}\sin 2x - \dfrac{1}{2}\cos 2x = \tan\dfrac{x}{2}\left(2\cos^2\dfrac{x}{2}\right)$

$$\sin\left(2x - \frac{\pi}{6}\right) = \sin x$$

$$2x - \frac{\pi}{6} = n\pi + (-1)^n x$$

When, x is even, $x = 2k\pi + \dfrac{\pi}{6} \Rightarrow$ Solution $= \dfrac{\pi}{6}$

When, x is odd, $x = \dfrac{(2k-1)\pi}{3} + \dfrac{\pi}{18} \Rightarrow$ Solution $= \dfrac{7\pi}{18}, \dfrac{19\pi}{18}, \dfrac{31\pi}{18}$

Now, verify.

433. (b,d) Line perpendicular to $3x - 4y + 7 = 0$ is $4x + 3y + \lambda = 0$, passes (3, 4)

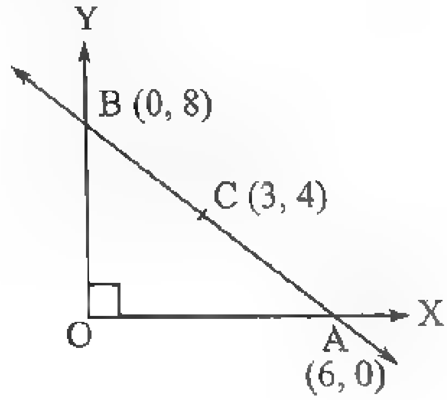
$4x + 3y - 24$

Orthocenter (0, 0)

Circumcenter = Mid-point of AB = (3, 4)

Centroid $\left(2, \dfrac{8}{3}\right)$

Incenter (2, 2)



(a) area $(\Delta OCG) = 0$ (co-linear point)

(b) area $(\Delta OCI) = 1$

(c) $OI = \sqrt{\dfrac{2}{3}}$

(d) $OC = 5$

434. (a,b,c) $S = 8$

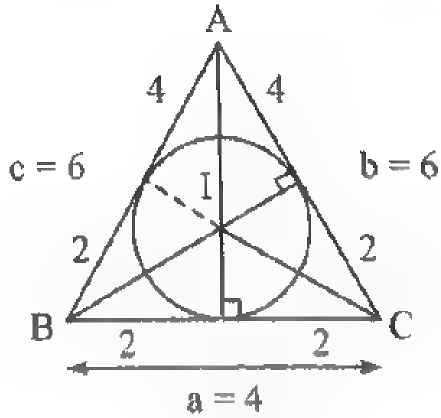
$$\Delta = \sqrt{8(2)(2)(4)} = 8\sqrt{2}$$

$$r = \frac{\Delta}{S} = \sqrt{2}$$

$$R = \frac{abc}{4\Delta} = \frac{6\cdot 6\cdot 4}{4\cdot 8\sqrt{2}} = \frac{9}{2\sqrt{2}} = \frac{9\sqrt{2}}{4}$$

$$\Delta R = 36 \neq a^2$$

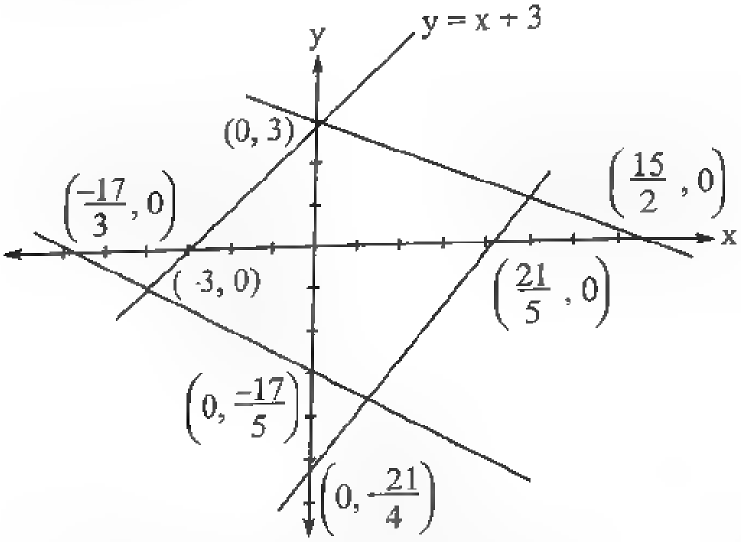
$$AI = \sqrt{2 + 16} = 3\sqrt{2}$$



$BI = \sqrt{4+2} = \sqrt{6}$ $\quad\therefore\quad AI : BI : CI = \sqrt{3} : 1 : 1$

$CI = \sqrt{4+2} = \sqrt{6}$

435. (b,d) $\quad -3 < \alpha < \dfrac{21}{5}$



436. (a,b,c,d) x^2+3x+2, x^2-x-10 and $x^2+x-4+y^2$ are in A.P. as well in G.P.

$\Rightarrow \quad x^2+3x+2 = x^2-x-10 = x^2+x-4+y^2$

$\Rightarrow \quad x = -3,\ y = 0$

Each number is equal to 2.

437. (b,c,d) Do yourself.

438. (a,b,c) If $\quad (a-2b-1)^2+(2a-3b-3)^2 = (a-2b-1)(2a-3b-3)$

$\Rightarrow \quad a-2b-1 = 0 = 2a-3b-3$

$\Rightarrow \quad a = 3,\ b = 1$

$\text{Ar.}(\Delta ABC) = \dfrac{1}{2}ab\sin c = \dfrac{3\sqrt{3}}{4} \Rightarrow \sin c = \dfrac{\sqrt{3}}{2} \Rightarrow \angle C = 60^\circ$ or $\angle C = 120^\circ$

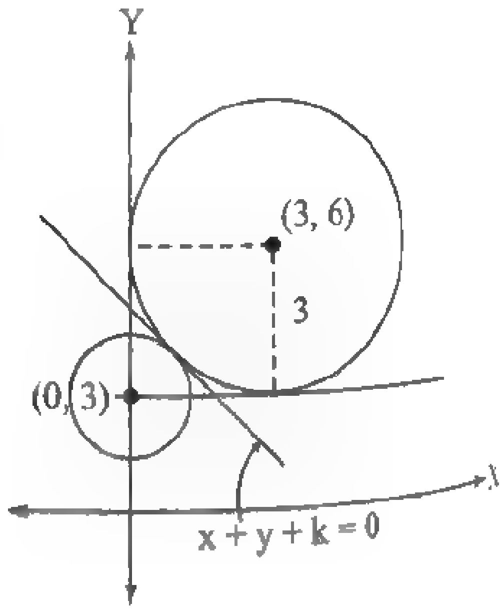
439. (a,b) $\quad x(y-3) = 0$

$\Rightarrow$ (0, 3) is the centre

$c_1c_2 = r_1 + r_2$

$\sqrt{9+9} = r+3$

$\Rightarrow \quad r = 3(\sqrt{2}-1) \simeq 1.2$



Transverse common tangent to both the circles

$$x+y+k = 0$$

applying $p = r \quad \Rightarrow \quad k = \pm 3\sqrt{2} - 9$

$$x+y+3\sqrt{2}-9 = 0 \quad \text{or}$$

$x+y-3\sqrt{2}-9 = 0$ (rejected)

440. (a,c) $m = n = 1$

Now, verify options.

441. (a,b,c)

(a) $D \geq 0 \Rightarrow \lambda^2 - 4(a^2 + a + 1) \geq 0 \Rightarrow \lambda^2 > 4\left[\left(a + \frac{1}{2}\right)^2 + \frac{3}{4}\right]$

$\lambda^2 \geq 3 \Rightarrow \lambda \geq \sqrt{3}$

(b) For $\lambda = 2$

$D = 4 - 4(a^2 + a + 1) \geq 0 \Rightarrow a \in [-1, 0]$

(c) $\frac{1}{2} = \frac{\lambda}{-1} = \frac{a^2 + a + 1}{6} \Rightarrow a^2 + a - 2 = 0$

$\Rightarrow (a+2)(a-1) = 0 \Rightarrow a = -2, 1$

(d) $\frac{-b}{2a} = 1 = \frac{-\lambda}{2} \Rightarrow \lambda = -2$

442. (c,d) $a = b = c$

For the least positive value of $y, x - 2 = 2 \rightarrow x = 4 - a$

Now, verify the options.

443. (a,b,c,d) CHTRNJV (II) (EE), A

(a) |–|–|–|–|–|–|–|

$7! \times {}^8C_5 \times \frac{5!}{2!\ 2!}$

(b) $\frac{12!}{5!}$

(c) $\frac{7!}{2!}$ (CHITRA) N, J, V, I, (E, E)

(d) 7! (IITJEE) < C, H, R, N, V, A.

444. (a,d)

(a) $T_n^3 + T_n + 1 = T_{n+1} - T_n$

$$\sum_{n=1}^{100} (T_n^3 + T_n + 1) = T_{101} - T_1 + 100 = T_{101} + 99$$

(c) $T_n^2 + 2 = \frac{T_{n+1}}{T_n}$

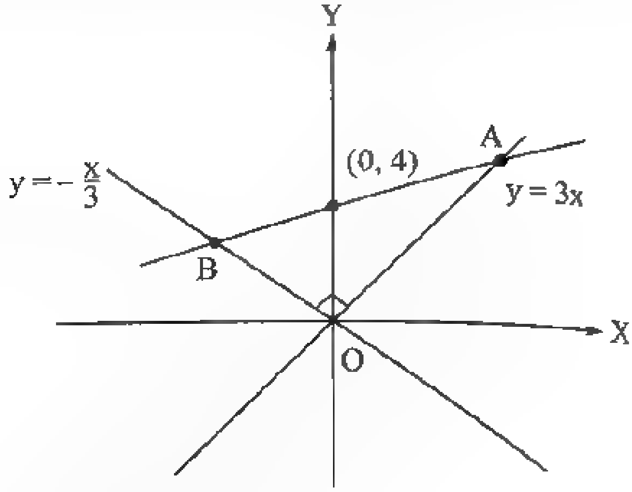
$$\prod_{n=1}^{100} (T_n^2 + 2) = \prod_{n=1}^{100} \frac{T_{n+1}}{T_n} = \frac{T_{101}}{T_1} = T_{101}$$

(d) $$\prod_{n=1}^{100} \left(\frac{T_{n+1}}{T_n} - T_n^2\right) = 2^{100}$$

445. (b,c) $a=3$

$(3x-y)(x+3y)=0$

$y=3x,\ y=\dfrac{-x}{3}$



(a) $m_1+m_2=3-\dfrac{1}{3}=\dfrac{8}{3}$

(b) $am_1+m_2=3\left(-\dfrac{1}{3}\right)+3=2$

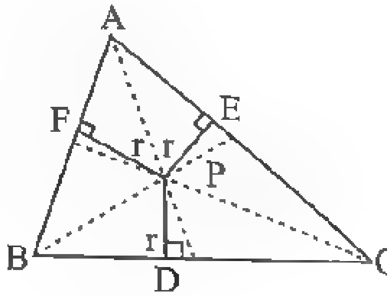
(c) $y=2x+4$

$A\equiv(4,12),\ B\equiv\left(\dfrac{-12}{7},\dfrac{-4}{7}\right)$

$$\text{Area }(\Delta AOB)=\frac{1}{2}\times\sqrt{16+144}\times\sqrt{\frac{144+16}{49}}=\frac{160}{2\times7}=\frac{80}{7}$$

446. (b,c,d)

P and Q are the incentre and circumcentre respectively.



$s=12$

$\Delta=\sqrt{12\cdot5\cdot3\cdot4}=12\sqrt{5}$

$r=\sqrt{5}$

$BD=s-b=4,\ CD=5$

$AE=s-a=3$

$\therefore\quad AP^2=9+5=14$

$BP^2=16+5=21$

$CP^2=25+5=30$

$$AQ=R=\frac{abc}{4\Delta}=\frac{7\cdot8\cdot9}{4\cdot12\sqrt{5}}=\frac{21\sqrt{5}}{10}$$

447. (a,b,d) $T_{r+1}={}^{105}C_r\left(2^{\frac{1}{5}}\right)^{105-r}\cdot7^{\frac{r}{7}}$

$={}^{105}C_r\,2^{\frac{105-r}{5}}\cdot7^{\frac{r}{7}}$

Clearly r should be a multiple of 7 and 5.

$r=0,35,70,105$

$\Rightarrow$ Number of rational terms = 4

Number of irrational terms = 102

Middle term = T_{53} and T_{54} are both irrational.

448. (b,c) $f(x)=3\cos^2x+9\sin^2x,\ x\neq\dfrac{n\pi}{2},\ n\in I=3+6\sin^2x$

Range of $f(x)=(3,9)$

449. (a,c,d)

(a) $^5C_2 \times {^3C_2} \times {^4C_2} \times {^3C_2} = 540$

(b) $^5C_2 \times {^3C_2} \times {^4C_2} \times {^3C_2} \times 4 = 2160$

(c) $P = 2^3 \times 3^1 \times 5^2 \times 7^1$

$\therefore$ Number of divisors of P which are divisible by $12 = 2\times 3\times 2 = 12$

(d) $\dfrac{P}{12} = 2^1 \times 5^2 \times 7^1$

Product of divisors of P which are divisible by $12 = (12)^{12}\left(\dfrac{P}{12}\right)^6 = (12P)^6$.

450. (b,c) $(\sqrt{2}\tan x)^3 + (\ 3\sqrt{2}\cot x)^3 + (-1)^3 = 18$

$\Rightarrow \sqrt{2}\tan x - 3\sqrt{2}\cot x - 1 - 0 \qquad \Rightarrow \sqrt{2}\tan^2 x - \tan x - 3\sqrt{2} - 0$

$\tan x = \dfrac{1 \pm \sqrt{1+24}}{2\sqrt{2}} - \dfrac{1\pm 5}{2\sqrt{2}}$

$\therefore \quad \tan x = \dfrac{3}{\sqrt{2}} \quad$ or $\quad -\sqrt{2}$

$\Rightarrow \quad 2\tan^2\alpha + \sqrt{2}\tan\alpha - 12, 2$

451. (a,c) $-6\alpha + 3\alpha^2 + 3\beta = 2\beta$

$3\alpha^2 \quad 6\alpha + \beta = 0$

$\because \quad \alpha$ is real. $\quad \therefore \ D \geq 0$

$36 - 12\beta \geq 0 \qquad \Rightarrow \ \beta \leq 3$

For $\beta - 3, 3\alpha^2 - 6\alpha + 3 = 0 \qquad \Rightarrow \qquad \alpha = 1$

$\beta = 2, 1$ (rejected)

$\therefore$ A.P. is $-6, 3, 12, \ldots\ldots\ldots$

Now, verify the options.

452. (a,d) $y(3^{|x|} - 1) + 2|x|\,(2^y - 1) - 0$

Either $|x| = 0$ or $y = 0$

$\Rightarrow \quad y = 0 \qquad \Rightarrow \quad (3x^2 - 1)^2 = x^2 + 1$

$\Rightarrow \quad 9x^4 - 6x^2 = x^2 \quad \Rightarrow \quad x^2 = 0, \ \dfrac{7}{9}$

$\therefore \quad x = 0$ (rejected), $\qquad \pm\dfrac{\sqrt{7}}{3}$

453. (a,c)

(a) $f(x) = \ln(\tan \pi[x] + |x^2 + 2x - 3|)$

$\therefore \quad [x] \in I \qquad \Rightarrow \qquad \tan \pi[x] = 0,$

and $|x^2 + 2x - 3| = |(x+1)^2 - 2| \in [0, \infty)$

So, $f(x) \in R \qquad \Rightarrow \qquad f(x)$ is surjective.

(b) $g(x)=\dfrac{x^2+2x-3}{x-1}$, $x\neq 1$

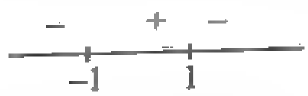
$g(x)=\dfrac{(x-1)(x+3)}{(x-1)}$, $x\neq 1$

$g(x)=x+3 \quad \therefore \quad g(x)\neq 4 \qquad (\because\ x\neq 1)$

So, range of $g(x)$ is $R-\{4\}$.

$\Rightarrow$ $g(x)$ is not surjective.

(c) $h(x)=\ln\left(\dfrac{1-x}{1+x}\right)$, $\quad \dfrac{1-x}{1+x}>0$



$\Rightarrow \quad D_h=(-1,1)$

$\because$ $\dfrac{1-x}{1+x}$ take all value between $(0,\ \infty)$

So, range of $h(x)=R$

$\Rightarrow$ $h(x)$ is surjective.

(d) $k(x)\sqrt{[x]+[-x]+1}+\sqrt{\{x\}+\{-x\}+1}$

Domain of $k(x)$ is R

$x\notin I \quad \Rightarrow \quad [x]+[-x]=-1 \quad$ and $\quad \{x\}+\{-x\}=1$

$\Rightarrow \quad k(x)=\sqrt{2}$

$x\in I \quad \Rightarrow \quad [x]+[-x]=0 \quad$ and $\quad \{x\}+\{-x\}=0$

$\Rightarrow \quad k(x)=2$

So, range of $k(x)=\{\sqrt{2},2\}$

So, $k(x)$ is not surjective.

454. (a,b,c) $\quad b=\dfrac{\alpha^2+\beta^2}{\alpha\beta}=\dfrac{(\alpha+\beta)-2\alpha\beta}{\alpha\beta}=\dfrac{25-10}{5}=3$

$t=x^2-4x+9-\dfrac{1}{5}+\dfrac{1}{x^2-4x+9}$

$t=(x-2)^2+5-\dfrac{1}{5}+\dfrac{1}{(x-2)^2+5}$

$t_{\text{min.}}=5 \qquad (\text{at } x=2)$

(a) Minimum value of $b+t=3+5=8$

(b) $\log_{1/5}5=-1$, maximum

(c) $y=\cot^{-1}(\log_5 t), t\geq 5$

$\Rightarrow \quad \log_5 t\in[1,\infty)$

$\Rightarrow \quad \cot^{-1}(\log_t 5)\in\left(0,\dfrac{\pi}{4}\right]$

(d) $y=\cot^{-1}(\log_{1/5}(t))$

$\log_{1/5} t \in (-\infty, -1]$

$\Rightarrow \quad \cot^{-1}(\log_{1/5}(t)) \in \left[\frac{3\pi}{4}, \pi\right)$

455. (a,b,d) $x^3 - px^2 + qx - 7 = 0$ with roots α, β, γ

$\alpha+\beta+\gamma = p$

$\alpha\beta+\beta\gamma+\gamma\alpha = q$

$\alpha\beta\gamma = 7$

$\gamma = 7 \qquad (\because \alpha\beta = 1)$

So, $\quad 1+7(\beta+\alpha) = q$

$1+7(p-7) = q$

$7p-48 = q$

$\because \quad p \le 9$

When, $\quad p=9 \Rightarrow q=15$

$p=8 \Rightarrow q=6$ (Not satisfied given condition)

So, $\quad \alpha=1,\ \beta=1,\ \gamma=7$

So,

(a) $|p+q| = 24$

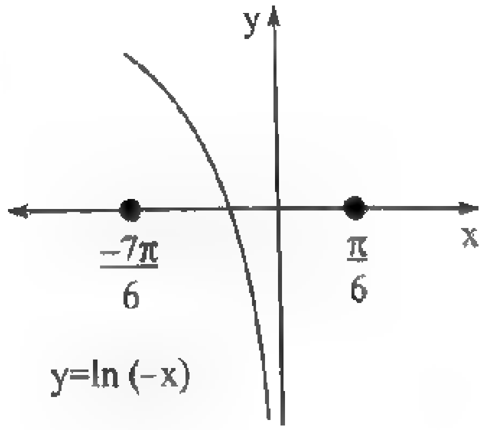
(b) $p-q = 9-15 = -6$

(d) $\tan^{-1}\alpha + \tan^{-1}\gamma + \tan^{-1}\left(\frac{4}{3}\right)$

$$\frac{\pi}{4} + \pi + \tan^{-1}\left(\frac{7+\frac{4}{3}}{1-7\times\frac{4}{3}}\right) \qquad (\because xy > 1)$$

$$\frac{\pi}{4} + \pi + \tan^{-1}(-1) = \pi$$

456. (b,c) $\quad f(x) = \tan^{-1}\left(\frac{2x}{\sqrt{9-4x^2}}\right) - \cos^{-1}\left(\frac{x}{3}\right)$



Domain of $f(x)$ is $x \in \left(\frac{-3}{2}, \frac{3}{2}\right)$

$f(x) = \sin^{-1}\left(\frac{2x}{3}\right) - \cos^{-1}\left(\frac{x}{3}\right)$

When, $x \in \left(\frac{-3}{2}, \frac{3}{2}\right)$

$$\left(\frac{-\pi}{2}, \frac{\pi}{2}\right) - \left(\frac{2\pi}{3}, \frac{\pi}{3}\right)$$

Range of $f(x)$

$$\left(\frac{-7\pi}{6} + \frac{\pi}{6}\right)$$

Number of solution of $f(x) = \ln(-x)$

has only one solution

$f(x) - k = 0$ has number of integral solution

$k = \{-3, -2, -1, 0\} \quad \Rightarrow \quad$ 4 value

457. (a,c) Least value of $f(x)$ is $\frac{-D}{4a} = \frac{-81}{4}$

Least value of $\tan^{-1}(22 + [f(x)])$

$$\tan^{-1}\left(22 + \left[-\frac{81}{4}\right]\right)$$

$$\tan^{-1}(22 - 21) = \frac{\pi}{4}$$

Largest integral value of k for which equation

$$\text{sgn}(f(x) + k) = 0 \text{ has a solution}$$

$\because \quad f(x)$ has min. value -20.25

If $k > 20.25$, then sgn $(f(x) + k)$ is always positive.

So, largest integral value of k is 20.

458. (c,d) $a = 5$

$T_{r+1} = 5 \cdot r^{n-1} = 5 \cdot 2^8 \cdot 3^{16}$

$r^{n-1} = (2 \cdot 3^2)^8 = ((2 \cdot 3^2)^2)^4 = ((2 \cdot 3^2)^4)^2 = ((2 \cdot 3^2)^8)^1$

$\therefore$ Possible common ratio of the G.P. are

$2 \cdot 3^2, (2 \cdot 3^2)^2, (2 \cdot 3^2)^4, (2 \cdot 3^2)^8$

459. (a,b,c) $x f(x) - 1 \equiv (x-1)(x-2)(x-3)(x-4)(x-\alpha)$

Put $x = 0 \Rightarrow \alpha = \frac{1}{24}$

$$\therefore \quad f(x) = \frac{(x-1)(x-2)(x-3)(x-4)\left(x - \frac{1}{24}\right) + 1}{x}$$

$$\Rightarrow \qquad f(5)=\frac{4\times3\times2\times1\times\frac{119}{24}+1}{5}=24$$

Now, verify the options.

460. (a,c) $\qquad (1+x)^{2n}+(1+2x+x^2)^n=\sum_{r=0}^{2n}a_rx^r$

$$2(1+x)^{2n}=\sum_{r=0}^{2n}a_rx^r$$

So, $\qquad f(n)=\sum_{r=0}^{2n}a_r=2^{2n+1} \qquad \ldots(1)$

So, $\qquad \sum_{n=1}^{\infty}\frac{1}{f(n)}=\frac{1}{2^3}+\frac{1}{2^5}+\frac{1}{2^7}+\ldots\ldots$

$$=\frac{1/2^3}{1-\frac{1}{4}}=\frac{1/8}{3/4}=\frac{1}{6}$$

Largest value of p for which $f(5)$ is divisible by 2^p

$f(5)=2^{11}$

So, $\quad p=11$

461. (a,b,d) $\quad$ Let $\quad f(x)=ax^3+bx^2+cx+d$

$$\lim_{x\to0}(1+f(x))^{\frac{1}{x}}=e^{-1} \quad\Rightarrow\quad C=-1 \quad\text{and}\quad d=0$$

$$x^3f\left(\frac{1}{x}\right)=x^3\left(\frac{a}{x^3}+\frac{b}{x^2}+\frac{c}{x}+d\right)=a+bx+cx^2+dx^3$$

$$\lim_{x\to0}\left(x^3f\left(\frac{1}{x}\right)\right)^{\frac{1}{x}}=e^2 \qquad\Rightarrow\qquad \lim_{x\to0}(a+bx+cx^2+dx^3)^{\frac{1}{x}}=e^2$$

$\Rightarrow \quad a=1 \quad$ and $\quad b=2$

$f(x)=x^3+2x^2-x$

462. (a,c) $\qquad f(x)=\dfrac{\sin^{-1}(1-\{x\})\cos^{-1}(1-\{x\})}{\sqrt{2\{x\}}(1-\{x\})}$

$$f(0^+)=\lim_{x\to0^+}\frac{\sin(1-h)\cdot\cos^{-1}(1-h)}{\sqrt{2}\sqrt{h}(1-h)}=\frac{\pi}{2\sqrt{2}}\lim_{h\to0}\frac{\cos^{-1}(1-h)}{\sqrt{h}}$$

Let $\cos^{-1}(1-h)=\theta \qquad\Rightarrow\qquad 1-h=\cos\theta \qquad\Rightarrow\quad h=1-\cos\theta$

$$\therefore \qquad l=\frac{\pi}{2}\frac{\theta}{\sqrt{2}\sqrt{1-\cos\theta}}=\frac{\pi}{2}\lim_{x\to0}\frac{1}{\sqrt{2}}\frac{\theta\sqrt{1+\cos\theta}}{\sin\theta}=\frac{\pi}{2}$$

$$|||^{ly}\ f(0^-)=\lim_{h\to 0^-}\frac{\sin^{-1}(1-(1-h))\cos^{-1}(1-(1-h))}{\sqrt{2(1-h)}\,(1-(1-h))}$$

$$=\lim_{h\to 0}\frac{\sin^{-1}h\cdot\cos^{-1}h}{\sqrt{2}\,h}=\frac{\pi}{2\sqrt{2}}$$

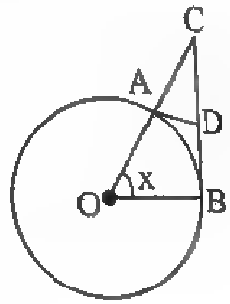
463. (a,b,d) $f(x)=\begin{cases}\cos^{-1}x, & -1\le x<0\\ \sin^{-1}x, & 0\le x\le 1\end{cases}$ and $g(x)=\begin{cases}\sin^{-1}x, & -1\le x<0\\ \cos^{-1}x, & 1\ge x\ge 0\end{cases}$

$$h(x)=\min\{f(x),g(x)\}=\begin{cases}g(x), & -1\le x<0\\ f(x), & 0\le x<\dfrac{1}{\sqrt{2}}\\ g(x), & \dfrac{1}{\sqrt{2}}\le x\le 1\end{cases}$$

$$h(x)=\begin{cases}\sin^{-1}x, & -1\le x<0\\ \sin^{-1}x, & 0\le x<\dfrac{1}{\sqrt{2}}\\ \cos^{-1}x, & \dfrac{1}{\sqrt{2}}\le x\le 1\end{cases}$$

$\Rightarrow$ $h(x)$ is continuous and not differentiable at $x=-1,\frac{1}{\sqrt{2}}$, 1 and $h_{\max}=h\left(\frac{1}{\sqrt{2}}\right)=\frac{\pi}{4}$

464. (a,c) $r=2$



$\Rightarrow$ $AD=r\tan\frac{x}{2}=BD$

Area $(\Delta OBC)=\frac{1}{2}r^2\tan x$ and Area $(\Delta OAB)=\frac{1}{2}r^2\sin x$

$$\lim_{x\to 0}\frac{\text{Area }(\Delta OBC)}{\text{Area }(\Delta OAB)}=1$$

$$\text{Area }(\Delta ADB)=\frac{1}{2}\cdot AD\cdot BD\sin(\pi-x)=\frac{1}{2}r^2\tan^2\frac{x}{2}\sin x$$

$$\lim_{x\to 0}\frac{\text{Area }(\Delta ABD)}{(\text{Area }(\Delta OAB))^3}=\lim_{x\to 0}\frac{\frac{1}{2}r^2\tan^2\frac{x}{2}\sin x}{\left(\frac{1}{2}r^2\sin x\right)^3}$$

$$=\lim_{x\to 0}\frac{4r^2\tan^2\frac{x}{2}}{r^6\sin^2 x}=\lim_{x\to 0}\frac{4\sin^2\frac{x}{2}}{16\cdot 4\sin^2\frac{x}{2}\cdot\cos^4\frac{x}{2}}=\frac{1}{16}$$

465. (b,c,d) $x^4+3x^3+2(1-a)x^2-3ax+a^2=0$

$\Rightarrow \quad a^2-(2x^2+3x)a+x^4+3x^3+2x^2=0$

$\Rightarrow \quad a^2-(x^2+2x+x^2+x)a+(x^2+2x)(x^2+x)=0$

$\Rightarrow \quad (x^2+x-a)(x^2+2x-a)=0$

$D_1=1+4a$ and $D_2=4+4a$

$D_1\ge 0$ and $D_2\ge 0 \Rightarrow \quad a>\dfrac{-1}{4}$

466. (c,d) $m={}^{41}C_{20}$; $\quad n={}^{40}C_{19}$

$m-n={}^{41}C_{20}-{}^{40}C_{19}$

$={}^{40}C_{20}+{}^{40}C_{19}-{}^{40}C_{19}$

$={}^{40}C_{20}=\dfrac{40!}{20!\cdot 20!}$ (Now verify each alternative)

467. (a,b) $f(3-x)=f(3+x)$

$\Rightarrow$ symmetric about $x=3$

$f(x_1)=f(x_2)=f(x_3)=f(x_4)=f(x_5)-0$

$\Rightarrow \quad x_3=3$

$x_1+x_2+x_3+x_4+x_5=15$

$\Rightarrow \quad f'(3)=0$

468. (b,c) $\left(\sqrt{y}+\dfrac{1}{2\sqrt[4]{y}}\right)^n$

First 3 coefficient are

$${}^nC_0,\ \frac{{}^nC_1}{2},\ \frac{{}^nC_2}{2^2}\ ;\ \text{Hence} \qquad 1+\frac{n(n-1)}{8}=n$$

$8+n^2-n=8n \quad\Rightarrow\quad n^2-9n+8 \quad\Rightarrow\quad n=8$ or 1 ($n=1$ is rejected)

$n=1$ is rejected $\quad\therefore\quad n=8$

$\therefore$ The given expansion is $\left[y^{\frac{1}{2}}+\dfrac{1}{2}y^{-\frac{1}{4}}\right]^8$

Where, $T_{r+1}=\dfrac{{}^8C_r}{2^r}\cdot y^{\frac{n-r}{2}}\cdot y^{-\frac{r}{4}}=\dfrac{{}^8C_r}{2^r}y^{\frac{2n-3r}{4}}=\dfrac{{}^8C_r}{2^r}\cdot y^{\frac{16-3r}{4}}$ (using $n=8$)

The terms where power of y is natural are

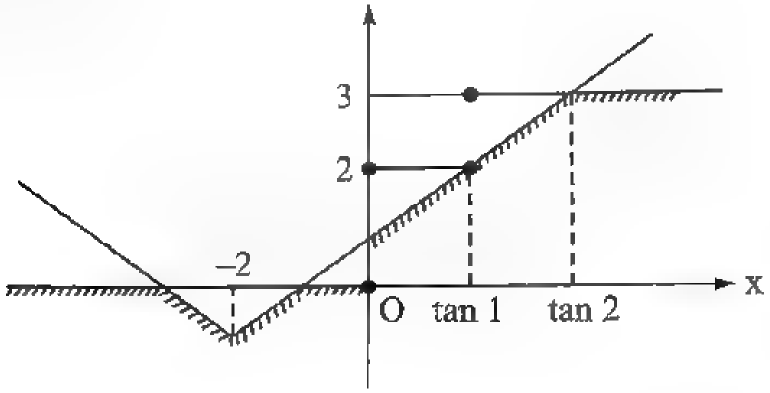
${}^nC_0\cdot y^4 \rightarrow$ First term where $r=0$

$\dfrac{{}^8C_4}{2^4}\cdot y^1 \rightarrow$ Fifth terms where $r=4$

469. (a,b,d) $\quad f(x)=\begin{cases} 0, & x\le 0 \\ 2, & 0<x<\tan 1 \\ 3, & x\ge \tan 1 \end{cases}$

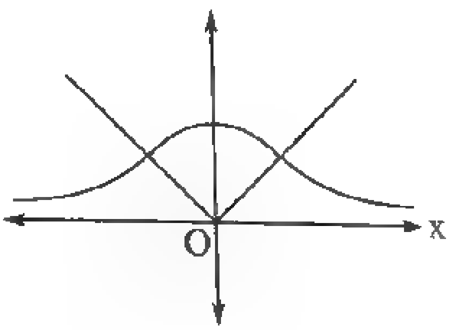
$g(x)=|x+2|-\tan 1$

Now, verify the options.



470. (b,c) $\quad f(x)=\lim\limits_{n\to\infty}\left(a^{\frac{1}{n}}+\ln b+\cos\dfrac{x}{\sqrt{n}}\right)^{n}, \qquad \ln b=-1 \quad\Rightarrow\quad b=e^{-1}$

$$=e^{\lim\limits_{n\to\infty}\left(\frac{a^{\frac{1}{n}}-1}{\frac{1}{n}}-\frac{\left(1-\cos\frac{x}{\sqrt{n}}\right)}{\frac{1}{n}}\right)}=e^{\ln a \;\frac{x^2}{2}}=ae^{\frac{-x^2}{2}}$$



$\therefore \quad f(x)=|x| \quad\Rightarrow\quad ae^{\frac{-x^2}{2}}=|x|$

$$L=\lim_{x\to 0}\frac{ae^{\frac{-x^2}{2}}-a}{\left(\dfrac{1-\cos x}{x^2}\right)\cdot x^2}=-a$$

$\therefore \quad L+a=0 \quad$ and $\quad L+a+3be=3$

471. (a,d) $\quad f(x)=(\sqrt{\pi^2-1}\cos x+\sin x)(\cos x\cdot\cos(\operatorname{cosec}^{-1}\pi)+\sin x\cdot\sin(\operatorname{cosec}^{-1}\pi))$

$$=(\sqrt{\pi^2-1}\cos x+\sin x)\left(\cos x\cdot\frac{\sqrt{\pi^2-1}}{\pi}+\sin x\cdot\frac{1}{\pi}\right)$$

$$=\frac{1}{\pi}(\sqrt{\pi^2-1}\cos x+\sin x)^2$$

$\therefore \quad f(x)|_{\max}=\dfrac{\pi^2}{\pi}=\pi=M$

$f(x)|_{\max}=0=m$

Now, verify the options.

472. (a,b,c)

$f(x)$ is non-derivable at $x=0$

Now, it should be derivable at $x=\pm 1$.

Continuous at $x=1$, $a+b+c=0$ ...(1)

Derivable at $x=1$, $(3ax^2+2bx)|_{x=1}=1$

$$3a+2b=1 \quad \text{...(2)}$$

Continuous at $x=-1$, $a+b+c=0$

Derivable at $x=-1$, $\quad -3a-2b=-1$

$$f'(2)=0 \quad \Rightarrow \quad 12a+4b=0 \quad \Rightarrow \quad 3a+b=0 \quad \text{...(3)}$$

From eqns. (1), (2) and (3), we get

$$a=\frac{-1}{3},\ b=1 \ \text{ and } \ c=\frac{-2}{3}$$

Now, verify the options.

473. (a,d) $\quad A_1 \to 2l-1, \quad A_2 \to 2m+2, \quad A_3 \to 2h+3, \quad A_4 \to 2p$

$\therefore \quad 2l-1+2m+2+2n+3+2p=50$

$\Rightarrow \quad 2l+2m+2n+2p=46 \quad \Rightarrow \quad l+m+n+p=23, \quad l,m,n,p\ge 1$

$\quad l'+m'+n'+p'=19, \quad l',m',n',p'\ge 0$

$\therefore \quad$ Total number of ways of distribution $= {}^{22}C_3$

When A_4 receiving not more than 14 marbles

$$l+m+n+p=23$$
$$1 \quad 1 \quad 1 \quad 1$$
$$1 \quad 1 \quad 1 \quad 8$$

$l'+m'+n'+p'=12, l',m',n',p'\ge 0$

Number of ways of distribution when A_4 receiving 16 or more marbles $= {}^{15}C_3$

$\therefore$ Number of ways when A_4 receiving not more than 14 marbles $= {}^{22}C_3 - {}^{15}C_3 = 1085$

474. (a,b,c) $\quad a$, b and $b-2$ are in G.P.

$\therefore \quad b^2=a(b-2)$

$\Rightarrow \quad b^2-ab+2a=0$

For b to be real $D\ge 0$

$$a^2-8a\ge 0 \quad \Rightarrow \quad a\ge 8 \quad \text{or} \quad a\le 0$$

Option (a) and (b)

$$a\in[1,8] \quad \Rightarrow \quad a=8$$

For $\quad a=8, \quad b=4$

G.P. $\quad 8, 4, 2, \ldots\ldots\ldots$

$$r = \frac{1}{2} \quad \text{and} \quad S_\infty = \frac{8}{1-\frac{1}{2}} = 16$$

Option (c) and (d)

$$a \in [8, 11]$$

For $\quad a = 9, \quad b^2 - 9b + 18 = 0 \quad \Rightarrow \quad b = 3, 6$

G.P. $\quad 9, 3, 1, \ldots\ldots \quad$ or $\quad 9, 6, 4, \ldots\ldots$

$$\downarrow \qquad\qquad\qquad \downarrow$$

$$S_\infty = \frac{9}{1-\frac{1}{3}} = 27 \qquad\qquad S_\infty = \frac{9}{1-\frac{2}{3}} = 27$$

475. (b,c) $\quad x^2 - 3x - 2 = 0 \begin{cases} \tan\alpha \\ \tan\beta \end{cases}$

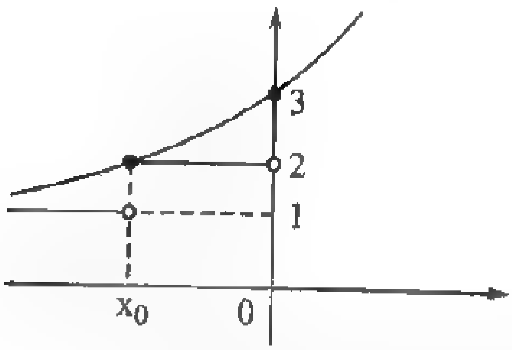
$$\tan\alpha, \tan\beta = \frac{3 \pm \sqrt{17}}{2}$$

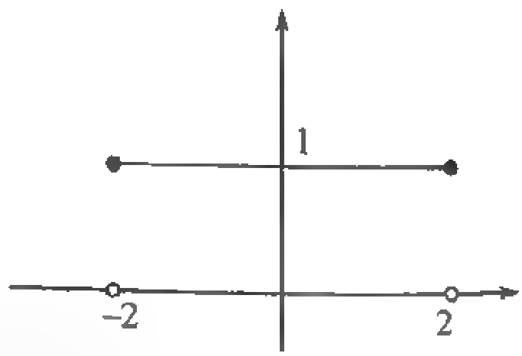
$$\therefore \quad \tan\alpha = \frac{3-\sqrt{17}}{2}, \qquad \tan\beta = \frac{3+\sqrt{17}}{2}$$

$$\tan(\beta - \alpha) = \frac{\sqrt{17}}{1+(-2)} = -\sqrt{17} \quad \Rightarrow \quad \beta - \alpha \in \left(\frac{\pi}{2}, \pi\right)$$

$$\tan 2\alpha = \tan[(\alpha+\beta)+(\alpha-\beta)] = \frac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta)\tan(\alpha-\beta)} = \frac{1+\sqrt{17}}{1-\sqrt{17}}$$

476. (a,b,c) $\quad f(x) = [2^x + 2^{x/2} + 1] \quad ; \quad g(x) = \left[\frac{9}{x^2+5}\right]$





$f(x)$ is discontinuous at 2 points in $(-\infty, 0]$

and $\quad g(x)$ is discontinuous at 2 points in $(-\infty, \infty]$

$$f(x)\cdot g(x) = [2^x + 2^{x/2} + 1]\left[\frac{9}{x^2+5}\right] = \begin{cases} 0 & x \in (-\infty, -2) \cup (2, \infty) \\ 1 & x = -2 \\ 2 & \\ 3 & \\ 7 & x = 2 \end{cases}$$

477. (a,b,d) $\int_0^x t\, f(x-t)\,dt = e^{2x}-1$

Using King

$$\int_0^x (x-t)\, f(t)\,dt = e^{2x}-1$$

$$x\int_0^x f(t)\,dt - \int_0^x t\, f(t)\,dt = e^{2x}-1$$

Differentiate both sides

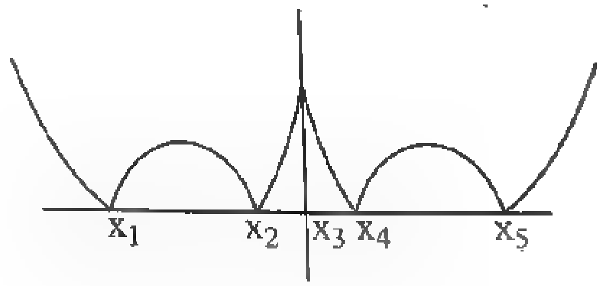
$$x\, f(x) + \int_0^x f(t)\,dt - x\, f(x) = 2e^{2x}-2 \quad \Rightarrow \quad f(x) = 4e^{2x} \quad \Rightarrow \quad f(0)=4$$

478. (b,c,d) $\lim\limits_{x\to 1} f(x) = e^{\lim\limits_{x\to 1}\frac{\ln(c^2+c+1)\tan^2(x-1)}{\ln^2(1+(x-1))}} = c^2+c+1$

$\because \quad c^2+c+1 \neq 3c$

$\Rightarrow \quad c \neq 1$

479. (b,c) $|x_1|+|x_2|+|x_3|+|x_4|+|x_5| = 10$



$\Rightarrow \quad |x_4|+|x_5| = 5$

$\therefore \quad x_4 + x_5 - 5$

$\therefore$ Roots of $x^2 - px + q$ are 1, 4 and 2, 3

$\therefore \quad f(x) = x^2 - 5x + 4 \quad$ or $\quad x^2 - 5x + 6$

$\therefore \quad p + q = 9 \quad$ or $\quad 11$

480. (a,b,c,d) (a) $I - \int_{-\pi}^{\pi} \underbrace{(\cos 2x\cdot\cos 2^2x\cdot\cos 2^3x\cdot\cos 2^4x\cdot\cos 2^5x)}_{f(x)}\,dx$

$$I = 2\int_0^{\pi} f(x)\,dx \qquad [f(x) \text{ is given}]$$

$$I = 2\cdot 2\int_0^{\frac{\pi}{2}} f(x)\,dx \qquad (\text{using Queen})$$

$$I = 4\int_0^{\frac{\pi}{2}} f(x)\,dx = 4I_1$$

Now, $\quad I_1 = \int_0^{\frac{\pi}{2}} f(x)\,dx$

Using King

$$I_1 = -I_1$$

$\therefore \quad I_1 = 0$

$\Rightarrow \quad I = 0$

481. (a,b,d) $\because \; a^2+b^2+c^2+ab+bc+ca = \frac{1}{2}[(a+b)^2+(b+c)^2+(c+a)^2] \le 0$

$\therefore \quad a=b=c=0$

$\therefore \quad f(x)=0$ which is always continuous and derivable.

482. (a,b,c) Slope of BC is -2 or $\frac{1}{2}$

$\Rightarrow \quad BC: 2x+y=7 \quad$ or $\quad BC: x-2y+4=0$

$A\,(0,0),\; B\,(7,\;7),\; C\left(\frac{-7}{5},\frac{49}{5}\right) \quad \Rightarrow \quad \Delta = \frac{147}{5}$

$A\,(0,0),\; B\left(\frac{-4}{3},\frac{4}{3}\right),\; C\left(\frac{-4}{15},\frac{28}{15}\right) \quad \Rightarrow \quad \Delta = \frac{16}{15}$

483. (a,c,d) $\int \frac{3\sin^2 x\cos x}{x^3}dx - 3\int \frac{\sin^3 x}{x^4}dx$

$$= \int \frac{3\sin^2 x\cos x}{x^3}dx - 3\left(\sin^3 x\left(\frac{-1}{3x^3}\right) - \int 3\sin^2 x\cos x\left(\frac{-1}{3x^3}\right)dx\right) + C = \frac{\sin^3 x}{x^3}+C$$

$\therefore \quad f(x)=\frac{\sin^3 x}{x^3}$

(a) $\lim\limits_{x\to 0}\frac{\int_0^x t\cdot\frac{\sin^3 t}{t^3}\cdot dt - 2x^2}{1-\cos x} = \lim\limits_{x\to 0}\frac{\frac{\sin^3 x}{x^2}-4x}{\sin x} = \lim\limits_{x\to 0}\frac{\frac{\sin^3 x}{x^3}-4}{\frac{\sin x}{x}} = -3$

(b) $\lim\limits_{x\to 0}\frac{\frac{\sin x}{x}-x^2}{x^2} = \lim\limits_{x\to 0}\frac{\sin x - x^3}{x^3} = \text{D.N.E.}$

(c) $h(x) = x\cdot\frac{\sin x}{x} = \sin x \quad \Rightarrow \quad \int_0^{\pi}\sin^4 dx = \frac{3\pi}{8}$

(d) $\int_0^{\frac{\pi}{2}} e^{\sin x}(\cos x\cos x + (-\sin x))dx = (e^{\sin x}\cdot\cos x)_0^{\pi/2} = 0-1$

484. (a,b,d) $f(x) = 5x^2-10x+3 = 5(x-1)^2-2 = g(x)$

$f(x) = a\,(x-1)^2-2$

$\lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0^+}[a(x-1)^2-2] = 3$

$\Rightarrow \quad a-2=3 \quad \Rightarrow \quad a=5$

$\therefore \quad g:[1,\infty)\to[-2,\infty)$

$g(x)=5(x-1)^2-2$

(a) $g'(x)=10(x-1) \quad \Rightarrow \quad g'(1)=0$

(b) Domain of $g(g(x))$

$g(x)\ge 1 \quad \Rightarrow \quad 5(x-1)^2-2\ge 1$

$x\ge 1+\sqrt{\frac{3}{5}}$

$$\therefore \quad x\in\left[1+\sqrt{\frac{3}{5}},\infty\right)\equiv\left[1+\sqrt{\frac{p}{q}},\infty\right)$$

$\Rightarrow \quad q-p=2$

(c) $g(x)=g^{-1}(x)=x$

$5(x^2 \quad 2x+1) \quad 2=x \quad \Rightarrow \quad 5x^2-11x+3=0$

$$\Rightarrow \quad x=\frac{11\pm\sqrt{121-60}}{10}$$

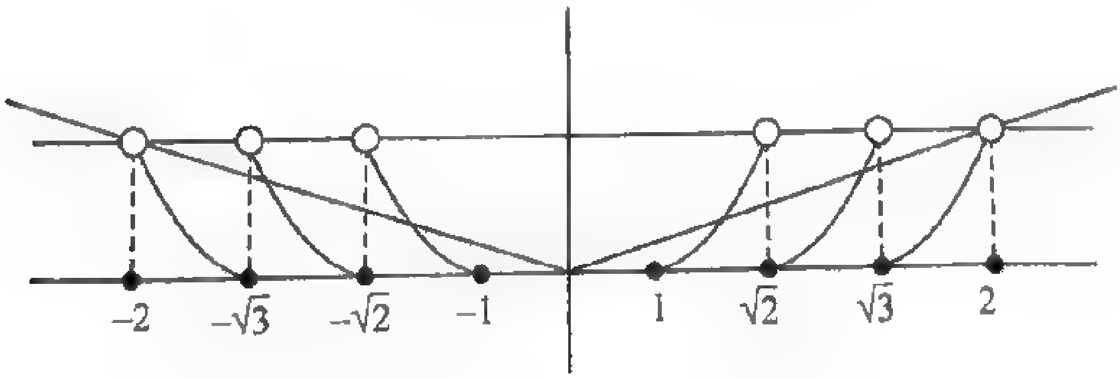
$$\Rightarrow \quad x=\frac{11\pm\sqrt{61}}{10} \begin{cases} \dfrac{11+\sqrt{61}}{10} & \text{(only one solution)} \\ \dfrac{11-\sqrt{61}}{10} & \text{(rejected)} \end{cases}$$

(d) $\frac{d}{dx}[90(g^{-1}(x))]|_{x=43}=\frac{90}{g'(4)}=\frac{90}{10(3)}=3$

$g(x)=43 \quad \Rightarrow \quad 5(x-1)^2-2=43 \quad \Rightarrow \quad x-1=3 \quad \Rightarrow \quad x=4$

485. (b,d)

$$f(x)=\{x^2-1\}[|x|]=\begin{cases}0, & -1<x<1\\ \{x^2-1\}, & x\in(-2,-1]\cup[1,2)\end{cases}$$

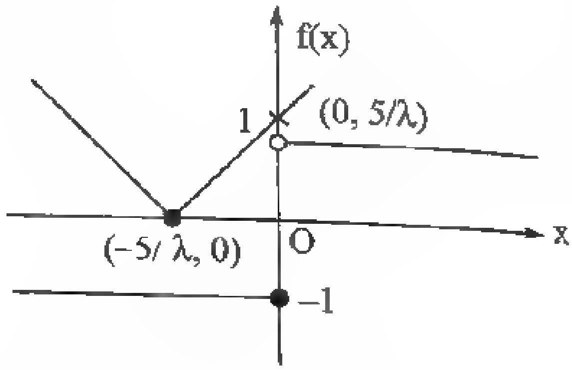


486. (a,c,d) $f(x)=\lim_{n\to\infty}(-n)\left(\left|2\tan^{-1}x-\frac{1}{n}\right|-2|\tan^{-1}x|\right)$

$$=\lim_{n\to\infty}\frac{(-n)\left[\left(2\tan^{-1}x-\frac{1}{n}\right)^2-4(\tan^{-1}x)^2\right]}{\left|2\tan^{-1}x-\frac{1}{x}\right|+2|\tan^{-1}x|}$$

$$=\lim_{n\to\infty}\frac{(-n)\left(\frac{-4\tan^{-1}x}{n}+\frac{1}{n^2}\right)}{\left|2\tan^{-1}x-\frac{1}{n}\right|+2\ \tan^{-1}x|}=\frac{4\tan^{-1}x}{|4\tan^{-1}x|}=\frac{\tan^{-1}x}{|\tan^{-1}x|},x\neq 0$$

$$f(x)=\begin{cases}\dfrac{\tan^{-1}x}{\tan^{-1}x|}, & x\neq 0\\ -1, & x=0\end{cases}$$



(a) $f(x)$ is discontinuous at $x=0$

(b) $|f(x)|$ is a continuous function.

(c) $f(1)+f(2)=2$

(d) $f(x)=\left|x+\frac{5}{\lambda}\right|$

For the existence of the solution of the equation $\frac{5}{\lambda}<1 \Rightarrow \lambda>5$

487. (a,b,c) $S=\{14, 15, 16, \ldots\ldots\ldots\ldots, 22\}$

Sum of the least and greatest number must be a perfect square *i.e.*, 36

($\because$ number of divisors of their sum is odd.)

(i) $14, \ldots\ldots\ldots, 22 \to 2^7=128$

(ii) $15, \ldots\ldots\ldots, 21 \to 2^5=032$

(iii) $16, \ldots\ldots\ldots, 20 \to 2^3=008$

(iv) $17, \ldots\ldots\ldots, 19 \to 2^1=\underline{002}$

170

$N=170=2\times 5\times 17$

488. (a,c,d) $f(f(x-2))=(x^2+3)^2+1$

$x-2\to t$

$$f(f(t))=((t+2)^2+3)^2+1=(t^2+4t+7)^2+1=(t^2+4t+5+2)^2+1$$

$$f(f(t))=(t^2+4t+5)^2+4(t^2+4t+5)+5$$

$\therefore\quad f(x)=x^2+4x+5$

(a) $f(x)=(x+2)^2+1 \Rightarrow$ Least value of $f(x)$ is 1.

(c) $\frac{d(f(f(x)))}{dx}=2((t+2)^2+3)\cdot(2(t+2))$

$$\left.\frac{d(f(f(x)))}{dx}\right|_{x=0}=2\cdot7\cdot4=56$$

(d) $\int\frac{dx}{(x+2)^2+1}=\tan^{-1}(x+2)+C$

$\therefore\quad g(x)=\tan^{-1}(x+2)\;\Rightarrow\; g(0)+g(1)=\tan^{-1}2+\tan^{-1}3=\frac{3\pi}{4}$

489. (b,d) Clearly, $P(x)$ is a polynomial of degree three with leading coefficient 2.

Clearly, $P(x)=2x^3$

$\therefore\quad P(4)=128$ and area $=\int_0^2 2x^3\,dx=\left(\frac{x^4}{2}\right)_0^2=8$

490. (a,b) $\because\quad \frac{1}{x}+\frac{1}{x-1}+\frac{1}{x-2}=1\;\Rightarrow\; x^3-6x^2+8x-2=0$ has roots a, b, c.

$\therefore\quad x^3-6x^2+8x-2=(x-a)(x-b)(x-c)$

$\therefore\quad (1-a)(1-b)(1-c)=1-6+8-2=1$

And $abc=2$

491. (a,b,c,d)

$\because\quad a_{ji}=(j^2+i^2-ji)(i-j)=-a_{ij}$

$\therefore\quad A$ is skew-symmetric matrix.

$\therefore\quad$ tr. (A) = 0 | A |

492. (a,d) Clearly, $f(x)$ and $g(x)$ is defined if $-1\le\frac{[x]}{\{x\}}<1$

$\therefore\quad 0\le\{x\}<1$ and $[x]\in I$

$\therefore\quad 0<x<1\;\Rightarrow\;\frac{[x]}{\{x\}}=0$

$\therefore\quad A=C=(0,1)$ and $f(x)=0$ and $g(x)=\frac{\pi}{2}\forall\, x\in(0,1)$

B and D are co-domain

$\therefore\quad$ Need not to be singleton sets.

$f(x)+g(x)=\frac{\pi}{2}\forall\, x\in(0,1)$

$\therefore\quad$ No. of integral solution = 0.

493. (a,c) $I_n=2\int_0^1 x\left(1+\frac{x^2}{2}+\frac{x^4}{4}+\ldots\ldots+\frac{x^{2n}}{2n}\right)dx$ $\quad\left(\because\int_{-1}^{1}(\text{odd})\,dx=0\right)$

$$= 2\left(\frac{x^2}{2} + \frac{x^4}{2\cdot 4} + \frac{x^6}{4\cdot 6} + \ldots\ldots + \frac{x^{2n+2}}{2n\,(2n+2)}\right)_0^1$$

$$= 1 + \frac{1}{2}\left[\left(1-\frac{1}{2}\right) + \left(\frac{1}{2}-\frac{1}{3}\right) + \ldots\ldots + \left(\frac{1}{n}-\frac{1}{n+1}\right)\right]$$

$$= 1 + \frac{1}{2}\left(1-\frac{1}{n+1}\right)$$

$$I_2 = 1 + \frac{1}{2}\left(1-\frac{1}{3}\right) = \frac{4}{3} \quad \text{and} \quad I_\infty = \frac{3}{2}$$

494. (b,c) $\because \quad g'(x) = 2xf'\left(\frac{x^2}{2}\right) - 2x\,f'(6-x^2) = 2x\left[f'\left(\frac{x^2}{2}\right) - f'(6-x^2)\right]$

$\because \quad f''(x) > 0 \Rightarrow f'(x)$ is increasing function.

$\therefore \quad g(x)$ is increasing when $g'(x) > 0$

$\Rightarrow \quad 2x\left[f'\left(\frac{x^2}{2}\right) - f'(6-x^2)\right] > 0$

(i) If $x > 0$, then $f'\left(\frac{x^2}{2}\right) > f'(6-x^2) \Rightarrow \frac{x^2}{2} > 6 - x^2 \Rightarrow \frac{3x^2}{2} > 6$

$\Rightarrow x^2 > 4 \Rightarrow x > 2 \Rightarrow x \in (2, \infty)$

(ii) If $x < 0$, then $f'\left(\frac{x^2}{2}\right) < f'(6-x^2) \Rightarrow \frac{x^2}{2} < 6 - x^2 \Rightarrow x^2 < 4 \Rightarrow -2 < x < 2$

$\therefore \quad x \in (-2, 0)$

$\therefore \quad g$ increases for $x \in (-2, 0) \cup (2, \infty)$ and g decreases for $x \in (-\infty, -2) \cup (0, 2)$

495. (b,c,d)

(a) For linearly dependent $\frac{f(x)}{x^2} = \frac{2}{3} \Rightarrow 3f(x) - 2x^2 = 0$

Let $g(x) = 3f(x) - 2x^2$

$\therefore \quad g(0) = 3f(0) > 0$ and $g(1) = 3f(1) - 2$, may be positive or negative

(b) $\frac{f(x)}{x^2} = \frac{3}{2} \Rightarrow 2f(x) - 3x^2 = 0$

Let $g(x) = 2f(x) - 3x^2$

$g(0) = 2f(0) > 0$ and $g(1) = 2f(1) - 3 < 0$

$\therefore \quad \vec{a}$ and $\vec{b}$ are linearly dependent.

(c) $\frac{\int_0^{1-x} f(t)\,dt}{x} = \frac{3}{2} \Rightarrow 2\int_0^{1-x} f(t)\,dt - 3x = 0$

Let $g(x)=2\int_0^{1-x} f(t)\,dt-3x$

$\therefore \quad g(0)=2\int_0^1 f(t)\,dt>0$ and $g(1)=-3<0$

$\therefore \quad \vec{a}$ and $\vec{b}$ are linearly dependent.

Similarly for (d) also vectors are linearly dependent.

496. (a,b) Total matrices $=5^4$

For symmetric $=5^3$ and for skew-symmetric $=$ diagonals can be filled in one ways

$= 5$ matrices

1 matrix (*i.e.*, null matrix) is common.

$$\therefore \quad \text{Probability}=\frac{5^3+5-1}{5^4}=\frac{1}{5}+\frac{1}{5^3}-\frac{1}{5^4}=\frac{129}{625}=0.203$$

497. (b,d) Let $f(x)=e^{-x}(\sin^4 ax+\cos^2 x)$

$$\therefore \quad \int_0^{n\pi} f(x)dx-\int_0^{\pi} f(x)dx+\underbrace{\int_{\pi}^{2\pi} f(x)\,dx}_{x=\pi+t}+\underbrace{\int_{2\pi}^{3\pi} f(x)\,dx}_{x=2\pi+t}+\ldots\ldots+\underbrace{\int_{(n-1)\pi}^{n\pi} f(x)\,dx}_{x=(n-1)\pi+t}$$

$$=\int_0^{\pi} f(x)dx\,(1+e^{-\pi}+e^{-2\pi}+\ldots\ldots+e^{-(n-1)\pi})\text{ for } a\in I$$

$$\therefore \quad L=1+e^{-\pi}+e^{-2\pi}+\ldots\ldots+e^{-(n-1)\pi}\ \forall\ a\in I$$

$$\therefore \quad \lim_{n\to\infty} L=\frac{1}{1-e^{-\pi}}>1\ \forall\ a\in I$$

498. (a,d) Clearly L should be parallel to both the planes P_1 and P_2.

$\therefore \quad L$ is perpendicular to both normals $\vec{n}_1=\hat{i}+3\hat{j}-\hat{k}$ and $\vec{n}_2=3\hat{i}-\hat{j}+\hat{k}$

$$\therefore \quad \vec{n}_1\times\vec{n}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 3 & -1\\ 3 & -1 & 1\end{vmatrix}=2\hat{i}-4\hat{j}-10\hat{k}$$

$\therefore \quad$ Dr's of $L=(1,-2,-5)$

$\therefore \quad$ Equation of L is $\dfrac{x}{1}=\dfrac{y}{-2}=\dfrac{z}{-5}$

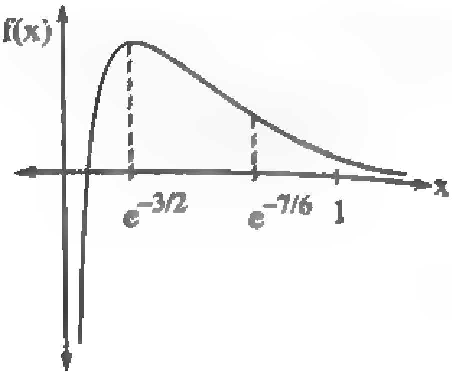
$\therefore \quad$ Point $(1,-2,-5)$ and $(-1,2,5)$ lie on L.

499. (a,b,c)

$$f(x)=\frac{2+\ln x}{x^2}$$

$$f'(x)=\frac{-2\ln x}{x^3}-\frac{3}{x^3}$$

$$f''(x)=\frac{6\ln x}{x^4}+\frac{7}{x^4}$$

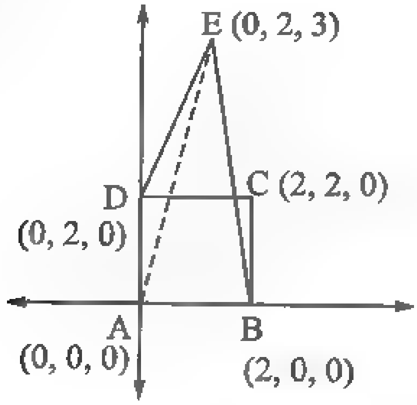


Now, verify the options.

500. (a,c) Plane ABE is $\begin{vmatrix} x & y & z \\ 0 & 2 & 3 \\ 2 & 0 & 0 \end{vmatrix}=0$

$$x(0)-y(-6)+z(-4)=0$$

$$\Rightarrow \quad 3y-2z=0$$



501. (a,b,d) Do yourself.

502. (b,c,d) $f(x)=-x\cos x$

503. (b,c,d) Do yourself

504. (b,c)

$$\left(\frac{x-y}{\sqrt{2}}\right)^2=2\sqrt{2}\left(\frac{x+y-2}{\sqrt{2}}\right)$$

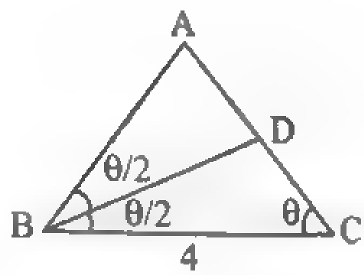
$Y^2=2\sqrt{2}X \Rightarrow 4a=2\sqrt{2} \Rightarrow$ Latus rectum.

Now, verify other options.

505. (a,d)

$$\frac{BD}{\sin\theta}=\frac{4}{\sin\frac{3\theta}{2}} \Rightarrow BD=\frac{4\sin\theta}{\sin\frac{3\theta}{2}} \quad \ldots (1)$$

$$\frac{BD}{\sin 2\theta}=\frac{AD}{\sin\frac{\theta}{2}}=\frac{4}{\sin 2\theta+\sin\frac{\theta}{2}} \quad \ldots (2)$$



From eqns. (1) and (2)

$$\frac{4\sin\theta}{\sin\frac{3\theta}{2}}=\frac{4\sin 2\theta}{\sin 2\theta+\sin\frac{\theta}{2}} \Rightarrow \sin 2\theta+\sin\frac{\theta}{2}=2\sin\frac{3\theta}{2}\cos\theta$$

$\Rightarrow \quad \sin 2\theta + \sin\frac{\theta}{2} = \sin\frac{5\theta}{2} + \sin\frac{\theta}{2}$

$\therefore \quad 2\theta + \frac{5\theta}{2} = \pi \quad \Rightarrow \quad \theta = \frac{2\pi}{9} = 40^\circ$

$\therefore \quad \angle A = 100^\circ$

Now, verify the options.

506. (a,c,d) Do yourself

507. (a,b,c,d) Given expression can be written as

$(a-2b)^2 + (2b-c-d)^2 + (c-d)^2 - 0$

$\therefore \quad a = 2b, 2b = c+d$ and $c = d$

$\therefore \quad b = c$

$\therefore \quad a = 2b = 2c = 2d$

$\therefore \quad$ Determinant $= ad - bc = 2d\cdot d - d\cdot d = d^2 = c^2 = b^2 = \frac{a^2}{4}$

508. (a,b,c,d)

$\because \quad P(3, 4)$ is foot of perpendicular from $S(0, 0)$ on $3x + 4y - 25 = 0$ which is also on ellipse

$\therefore \quad P$ is vertex of ellipse.

$\therefore \quad$ Distance between focus and directrix $= a - ae = 5$

$\Rightarrow \quad a - \frac{a}{2} = 5 \quad \Rightarrow \quad a = 10$

$\because \quad b^2 = a^2(1-e^2) = 100\left(1-\frac{1}{4}\right) - 75$

Clearly AB will be latus rectum of ellipse.

$\therefore \quad AB = \text{length of } LR = \frac{2b^2}{a} = 15$

$\therefore \quad$ Focal length $= 2ae = 10$

Mid-point of vertex and centre will focus.

$\therefore \quad$ centre $= (-3, -4)$

509. (a,b,c,d)

(a) Number of ways $= {}^6C_3 \times {}^4C_2 \times 5! \times 5! = (5!)^3$

(b) Number of ways $= {}^6C_1 \times 9! = 6 \times 9!$

(c) Number of ways $= (6+1)! \times 4! = 7!4!$

(d) Number of ways $= {}^{10}C_6 \times 1 \times 4! = {}^{10}C_4 \times 4! = {}^{10}P_4$

510. (a,b,c,d)

(a) Let $g(x) = e^x f(x) \quad \therefore \quad g(\alpha) = g(\beta) = 0$

According to Rolle's theorem, $g'(x) = e^x f(x) + e^x f'(x) = 0$

$\Rightarrow \quad f(x) + f'(x) = 0$ will have at least one real root.

(b) Let $h(x)=e^{-x}f(x)$ similarly from Rolle's theorem

$f(x)-f'(x)$ will have at least one real root.

(c) and (d) $f'(x)=0$ has at least one real root $\gamma\in(\alpha,\beta)$

$\therefore$ $f(x)f'(x)=0$ will have roots as α,γ,β

$\therefore$ Its derivative $(f'(x))^2+f(x)f''(x)=0$ will have at least two real roots.

511. (a,b,d) Let $C=(\lambda_1+1, 2\lambda_1+2, 3\lambda_1+3)$ and $D=(\lambda_2+1, 2\lambda_2+2, 3\lambda_2+3)$

$\therefore$ $CD=\sqrt{14}(\lambda_1-\lambda_2)=\sqrt{14}$ $\Rightarrow$ $\lambda_1=\lambda_2+1$

$\Rightarrow$ Let centroid $(\alpha,\beta,\gamma)=\left(\dfrac{5+2\lambda_2}{4},\dfrac{10+4\lambda_2}{4},\dfrac{9+6\lambda_2}{4}\right)$

$\therefore$ Locus is $\dfrac{4x-5}{2}=\dfrac{4y-10}{4}=\dfrac{4z-9}{6}$

$\therefore$ Dr's of line 1, 2, 3 and point $\left(\dfrac{3}{2},3,3\right)$ satisfies the line

$\therefore$ Locus is $\dfrac{x-\frac{3}{2}}{1}=\dfrac{y-3}{3}=\dfrac{z-3}{3}$

512. (a,b,d) $f(x)=x^3-x^2+x+1$ $\Rightarrow$ $f'(x)=3x^2-2x+1>0$, f is increasing.

$$\therefore \quad g(x)=\begin{cases} f(x)-x^3-x^2+x+1, & 0\le x\le 1\\ 3-x, & 1\le x\le 2\end{cases}$$

Clearly $g(x)$ is continuous for all $x\in[0,2]$

$$\text{and} \quad g'(x)=\begin{cases} 3x^3-2x+1, & 0<x<1\\ -1, & 1<x<2\end{cases}$$

$\therefore$ $g'(1^-)=2$ and $g'(1^+)=-1$

$\therefore$ g is non-derivable at $x=1$

513. (a,c)

$$f(x)=\sin^{-1}\left(\frac{2x}{1+x^2}\right)=\begin{cases} 2\tan^{1}x, & -1\le x\le 1\\ \pi-2\tan^{-1}x, & x\ge 1\\ -\pi-2\tan^{-1}x, & x\le -1\end{cases}$$

$$g(x)=\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)=\begin{cases} 2\tan^{-1}x, & x\ge 0\\ -2\tan^{-1}x, & x<0\end{cases}$$

$$h(x)=\tan^{-1}\left(\frac{2x}{1+x^2}\right)=\begin{cases} \pi+2\tan^{1}x, & x<-1\\ 2\tan^{-1}x, & -1<x<1\\ 2\tan^{-1}x-\pi, & x>1\end{cases}$$

514. (a,c) $\tan^{-1} x = t \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

and $f(x)$ is defined when $0 \le t < \frac{\pi}{2}$, where $f'(x) > 0$ so injective.

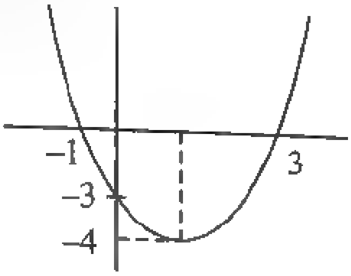
Also, at $t = 0 \to f(x) = \sqrt{0} + \sqrt{\pi} = \sqrt{\pi}$

at $t = \frac{\pi}{2} \to f(x) = \sqrt{\frac{\pi}{2}} + \sqrt{\frac{\pi}{2}} = 2\sqrt{\frac{\pi}{2}} = \sqrt{2\pi}$

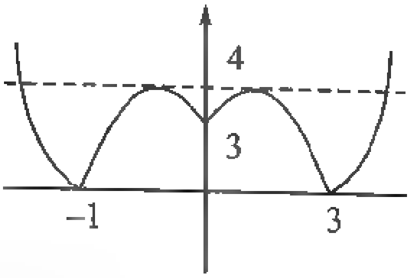
515. (b,c) $S_n = \tan^{-1}\left(\sum_{r=1}^{n} \frac{\sin(r-(r-1))}{\cos r \cdot \cos(r-1)}\right) = \tan^{-1}\left(\sum_{r=1}^{n} (\tan r - \tan(r-1))\right)$

$\therefore \quad S_n = \tan^{-1}(\tan(n))$

516. (b,d) $f(x)$:



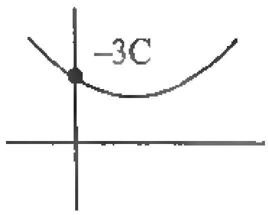
$\therefore \quad |f(|x|)|$.



517. (b,c,d) $T_n = \frac{2}{n(n+1)} = 2\left(\frac{1}{n} - \frac{1}{n+1}\right)$

$S_n = 2\left(1 - \frac{1}{n}\right) < 2$

518. (a,b,c) $4a + 4b - 3c > 0 \to f(2) > 0$



519. (a,c) Let $\sin x = t \in [-1, 1]$

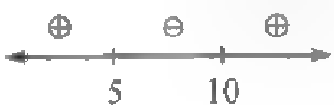
$\therefore \quad f(x) \ge 0 \quad \Rightarrow \quad k \ge t - t^2 \,\forall\, t \in [-1, 1] \to k \ge \frac{1}{4}$

and $f(x) \le 0 \quad \Rightarrow \quad k \le t - t^2 \,\forall\, t \in [-1, 1] \to k \le -2$

520. (a,b,c,d)

$\lim_{x\to\infty} \cot^{-1}(x) = 0$ and $\lim_{x\to-\infty} \cot^{-1} x = \pi$

and $(x-5)(x-10)$



521. (a,b,c) $f'(x)=3x^2+6x+(4-k)\geq 0\ \forall\ x\in R$

$\Rightarrow$ $D\leq 0 \Rightarrow 36-12(4-k)\leq 0 \Rightarrow k\leq 1$

$\therefore$ $k\in(-\infty,1]$

522. (a,b,c)
$$\max.\left\{x,\frac{1}{x}\right\}=\begin{cases}x, & x\in[1,\infty)\\ \frac{1}{x}, & x\in(0,1)\\ x, & x\in[-1,0)\\ \frac{1}{x}, & x\in(-\infty,-1)\end{cases}$$

and
$$\min.\left\{x,\frac{1}{x}\right\}=\begin{cases}\frac{1}{x}, & x\in[1,\infty)\\ x, & x\in(0,1)\\ \frac{1}{x}, & x\in[-1,0)\\ x, & x\in(-\infty,-1)\end{cases}$$

$\therefore$
$$f(x)=\begin{cases}x^2, & x\geq 1\\ \frac{1}{x^2}, & 0<x<1\\ x^2, & -1\leq x<0\\ \frac{1}{x^2}, & x<-1\\ 1, & x-0\end{cases}$$

$\therefore$ $\lim\limits_{x\to 0^+} f(x)\neq 0,\ \lim\limits_{x\to 0^-} f(x)=0$

$\therefore$ $f(x)$ is discontinuous at $x=0$.

523. (b,c) $h(x)=\int(\int(g''(x)+C)dx)dx=\int(g'(x)+Cx+d)dx=g(x)+Cx^2+dx+e$

$h(x)-g(x)=f(x)=Cx^2+dx+e$ has roots 1 and 3

$\therefore$ $f(x)=c(x-1)(x-3)$

$\therefore$ $f(0)=6$

$\therefore$ $c=2$

$\Rightarrow$ $f(x)=2(x-1)(x-3)=2(x^2-4x+3)$

$f'(x)=2(2x-4)$

$\therefore$ $f(x)$ is decreasing in $(-\infty,2)$.

$f(4)=6$ and $f(2)=-2$

524. (a,d) Clearly $P(x)$ is quadratic equation. Let $P(x)=ax^2+bx+c$

$\therefore$ $ax^2+bx+c+a(4x^2)+b(2x)+c=5x^2-18$

$\therefore$ $5a=5 \Rightarrow a=1$

$3b=0 \Rightarrow b=0$

$2c=-18 \Rightarrow c=-9$

$\therefore \quad P(x) = x^2 - 9$

Now, verify options.

525. (a,b,c,d) $f(x) = 2 + \frac{x^2}{2}\int_{-1}^{1} t\, f(t)\,dt + \frac{9x}{14}\int_{-1}^{1} f(t)\,dt$

Let $A = \frac{1}{2}\int_{-1}^{1} t\, f(t)\,dt$ and $B = \frac{9}{14}\int_{-1}^{1} f(t)\,dt$

$\therefore \quad f(x) = Ax^2 + Bx + C$

$\therefore \quad A = \frac{1}{2}\int_{-1}^{1} t(At^2 + Bt + 2)\,dt = \int_{0}^{1} Bt^2\,dt = \frac{B}{3} \quad \Rightarrow \quad 3A = B$

and $B = \frac{9}{14}\int_{-1}^{1} (At^2 + Bt + 2)\,dt = \frac{9}{14} \times 2\left(\frac{At^3}{3} + 2t\right)_0^1$

$\Rightarrow \quad 3A = \frac{9}{7}\left(\frac{A}{3} + 2\right) = \frac{3}{7}(A + 6)$

$\Rightarrow \quad 21A = 3A + 18 \quad \Rightarrow \quad A = 1$ and $B = 3$

$\therefore \quad f(x) = x^2 + 3x + 2$

Now, verify options.

526. (a,b,c) Clearly $g(x) = f^{-1}(x)$

$\therefore \quad g'(f(x)) = \frac{1}{f'(x)}$

$\Rightarrow \quad g'(3) = \frac{1}{f'(0)}$

$\therefore \quad f(x) = e^{2x} + 2(x+3)e^{2x} + 2e^{2x}\int_{0}^{x}\frac{dt}{\sqrt{t^6+1}} + e^{2x}\cdot\left(\frac{1}{\sqrt{x^6+1}}\right)$

$\therefore \quad f'(0) = 1 + 6 + 0 + 1 = 8$

$\therefore \quad g'(3) = \frac{1}{8}$

527. (a,c,d) If $f(x)$ has absolute minimum at $x = 1$, then $\lim\limits_{x \to 1^{-1}} f(x) \geq f(1)$

$\Rightarrow \quad a - 1 \geq 1 \quad \Rightarrow \quad a \geq 2$

If $f(x)$ has absolute maximum at $x = 3$, then $f(0) \leq f(3) \quad \Rightarrow \quad a \leq 3$

If $f(x)$ has absolute maximum at $x = 3$, then $f(0) \geq f(3) \quad \Rightarrow \quad a \geq 3$

528. (a,b) $\frac{d}{dx}(P(x)) + (x-1)^3 - (P(x)+1) \geq 0$

$\Rightarrow \quad e^{-x}\left(\frac{dP(x)}{dx} - P(x) + x^3 - 3x^2 + 3x - 2\right) \geq 0$

$$\Rightarrow \quad \frac{d}{dx}(P(x)e^{-x}) - \frac{d}{dx}e^{-x}x^3 - 3\frac{d}{dx}xe^{-x} - \frac{d}{dx}e^{-x} \geq 0$$

$$\Rightarrow \quad \frac{d}{dx}(P(x)-(x^3+3x+1))e^{-x} \geq 0$$

Let $g(x) = (P(x)-(x^3+3x+1))e^{-x}$ is increasing

$$g(x) \geq g(0) \quad \Rightarrow \quad (P(x)-(x^3+3x+1))e^{-x} \geq 0 \,\forall\, x \geq 0$$

but $\quad P(x) \leq x^3+3x+1 \,\forall\, x \geq 0$

$\Rightarrow \quad P(x) = x^3+3x+1 \,\forall\, x \geq 0$

529. (a,b,d)

(a) $\quad y = f(x)$ is not a constant function.

$\Rightarrow \quad f(x)$ is monotonic in some interval.

(b) $\quad f(-3) \geq -3$ and $f(0) \leq 3$

$$\Rightarrow \quad f'(c) = \frac{f(0)-f(-3)}{3} \leq 2 \qquad \exists\, c \in (-3, 0)$$

(d) $\quad f'(c_1) \leq 2 \,\exists\, c_1 \in (-3, 0)$

$f'(c_2) = 0 \,\exists\, c_2 \in (1, 3)$

$1 > c_2 - c_1 < 6$

$$\Rightarrow \quad f''(c) = \frac{f'(c_2) - f'(c_1)}{c_2 - c_1} \,\exists\, c \in (c_1, c_2) = \frac{-f'(c_1)}{c_2 - c_1} \geq -2$$

530. (a,b,c) $\quad f(x) = \frac{1}{2^2}\sec^2\frac{x}{2} + \frac{1}{2^4}\sec^2\frac{x}{2^2} + \frac{1}{2^6}\sec^2\frac{x}{2^3} + \ldots\ldots \infty$

Let $g(x) = \int f(x)\,dx$

$$= \frac{1}{2}\tan\frac{x}{2} + \frac{1}{2^2}\tan\frac{x}{2^2} + \frac{1}{2^3}\tan\frac{x}{2^3} + \ldots\ldots \infty + C$$

$$= \lim_{n\to\infty}\frac{1}{2^n}\cot\frac{x}{2^n} - \cot x + C$$

$$= \frac{1}{x} - \cot x + C \quad \Rightarrow \quad f(x) = \operatorname{cosec}^2 x - \frac{1}{x^2}$$

$$\Rightarrow \quad f'(x) = -2\operatorname{cosec}^2 x \cot x + \frac{2}{x^3}$$

531. (b,c) $\quad G_n = \left(\prod_{k=1}^{n}\sin\frac{k\pi}{2n}\right)^{1/n}$

$$\ln(G_n) = \lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\ln\left(\sin\frac{k\pi}{2n}\right)$$

$$= \int_0^1 \ln\left(\sin\frac{\pi x}{2}\right)dx$$

$$= \frac{2}{\pi}\int_0^{\pi/2} \ln(\sin\theta)\,d\theta = -\ln 2$$

$$\therefore \quad \lim_{n\to\infty} G_n = \frac{1}{2}$$

532. (a,b,c) Let $g(x) = f^{-1}(x) \Rightarrow g(x) = x^3 + x^2 + x + 1$

$$f'(x) = \frac{1}{g'(f(x))} \quad \text{and} \quad f''(x) = \frac{-g''(f(x))}{(g'(f(x)))^3}$$

$$\Rightarrow \quad f'(0) = \frac{1}{g'(f(0))} = \frac{1}{g'(-1)} = \frac{1}{2}$$

$$\Rightarrow \quad f''(0) = \frac{-g''(-1)}{\left(\frac{1}{2}\right)^3} = 32$$

$$I = \int_0^4 f(x)\,dx$$

put $x = g(t)$

$$I = \int_{-1}^{1} t(3t^2 + 2t + 1)\,dt = \frac{4}{3}$$

533. (a,b,c,d) $p^n = \begin{bmatrix} 1 & n & 0 \\ 0 & 1 & 0 \\ 0 & n & 1 \end{bmatrix}$ and $B^n = QP^nQ^T$

$\Rightarrow$ det. $(A) = 1$ and det. $(B) = 1$

534. (a,d) $[f(2) \quad f(1) \quad f(0)]\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = [2x + y + 2]$

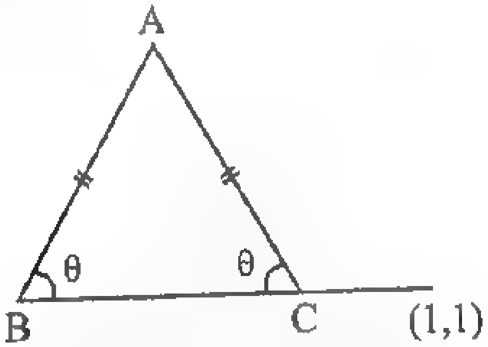
$\Rightarrow$ $(f(2) - 2)x + (f(1) - 1)y + (f(0) - 2) = 0 \ \forall\ x, y \in R$

$\Rightarrow$ $f(2) = 2,\ f(1) = 1$ and $f(0) = 2$

$\Rightarrow$ $f(x) = x^2 - 2x + 2$

$$\text{Area} = \left|\int_0^1 ((x^2 - 2x + 2) - (2 - x))\,dx\right| = \left|\int_0^1 (x^2 - x)\,dx\right| = \frac{1}{6}$$

535. (b) Slope of BC is -1 or 1.



536. (a,c) $|adj\,(adj\,(adj\,A))| = |A|^8 = 256 \Rightarrow |A| = 2$

$$adj\,(adj\,(adj\,(A))) = |adj\,A|\,(adj\,A) = 4\,(adj\,A) = \begin{bmatrix} 16 & 0 & 4 \\ 5 & 4 & 0 \\ 1 & 4 & 3 \end{bmatrix}$$

$$\Rightarrow \quad adj\,(A) = \begin{bmatrix} 4 & 0 & 1 \\ \frac{5}{4} & 1 & 0 \\ \frac{1}{4} & 1 & \frac{3}{4} \end{bmatrix} \quad \Rightarrow \quad A^{-1} = \begin{bmatrix} 2 & 0 & \frac{1}{2} \\ \frac{5}{8} & \frac{1}{2} & 0 \\ \frac{1}{8} & 1 & \frac{3}{8} \end{bmatrix}$$

537. (a,c) Equation of chord of contact QR from $P(at^2, 2at)$ will be $y \cdot 2at = -2a\,(x + at^2)$

$\Rightarrow \quad y(-2at) = 2a\,(x + at^2)$ which is tangent to parabola at $y^2 = 4ax$ at $(at^2, -2at)$

and normal at $A\,(t)$ on $S_1 \quad \Rightarrow \quad y + tx = 2at + at^3$

$$\Rightarrow \qquad y = -tx + 2at + at^3 \qquad \dots (1)$$

$$\text{Tangent at } S_2 \text{ will be}: y = mx - \frac{a}{m} \qquad \dots (2)$$

$\therefore \qquad m = -t \quad$ and $\quad 2at + at^3 = -\dfrac{a}{m}$

$\therefore \qquad 2at + at^3 = \dfrac{a}{t} \quad \Rightarrow \quad t^4 + 2t^2 = 1 \quad \Rightarrow \quad (t^2 + 1)^2 = 2$

$\Rightarrow \quad t^2 + 1 = \sqrt{2} \quad \Rightarrow \quad t^2 = \sqrt{2} - 1$

538. (a,d) Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ and $B = \begin{bmatrix} a_{12} + a_{13} & a_{11} + a_{13} & a_{11} + a_{12} \\ a_{22} + a_{23} & a_{21} + a_{23} & a_{21} + a_{22} \\ a_{32} + a_{33} & a_{31} + a_{33} & a_{31} + a_{32} \end{bmatrix}$

$\therefore \quad C = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

$\therefore \quad C$ is symmetric matrix.

$\because \quad |C| = 0 - 1(0-1) + 1(1-0) = 2$

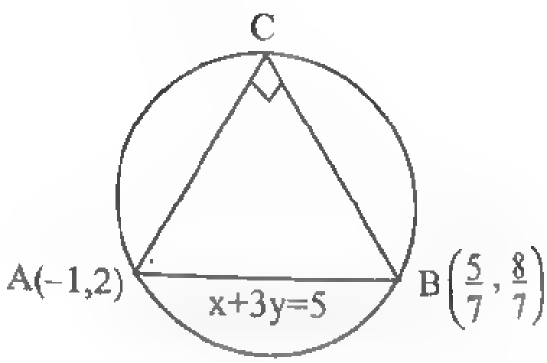
$\therefore \quad |B| = |C|\,|A| = 2|A|$

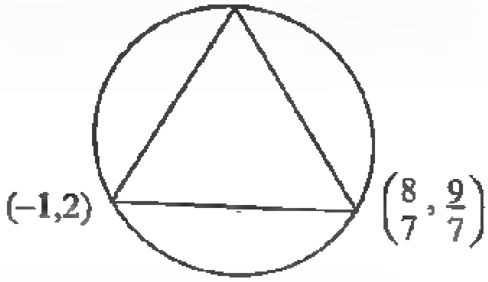
539. (a,c,d) (i) If $\dfrac{-1}{2} \times a = -1 \quad \Rightarrow \quad a = 2$

$\therefore \quad \text{Centre} = \left(\dfrac{-1}{7}, \dfrac{11}{7}\right)$

(ii) If $\dfrac{-1}{3} \times a = -1 \quad \Rightarrow \quad a = 3$

$\text{Centre} = \left(\dfrac{1}{14}, \dfrac{23}{14}\right)$





540. (a,c,d) $\because \quad f(x+y) = 2^x f(y) + 4^y f(x)$ ...(1)

Replace x by y and y by x, we get

$$f(y+x) = 2^y f(x) + 4^x f(y) \quad \text{...(2)}$$

$\therefore \quad 2^x f(y) + 4^y f(x) = 2^y f(x) + 4^x f(y)$

$$f(x)(4^y - 2^y) = (4^x - 2^x) f(y)$$

$\Rightarrow \quad \dfrac{f(x)}{4^x - 2^x} = \dfrac{f(y)}{4^y - 2^y} = k \quad \text{(let)}$

$\therefore \quad f(x) = k(4^x - 2^x)$

$\therefore \quad f'(x) = k(4^x \ln 4 - 2^x \ln 2)$

$f'(0) = k \ln 2 = \ln 2 \quad \Rightarrow \quad k = 1$

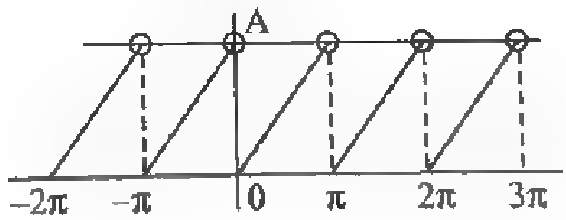
$\therefore \quad f(x) = 4^x - 2^x$

$\therefore \quad f(4) = 4^4 - 2^4 = 256 - 16 = 240$

$$f(x) = \left(2^x - \frac{1}{2}\right)^2 - \frac{1}{4}$$

$\therefore$ minimum value of $f(x)$ is $\dfrac{-1}{4}$.

541. (a,b,c,d) Graph of $y = f(x)$



Clearly, range of $f(x)$ is $[0, \pi)$

$$\int_0^{2\pi} f(x)\,dx = 2 \times \frac{1}{2} \times \pi \times \pi = \pi^2$$

542. (a,c) Let $k = \int_0^1 f(t)\,dt$

$\therefore \quad g(x) = x - k$

$\therefore \quad f(x) = \dfrac{x^3}{2} + 1 - x\int_0^x (t-k)\,dt = \dfrac{x^3}{2} + 1 - x\left(\dfrac{x^2}{2} - kx\right) = 1 + kx^2$

$\therefore \quad k = \int_0^1 f(t)\,dt = \left(t + \dfrac{kt^3}{3}\right)_0^1 \quad \Rightarrow \quad k = 1 + \dfrac{k}{3} \quad \Rightarrow \quad k = \dfrac{3}{2}$

$\therefore \quad f(x) = 1 + \dfrac{3x^2}{2}$

$\therefore \quad f(|x|) = 1 + \dfrac{3x^2}{2}$ which is always derivable.

543. (b,c,d) The required point of intersection of three planes

$$x - 2y + z - 1 = 0 \qquad \cdots (1)$$
$$x + 2y - 2z - 5 = 0 \qquad \cdots (2)$$
$$x + y - 2z - 7 = 0 \qquad \cdots (3)$$

Solving eqns. (1), (2) and (3), we get (1, 2, –4).

544. (b,c) $\because \quad ||z_1| - |z_2|| \le |z_1 + z_2|$

$$\Rightarrow \left| |z| - \frac{25}{|z|} \right| < 24 \quad \Rightarrow -24 \le |z| - \frac{25}{|z|} \le 24$$

(i) $|z|^2 - 24|z| - 25 \le 0$

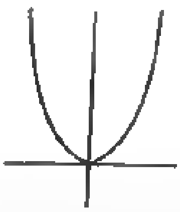
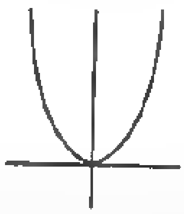
$\Rightarrow \quad (|z| - 25)(|z| + 1) \le 0 \quad \Rightarrow |z| \le 25$

(ii) $|z|^2 + 24|z| - 25 \ge 0 \rightarrow (|z| + 25)(|z| - 1) \ge 0 \rightarrow |z \ge 1$

$\therefore \quad 1 \le |z| \le 25$

545. (a,d) $A = \begin{bmatrix} 0 & 1 & 1 \\ x & 0 & 1 \\ 2x & 4x & 0 \end{bmatrix}, \quad f(x) = 4x^2 + 2x$

(a) $\displaystyle\int_{-1}^{1} f(x)\,dx = \int_{-1}^{1} (4x^2 + 2x)\,dx = \frac{4}{3} \times 2 \times 1 = \frac{8}{3}$

(b)  (c) 

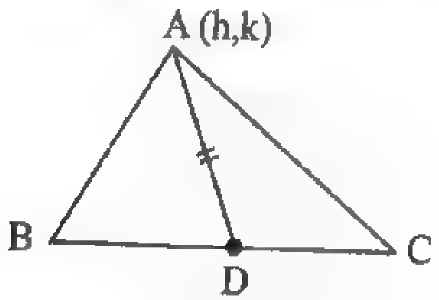
(d) $\displaystyle\int_0^1 \frac{dx}{(2x+1)^2} = -\left[\frac{1}{(2x+1)\cdot 2}\right]_0^1 = -\left[\frac{1}{6} - \frac{1}{2}\right] = \frac{1}{3}$

546. (a,b,c)

(a) $b^2 + c^2 = 2a^2 \quad \Rightarrow \quad 2b^2 + 2c^2 - a^2 = 3a^2$

$L = \dfrac{1}{2}\sqrt{2b^2 + 2c^2 - a^2} = \dfrac{1}{2}\sqrt{3a^2} = \sqrt{3}$

(b) AD is constant, then locus of A is circle with centre D and radius AD = 'L'



(c) $\cos\theta = \frac{b^2+c^2-a^2}{2bc}$ = positive

(d) Let $\cot B$, $\cot A$, $\cot C$ A.P.

$$\frac{\frac{2R(c^2+a^2-b^2)}{2abc}}{b^2}, \quad \frac{\frac{2R(b^2+c^2-a^2)}{2abc}}{a^2}, \quad \frac{\frac{2R(a^2+b^2-c^2)}{2abc}}{c^2} \rightarrow \text{A.P.}$$

547. (b,c,d) $3I = 5A^2 - A^3$

$A^3 - 5A^2 + 3I = 0$

tr. $(A) = 5 = a+3, \; a = 2$

$|A| = -3, 4+2c+3b-1-6c-4b = -3, b+4c = 6$

and $a+2+2a-2b-1-3c = 0$, $2b+3c = 7$, $2b+8c = 12$, $c = 1, \; b = 2$

548. (a,b,c,d)

(a) L.M.V.T. in [0, 4] $\Rightarrow$ $f'(C_1) = \frac{2-0}{4-0} = \frac{1}{2}$

(b) Rolle's in [4, 8] $\Rightarrow$ $f'(C_2) = 0$ and $\frac{1}{10}$ lie between 0 and $\frac{1}{2}$

(c) From I.V.T. :

$f(C_2) = \frac{1}{4}, C_2 \; (0,4)$ and from (A) $f'(C_1) = \frac{1}{2}$

$f'(C_1) \cdot f(C_2) = \frac{1}{8}$

(d) Let $g(x) = \int_0^{x^3} f(t)\,dt \quad \Rightarrow \quad g'(x) = 3x^2 f(x^3)$

Using L.M.V.T. in [0, 1] and [1, 2]

$g'(C_1) = \frac{g(1)-g(0)}{1-0}$ and $g'(C_2) = \frac{g(2)-g(1)}{2-1}$

$C_1 \in (0,1)$ and $C_2 \in (1,2)$

$g'(C_1) + g'(C_2) = g(2) - g(0)$

$3(C_1^2 f(C_1^3) + C_2^2 f(C_2^3)) = \int_0^8 f(t)\,dt$

549. (a,b) $\Delta = 0, \quad a = 5$

$$\int_0^{-10} f(x)\,dx = \int_0^{-5} f(x)\,dx + \int_{-5}^{-10} f(x)\,dx = \int_{-5}^{-10} f(x+5)\,dx + \int_{-5}^{-10} f(x)\,dx = 2\int_{-5}^{-10} dx = -10$$

550. (b,d)

(a) $f(x) = \int_0^x t\,g'(t)\,dt$

$f'(x) = xg'(x) \leq 0 \qquad \forall \; x \geq 0$

(b) $f(x)$ is differentiable $\Rightarrow$ $f(x)$ is continuous.

(c) $f(x)=\int_0^x t\,g'(t)\,dt = x\,g(x)-\int_0^x g(t)\,dt \quad \forall\, x\ge 0$

(d) $f'(x)=xg'(x) \quad \forall\; x>0$

(a) and (c) are correct for $x\ge 0$.

551. (b,d)

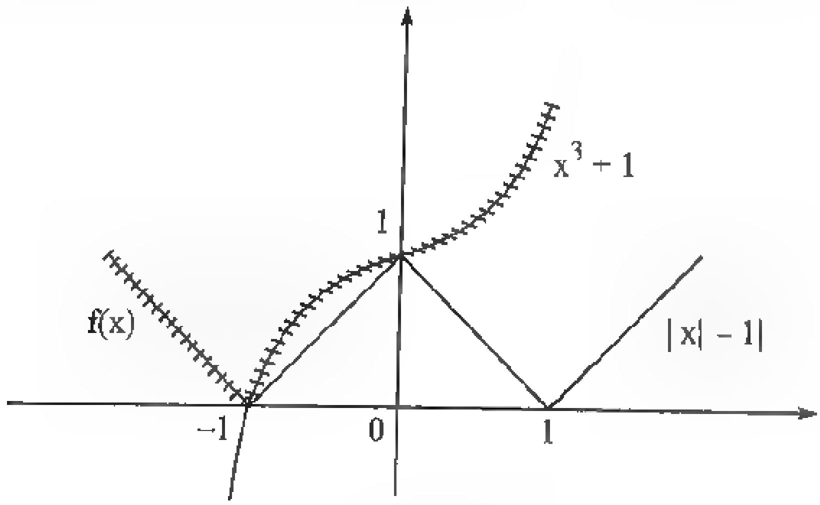
$(A^2-2I)B=0 \;\Rightarrow\; A^2=2I$ and $B=adj\ A=|A|\cdot\dfrac{A}{2}=-\sqrt{2}\,A \qquad \{\because\ |A|=-2\sqrt{2}\}$

$AB=A\,(adj\ A)=|A|\cdot I$

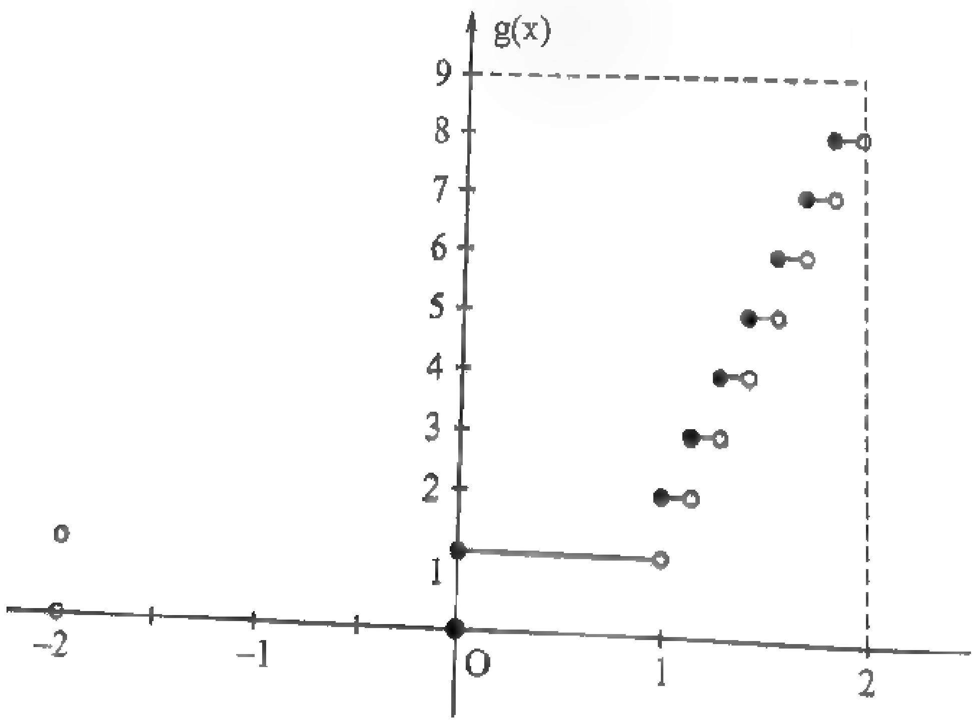
tr. $(AB)=3|A|=-6\sqrt{2}$

and det. $(A-\sqrt{2}B)-$ det. $(A+2A)=$ det. $(3A)=27(-2\sqrt{2})=-54\sqrt{2}$

552. (a,d) Clearly, $f(x)$ is continuous everywhere but non-derivable at $x=-1$ only.



$g(x)$ is discontinuous and non-derivable at exactly 8 points.



553. (a,c,d) $f''(x) \geq 0 \Rightarrow f(x)$ is concave upwards

and $f'(x)$ is increasing

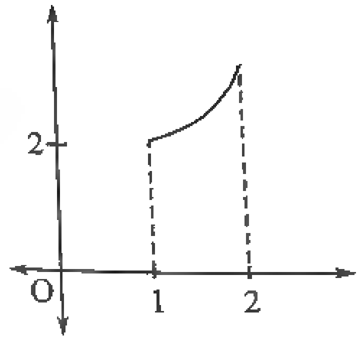
$$f'(2) \leq 1 \Rightarrow f'(x) \leq 1 \; \forall \; x \in [1, 2]$$

L.M.V.T. for $f(x)$ in $[1, x]$ where $x \in (1, 2]$

$$\frac{f(x)-f(1)}{x-1} = f'(c)$$

$$\Rightarrow \quad f(x)-2 \leq (x-1)$$

$$f(x) \leq x+1 \; \forall \; x[1, 2] \quad \Rightarrow \text{(a)}$$



Again, $\dfrac{f(2)-f(1)}{2-1} \leq f'(2) \Rightarrow f(2)-2 \leq f'(2)$

$$\Rightarrow \quad f'(2)-f(2) \geq -2 \quad \Rightarrow \text{(c)}$$

$$f(x) \leq x^2+1 \quad \Rightarrow \quad e^{f(x)} \leq e^{x^2+1}$$

$$\int_1^2 e^{f(x)} dx \leq \int_1^2 e^{x^2+1} dx \quad \Rightarrow \text{(d)}$$

554. (b,c,d) $\displaystyle\int \frac{(\sec^2 x-1)\tan x}{\tan^2 x+2} dx = \int \left(\frac{\sec^2 x \tan x}{\sec^2 x+1} - \frac{\tan x}{\sec^2 x+1}\right) dx$

Put $\sec x = t \Rightarrow \sec x \tan x \, dx = dt$

$$\int \left(\frac{t}{t^2+1} - \frac{1}{t(t^2+1)}\right) dt = \int \left(\frac{t}{t^2+1} - \left(\frac{1}{t} - \frac{t}{t^2+1}\right)\right) dt = \int \left(\frac{2t}{t^2+1} - \frac{1}{t}\right) dt$$

$$= \ln(t^2+1) - \ln t + C = \ln\left(\frac{\sec^2 x+1}{\sec x}\right) + C = \ln\left(\frac{1+\cos^2 x}{\cos x}\right) + C = \ln\left(\frac{2-\sin^2 x}{\cos x}\right) + C$$

$g(x) = \sin^2 x$

Now, verify the options.

555. (a,b) $\displaystyle P(t) = \lim_{n\to\infty} \sum_{r=2}^{n} \left(\frac{\sqrt{t^{2r-3}} - \sqrt{t^{2r-1}}}{(\sqrt{t^{2r-1}}+1)(\sqrt{t^{2r-3}}+1)}\right) = \lim_{n\to\infty} \sum_{r=2}^{n} \frac{(\sqrt{t^{2r-3}}+1)-(\sqrt{t^{2r-1}}+1)}{(\sqrt{t^{2r-1}}+1)(\sqrt{t^{2r-3}}+1)}$

$$= \lim_{n\to\infty} \sum_{r=2}^{n} \left(\frac{1}{\sqrt{t^{2r-1}}+1} - \frac{1}{\sqrt{t^{2r-3}}+1}\right) = \lim_{n\to\infty} \left(\frac{1}{\sqrt{t^{2n-1}}+1} - \frac{1}{\sqrt{t}+1}\right)$$

For $t \in (0, 1)$

$$P(t) = 1 - \frac{1}{\sqrt{t}+1} = \frac{\sqrt{t}}{\sqrt{t}+1}$$

$$\therefore \quad P\left(\frac{1}{2}\right) = \sqrt{2}-1 \cong 0.414$$

556. (b,c,d) $8b^3 - a^3 - c^3 = 6abc$

$$a^3 + (-2b)^3 + c^3 = 3a(-2b)c$$

$\Rightarrow \quad a-2b+c=0 \Rightarrow a, b, c$ are in A.P.

$b=a^2-4=c-2$

$\Rightarrow \quad a=3, b=5, c=7$

Now, verify the options.

557. (a,d) $\quad I_1=\frac{\pi}{4}-\frac{1}{2}\ln 2$

$I_2=\frac{\pi}{4}+\frac{1}{2}\ln 2$

$I_3=\ln 2$

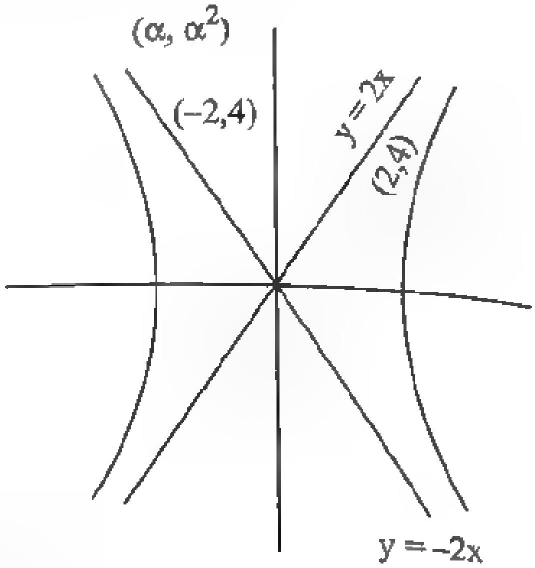
558. (a,b,d)

Solving $y=x^2$ and $y=2x$

$x^2=2x$

$x=2$

$x>2$ or $x<-2 \Rightarrow$ a,b,d



559. (a,c)

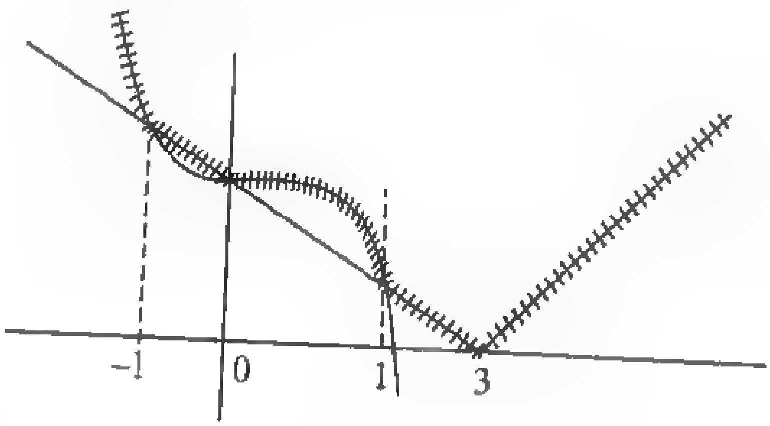
(a) and (b) probability $=\frac{1\times D_5}{6!}=\frac{44}{720}=\frac{11}{180}$

(c) and (d) $\quad$ Dearrangement of S_2, S_4, S_6 at $R_2, R_4, R_6=D_3=2$

Dearrangement of S_1, S_3, S_5 at $R_1, R_3, R_5=D_3=2$

$\therefore \quad$ probability $=\frac{2\times 2}{6!}=\frac{1}{180}$

560. (a,d) $\quad$ Clearly $f(x)$ is non-derivable at –1, 0, 1, 3



561. (a,c) $\quad \because \quad \int_0^{\pi} f(x)\,dx+\int_0^{\pi} f^{-1}(x)\,dx=\pi^2$

and $\int_0^{\pi} f(x)\,dx=\int_0^{\pi}(x+\sin x)\,dx=\left(\frac{x^2}{2}-\cos x\right)_0^{\pi}=\frac{\pi^2}{2}+2$

$\therefore \quad \left(\frac{\pi^2}{2}+2\right)+\int_0^{\pi} f^{-1}(x)\,dx=\pi^2$

$\therefore \quad I = \frac{\pi^2}{2} - 2$

$\therefore \quad 2 < I < 3$

$\because \quad \int_0^1 \frac{1}{1+x^3}\,dx < \int_0^1 1\,dx < 1 < I$

562. (c,d) Let image of $\left(t, \frac{1}{t}\right)$ in line $2x - y = 0$ be (h, k).

$$\therefore \quad \frac{h-t}{2} = \frac{k-\frac{1}{t}}{-1} = \frac{-2\left(2t-1\frac{1}{t}\right)}{5}$$

$$\Rightarrow \quad 5h = 3t + \frac{4}{t} \quad \text{and} \quad 5k = 4t + \frac{3}{t}$$

Eliminating t, we get

$$(3h-4k)(4h+3k) = -25$$

$$\Rightarrow \quad 12x^2 - 7xy - 12y^2 + 25 = 0$$

$$r = -7, \ s = -12, \ t = 25$$

563. (a,b) $Y = \text{surface area} = 2\left(\frac{|\vec{b}\times\vec{c}|}{[\vec{a}\ \vec{b}\ \vec{c}]} + \frac{|\vec{c}\times\vec{a}|}{[\vec{a}\ \vec{b}\ \vec{c}]} + \frac{|\vec{a}\times\vec{b}|}{[\vec{a}\ \vec{b}\ \vec{c}]}\right)$

$$= 2\left(\frac{1}{|\vec{a}|\cos\alpha} + \frac{1}{|\vec{b}|\cos\alpha} + \frac{1}{|\vec{c}|\cos\alpha}\right)$$

$$\therefore \quad \frac{2}{\cos\alpha} = 4 \Rightarrow \cos\alpha = \frac{1}{2}$$

564. (a,b) $4a^2 + 4a + 1 = 3b^2 + 3a^2$

$$\frac{(2a + 0\cdot b + 1)^2}{\sqrt{a^2+b^2}} = \sqrt{3}^2$$

Centre of the circle is (2, 0), radius is $\sqrt{3}$.

565. (a,c,d) $A_1 = \frac{2a+b}{3}$; $A_2 = \frac{a+2b}{3}$

$G_1 = a^{2/3}b^{1/3}$; $G_2 = a^{1/3}b^{2/3}$

$H_1 = \frac{3ab}{a+2b}$; $H_2 = \frac{3ab}{2a+b}$

$$\therefore \quad A_1H_2 = \frac{2a+b}{3}\cdot\frac{3ab}{2a+b} = ab$$

$$\therefore \quad G_1G_2 = ab$$

$$\therefore \quad A_2H_1 = \frac{a+2b}{3}\cdot\frac{3ab}{a+2b} = ab$$

566. (a,c) $2(\sin x+1)^2=\sin x(6)(\sin x+1)$

$$2\sin^2 x+4\sin x+2=6\sin^2 x+6\sin x$$

$$\Rightarrow \quad 4\sin^2 x+2\sin x-2=0$$

$$\Rightarrow \quad 2\sin^2 x+\sin x-1=0$$

$$\Rightarrow \quad (2\sin x-1)(\sin x+1)=0$$

$$\Rightarrow \quad \sin x=\frac{1}{2} \quad ; \qquad \sin x=-1 \quad \text{(rejected)}$$

$$\Rightarrow \quad x=\frac{\pi}{6}$$

G.P. $\frac{1}{2},\ \sqrt{2}\left(\frac{1}{2}+1\right),\ 6\left(\frac{1}{2}+1\right),\ \ldots\ldots$

$$\rightarrow \quad \frac{1}{2},\frac{3}{\sqrt{2}},9,\ldots\ldots\ldots\ldots$$

$$T_5=ar^4=\frac{1}{2}(3\sqrt{2})^4=81\times 2=162$$

$$S_n=\frac{\frac{1}{2}[(3\sqrt{2})^n-1]}{3\sqrt{2}-1}$$

567. (a,b)

(a) $$a_7=\Delta(0)=\begin{vmatrix} 0 & -1 & 3 \\ 1 & 2 & -3 \\ -3 & 4 & 0 \end{vmatrix}=21$$

(b) and (d) $$\Delta(1)=\begin{vmatrix} 2 & 1 & 4 \\ 4 & 3 & -2 \\ -2 & 5 & 2 \end{vmatrix}=132=\sum_{k=0}^{7} a_k$$

Now, $$\sum_{k=0}^{6} a_k=\sum_{k=0}^{7} a_k-a_7=132-21=111$$

(c) $$\Delta(-1)=\begin{vmatrix} 0 & -3 & 2 \\ -2 & 3 & -4 \\ -4 & 5 & -2 \end{vmatrix}=-32$$

568. (a,d) $\cos A+\cos B=4\sin^2\frac{C}{2}$

$$\Rightarrow \quad 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}=4\sin^2\frac{C}{2}$$

$$\Rightarrow \quad 2\sin\frac{C}{2}\cos\frac{A-B}{2}=4\sin^2\frac{C}{2}$$

$\Rightarrow \quad \cos\frac{A-B}{2} = 2\sin\frac{C}{2}$ ...(1)

$\Rightarrow \quad 2\sin\frac{A+B}{2}\cos\frac{A-B}{2} = 2\sin\frac{A+B}{2}\cdot 2\sin\frac{C}{2}$

$\Rightarrow \quad \sin A + \sin B = 2\cos\frac{C}{2}\cdot 2\sin\frac{C}{2}$

$\Rightarrow \quad \sin A + \sin B = 2\sin C \quad ; \quad \Rightarrow \quad a+b = 2c$ ....

Clearly option (b) is wrong.

Now, for options (c) and (d) : $\because \; a+b=2c$

$\therefore \quad a, \quad c, \quad b, \quad$ in A.P.

$\Rightarrow \quad -a, \quad -c, \quad -b \quad$ in A.P.

$\Rightarrow \quad s-a, \quad s-c, \quad s-b \quad$ in A.P.

$\Rightarrow \quad s(s-a), \quad s(s-c), \quad s(s-b) \quad$ in A.P.

$\Rightarrow \quad \frac{s(s-a)}{\Delta}, \quad \frac{s(s-c)}{\Delta}, \quad \frac{s(s-b)}{\Delta} \quad$ in A.P.

where $\quad s = \frac{a+b+c}{2} \quad$ and $\quad \Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$\Rightarrow \quad \sqrt{\frac{s(s-a)}{(s-b)(s-c)}}, \quad \sqrt{\frac{s(s-c)}{(s-b)(s-c)}}, \quad \sqrt{\frac{s(s-b)}{(s-a)(s-c)}} \quad$ in A.P.

$\Rightarrow \quad \cot\frac{A}{2}, \quad \cot\frac{C}{2}, \quad \cot\frac{B}{2} \quad$ in A.P.

$\Rightarrow \quad \tan\frac{A}{2}, \quad \tan\frac{C}{2}, \quad \tan\frac{B}{2} \quad$ in H.P.

569. (b,c) $\quad S_1P = S_2P \quad \Rightarrow \quad a - e\alpha = E\alpha - \frac{a}{2}$

Also, $\quad \alpha = \frac{ae + \frac{aE}{2}}{2}$

Eliminating α, we get $E^2 + 3eE + (2e^2 - 6) = 0 \quad \Rightarrow \quad E = \frac{\sqrt{e^2+24} - 3e}{2}$

570. (a,d)

$E_1 = \{(2,2),(2,3),(3,2),(2,5),(5,2),(3,5),(5,3),(3,3),(5,5)\}$

$E_2 = \{(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)\}$

$E_3 = \{(1,3),(3,1),(2,2)\}$

Now, $\quad P(E_1) = \frac{9}{36} \quad ; \quad P(E_2) = \frac{6}{36} \quad ; \quad P(E_3) = \frac{3}{36}$

Clearly $\quad P(E_1), P(E_2), P(E_3)$ are in A.P. $\quad \Rightarrow \quad$ (a) is correct.

$P(E_1 \cap E_2) = \frac{3}{36} = \frac{1}{12} \neq P(E_1)P(E_2) \Rightarrow \quad$ (b) is incorrect.

Now, $P(E_3 / E_1) = \dfrac{P(E_3 \cap E_1)}{P(E_1)} = \dfrac{1}{9}$ $\Rightarrow$ (c) is incorrect.

Also, $P(E_1 + E_2) + P(E_2 - E_3) = [P(E_1) + P(E_2) - P(E_1 \cap E_2)] + [P(E_2) - P(E_2 \cap E_3)]$

$$= \frac{9}{36} + \frac{12}{36} - \frac{3}{36} - \frac{1}{36} = \frac{17}{36} \quad \Rightarrow \quad \text{(d) is correct.}$$

571. (a,b,c,d)

Three planes meet at two points it means they have infinitely many solutions, so

$$\begin{vmatrix} 2 & 1 & 1 \\ 1 & -1 & 1 \\ \alpha & -1 & 3 \end{vmatrix} = 0$$

$\Rightarrow \quad 2(-3+1) - 1(3+1) + \alpha(1+1) = 0 \quad \Rightarrow \quad \alpha = 4$

$P_1 : 2x + y + z = 1$

$P_2 : x - y + z - 2$

$P_3 : 4x - y + 3z = 5$

P on XOY plane $= (1, -1, 0)$

(which can be obtained by putting $z = 0$ in any two of the given planes.)

Q on YOZ plane $\equiv \left(0, \dfrac{-1}{2}, \dfrac{3}{2}\right)$

(which can be obtained by putting $x - 0$ in any two of the given planes.)

$\therefore$ Straight line perpendicular to plane P_3 passing through P is :

$$\frac{x-1}{4} = \frac{y+1}{-1} = \frac{z}{3}$$

$$\vec{\mathbf{PQ}} = \hat{\mathbf{i}} - \frac{1}{2}\hat{\mathbf{j}} - \frac{3}{2}\hat{\mathbf{k}}$$

Projection of $\vec{\mathbf{PQ}}$ on x-axis $\Rightarrow \left|\dfrac{\vec{\mathbf{OP}} \cdot \hat{\mathbf{i}}}{|\hat{\mathbf{i}}|}\right| = 1$

Centroid of ΔOPQ is $\left(\dfrac{1}{3}, \dfrac{-1}{2}, \dfrac{1}{2}\right)$.

572. (a,b,c,d) $|z_1 z_2| = \left|\dfrac{c}{a}\right| = 1$, $\qquad |z_1 + z_2| = \left|\dfrac{-b}{a}\right| = 1$

$\Rightarrow \quad (z_1 + z_2) \times (\bar{z}_1 + \bar{z}_2) = 1 \quad \Rightarrow \quad (z_1 + z_2)\left(\dfrac{1}{z_2} + \dfrac{1}{z_2}\right) = 1$

$\Rightarrow \quad (z_1 + z_2)^2 = z_1 z_2$

$$\Rightarrow \quad \left(\frac{-b}{a}\right)^2 = \frac{c}{a} \qquad \Rightarrow \qquad b^2 = ac$$

$$|z_1 + z_2| = |z_1||1 + e^{i\theta}| = 2\cos\frac{\theta}{2} = 1 \quad \Rightarrow \quad \theta = \frac{2\pi}{3}$$

$$PQ = |z_2 - z_1| = |z_1||e^{i\theta} - 1| = 2\sin\frac{\theta}{2}$$

$$= 2\sin\frac{\pi}{3} = \sqrt{3}$$

573. (a,b)

(a) Given that $AB = O$, where det. $(A) \neq 0$ ... (1)

So, A^{-1} exists.

Now, pre-multiplying equation (1) with A^{-1}, we get

$$(A^{-1}A)B = A^{-1}O \Rightarrow B = O_{\text{null matrix}}$$

(b) Given, det. $(A) = 2$, det. $(B) = 3$, det. $(C) = 4$

So, det. $(3ABC) = 3^2$ det. (A) det. (B) det. $(C) = 9\ (2)\ (3)\ (4) = 216$

(As, A, B, C are square matrices of order 2.)

(c) Given, det. $(A) = \frac{1}{2}$ (order of matrix A is 3)

As, det. $(adj. A) = (\text{det. } A)^{n-1}$ ... (1)

place A by A^{-1} in equation (1) and take $n = 3$, we get

$$\det(adj.\ A^{-1}) = |A^{-1}|^2 = \frac{1}{|A|^2} = \frac{1}{\left(\frac{1}{2}\right)^2} = 4$$

(d) We know that skew symmetric matrix of odd order is singular. But, if order of skew symmetric matrix is even, then it need not be singular. For example,

$A = \begin{bmatrix} 0 & -4 \\ 4 & 0 \end{bmatrix}$ and det. $A = 16$ (non - singular).

574. (a,b,c,d) Centre of circle = (4, –5)

$$\text{Radius} = \sqrt{|-4+5i|^2 - (-40)} = \sqrt{4^2 + 5^2 + 40} = 9$$

Distance of centre (4, –5) from (–2, 3) is 10.

$\therefore \quad a = \max. |z - (-2+3i)| = 9 + 10 = 19$

$b = \min. |z - (-2+3i)| = 10 - 9 = 1$

Now, verify.

575. (a,d) Eccentricity of ellipse $= \frac{1}{\sqrt{2}}$

Let equation of ellipse be $\frac{x^2}{a^2} + \frac{y^2}{a^2\left(1-\frac{1}{2}\right)} = 1$ ...(1)

$$x^2 - y^2 = 2 \quad \text{...(2)}$$

As eqns. (1) and (2) intersect orthogonally, so

$$\left.\frac{dy}{dx}\right]_{(1)} \times \left.\frac{dy}{dx}\right]_{(2)} = -1 \text{ at point of intersection.}$$

Paragraph Type Questions

Solutions of Paragraph for question nos. 576 → (b) and 577 → (c)

$$(x-p)^2 + (y-q)^2 = x^2 + y^2 = r^2$$

$$\therefore \quad p^2 + q^2 = 2px + 2qy \quad \text{...(1)}$$

For the circle to intersect orthogonally

$$\left.\frac{dy}{dx}\right|^{C_1} \cdot \left.\frac{dy}{dx}\right|^{C_2} = -1$$

$$\Rightarrow \quad \left(\frac{-x}{y}\right)\left(\frac{-(x-p)}{y-q}\right) = -1$$

$$x^2 - px = qy - y^2 \Rightarrow x^2 + y^2 - px - qy = 0 \quad \text{...(2)}$$

From eqns. (1) and (2) $\Rightarrow$ $p^2 + q^2 = 2(x^2 + y^2) = 2r^2$ ...(3)

Now, $q = a \Rightarrow$ differential $2p + 0 = 4r\frac{dr}{dp}$

$$\therefore \quad \frac{dr}{dp} = \frac{p}{2r}$$

Now, $p + bq = 0 \Rightarrow$ put $p = -bq$ in eqn. (3)

$$(b^2+1)q^2 = 2r^2$$

$$(b^2+1)2q = 4r\frac{dr}{dq}$$

$$\therefore \quad \frac{dr}{dq} = \frac{(b^2+1)q}{2r}$$

Solutions of Paragraph for question nos. 578 → (c) and 579 → (b)

$f(x)=e^x(2x-1)-ax+a$

$f(0)=a-1<0$ and $f(1)=e>0$

Also, $f'(x)=e^x(2x+1)-a$

$f(x)\uparrow \forall x>1$ and $\downarrow \forall x<-1$

Now, $f(x_0)<0$ for only one $x_0 \in I$

$f(-1)\geq 0$

$$\frac{-3}{e}+2a\geq 0 \quad \Rightarrow \quad a\geq \frac{3}{2e}$$

$$\therefore \quad a\in\left[\frac{3}{2e},1\right)$$

$\Rightarrow \quad p+q+r=6$

Solutions of Paragraph for question nos. 580 → (a), 581 → (a) and 582 → (b)

$$f(x)-e^x\int_0^1 e^t f(t)dt=Ae^x$$

$$g(x)=\left(e^x+\int_0^1 e^t g(t)\,dt\right)+x=Be^x+x$$

$$A=\int_0^1 e^t Ae^t dt=\frac{A}{2}(e^2-1)$$

$$\therefore \quad A=0 \quad \Rightarrow \quad f(x)=0$$

$$B=\int_0^1 e^t(Be^t+t)dt=\frac{B}{2}(e^2-1)+1$$

$$B\left(1-\frac{e^2-1}{2}\right)=1$$

$$B=\frac{2}{3-e^2}$$

$$g(x)=\frac{2}{3-e^2}e^x+x$$

Solutions of Paragraph for question nos. 583 → (a) and 584 → (b)

Case-I : $\dfrac{\log(m_1+m_2)}{\log x}=2$

$$\therefore \quad m_1+m_2=x^2$$

$$f'(x)+\frac{f(x)}{x}=x^2$$

$$\text{I.F.} = e^{\ln x} = x$$

$$x\,f(x) = \frac{x^4}{4} + C$$

At $(1, 0)$ $\Rightarrow$ $C = \frac{-1}{4}$

$$f_1(x) = \frac{x^3}{4} - \frac{1}{4x} \quad \ldots(1)$$

Case-II : $\frac{\log(m_1 + m_2)}{\log x} = -2$

$\therefore$ $$m_1 + m_2 = \frac{1}{x^2}$$

$$f'(x) + \frac{f(x)}{x} = \frac{1}{x^2}$$

I.F. $= x$

$$x\,f(x) = \ln x + C$$

At $(1, 0)$ $\Rightarrow$ $C = 0$

$$f_2(x) = \frac{\ln x}{x}$$

Solutions of Paragraph for question nos. 585 → (b) and 586 → (b)

$$f(x) = \frac{\pi^2}{4} + \frac{\pi^2}{12}\underbrace{(x^2 + 6x + 8)}_{(x+3)^2 - 1} = \frac{\pi^2}{6} + \frac{\pi^2}{12}(x+3)^2$$

Using calculus, $(x+3)^2$ is increasing in $[-2, 2]$.

Hence, $$b\pi^2 = f(x)_{\max.} = f(2) = \frac{\pi^2}{6} + \frac{25\pi^2}{12} = \frac{9\pi^2}{4}$$

$$a\pi^2 = f(x)_{\min.} = f(-2) = \frac{\pi^2}{6} + \frac{\pi^2}{12} = \frac{\pi^2}{4}$$

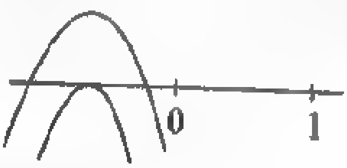
Hence, $$2\left(\frac{1}{6} + \frac{25}{12} + \frac{1}{6} + \frac{1}{12}\right) = 5$$

Solutions of Paragraph for question nos. 587 → (b) and 588 → (a)

(i) $c > 0$ not possible, think!

If $c < 0$, then for $g(x) < 0 \; \forall \; x \in (0, 1)$, vertex is negative.

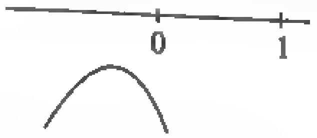
$\therefore$



$D \geq 0$, $g(0) \leq 0$ and $\frac{-b}{2a} \leq 0$

Solving we get $c=-1$.

or



$D<0 \Rightarrow c<-1$

$\therefore$ Range of c is $(-\infty,-1]$.

(ii) **Case 1 :** When both $f(x)$ and $g(x)$ are concave up *i.e.*, $c>1$

Possible graph to have $f(x)\le 0$ and $g(x)\ge 0$ to have unique solution.

Hence, $D_f=0 \Rightarrow 4c^2-4(c-1)(c+4)=0$

Hence, $c=\frac{4}{3}$

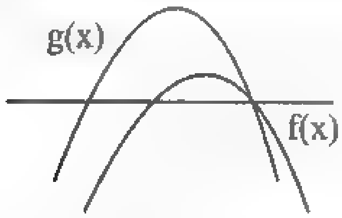
At $c=\frac{4}{3}$, $f(x)=(x+4)^2$ and $g(-4)>0$

$\therefore$ This case is possible.

Hence, $c=\frac{4}{3}$

Case 2 : Both $f(x)$ and $g(x)$ are concave down *i.e.*, $c<0$.

Possible graph to have $f(x)\le 0$ and $g(x)\ge 0$ to have unique solution.



Here $f(x)$ and $g(x)$ have a common root.

In this case, the value of c is $\frac{-3}{4}$.

Case-3: When one is concave up and another is concave down is rejected.

Hence, all possible values of c are $\frac{4}{3}$ and $\frac{-3}{4}$.

$\therefore$ Sum $=\frac{4}{3}+\left(\frac{-3}{4}\right)=\frac{7}{12}$.

Solutions of Paragraph for question nos. 589 → (b) and 590 → (a)

$$f(x)=e^{\lim\limits_{x\to 0}\left(\frac{\cos(\sqrt{y}\,x-1)}{yx^2}\right)x^2}$$

$$f(x)=e^{\frac{-x^2}{2}}$$

$$g(x)=\frac{1}{x^2}$$

Solutions of Paragraph for question nos. 591 → (d) and 592 → (b)

Replace x by $\frac{x}{2}$ and so on, we get $f(x)=3$.

Solutions of Paragraph for question nos. 593 → (c) and 594 → (b)

Range of (I) $=[0,\pi)$

Range of (II) $-\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$

Range of (III) $=\left(0,\frac{\pi}{4}\right]\ \ \left\{\frac{\pi}{8}\right\}$

Range of (IV) $=[\ \ \pi,\pi]$

Solutions of Paragraph for question nos. 595 → (c) and 596 → (d)

$$f(x)=ax^2+bx+c$$

$$g(x)-a\ln^2 x+b\ln x+c\ \forall\ x>0$$

Given, $g'(p)-0$

$$\Rightarrow\quad \frac{2a\ln p+b}{p}=0 \qquad\Rightarrow\qquad \ln p=\frac{-b}{2a}$$

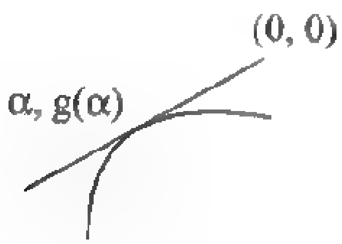
Also $g''(p^2)=0 \qquad\Rightarrow\qquad \dfrac{2a(1-\ln p^2)-b}{p^4}=0$

$$\Rightarrow\quad 1-\frac{b}{2a}=\ln p^2$$

$$\therefore\quad 1-\frac{b}{2a}=2\ln p=\frac{-b}{a}$$

$$\therefore\quad b=-2a$$

Also,



$$g'(\alpha) = \frac{g(\alpha)-0}{\alpha-0}$$

$$\frac{2a\ln\alpha+b}{\alpha} = \frac{a\ln^2\alpha+b\ln\alpha+c}{\alpha}$$

$$a\ln^2\alpha+(b-2a)\ln\alpha+c-b=0$$

For unique α, $D=0$

$(b-2a)^2-4a(c-b)=0$

$4b^2=4a(c-b)$

$4a=c+2a$

$\therefore \quad c=2a$

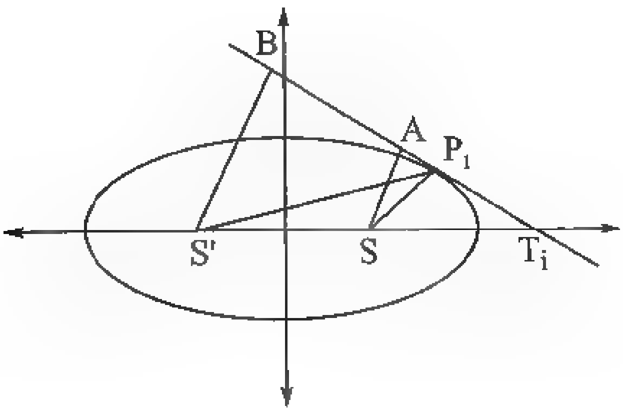
$\therefore \quad f(x)=a(x^2-2x+2)$

Now, proceed.

Solutions of Paragraph for question nos. 597 → (b) and 598 → (d)

(i) $SA \perp P_iT_i$

$S'B \perp P_iT_i$



$$A_1 = \text{Area}(\Delta SP_iT_i) = \frac{1}{2}\times SA\times P_iT_i$$

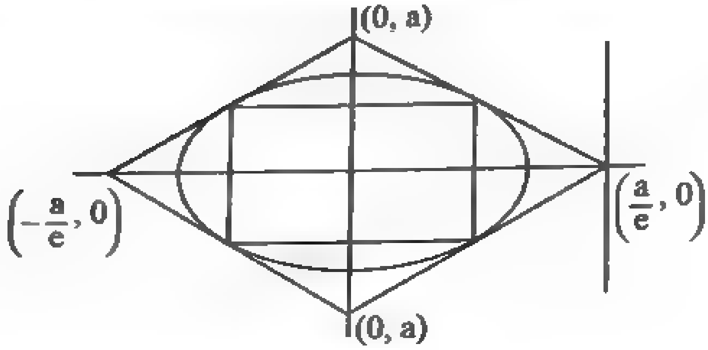
$$A_2 = \text{Area}(\Delta S'P_iT_i) = \frac{1}{2}\times S'B\times P_iT_i$$

$$\frac{A_1A_2}{(P_iT_i)^2} = \frac{b^2}{4} \qquad (SA\times S'B = b^2)$$

$$\sum\frac{A_1A_2}{(P_iT_i)^2} = 18 \quad \Rightarrow \quad \frac{nb^2}{4} = 18 \quad \Rightarrow \quad n=8$$

(ii) Tangent at $\left(ae, \dfrac{b^2}{a}\right)$

$$\frac{x\cdot ae}{a^2}+\frac{y\cdot b^2}{a\cdot b^2}=1$$



$$\Rightarrow \quad ex + y = a$$

$$A = \frac{2a}{e}\cdot\frac{2a}{2} = \frac{2a^2}{e} = 2\times 64\times\frac{8}{\sqrt{55}} \quad \Rightarrow \quad \lambda = 64$$

Solutions of Paragraph for question nos. 599 → (c) and 600 → (b)

$$f'(x) = 3ax^2 \quad \Rightarrow \quad f(x) = ax^3 + b$$

$$f(0) = b = 1 \quad \text{and} \quad f(1) = a + 1 = 2 \quad \Rightarrow \quad a = 1$$

Hence, $f(x) = x^3 + 1$

(i) Limit $= e^{\lim_{x\to 0}\frac{x^3}{\tan x - x}} = e^3$

(ii) D.I. $= \int_{-1}^{1}\frac{x^3+1}{\sqrt{x^2+7}}\,dx = 2\int_{0}^{1}\frac{1}{\sqrt{x^2+7}}\,dx = 2\left(\ln\left(x+\sqrt{x^2+7}\right)\right)_0^1 = 2\ln\left(\frac{\sqrt{8}+1}{\sqrt{7}}\right)$

$a + b + c = 8 + 7 + 0 = 15$

Solutions of Paragraph for question nos. 601 → (c) and 602 → (c)

Let the first A.P. be $a_1, a_2, a_3, \ldots\ldots, a_k$

and the second A.P. be $b_1, b_2, b_3, \ldots\ldots, b_k$

Given $\quad \frac{a_k}{b_1} = \frac{b_k}{a_1} = 4$

$$\Rightarrow \quad a_k = 4b_1 \quad \Rightarrow \quad a_1 + (k-1)d_1 = 4b_1 \quad \ldots(1)$$

$$b_k = 4a_1 \quad \Rightarrow \quad b_1 + (k-1)d_2 = 4a_1 \quad \ldots(2)$$

Given $\quad \frac{S_k}{S'_k} = 2 \quad \Rightarrow \quad \frac{2a_1 + (k-1)d_1}{2b_1 + (k-1)d_2} = 2$

$$\Rightarrow \quad \frac{a_1 + \overbrace{a_1 + (k-1)d_1}}{b_1 + \underbrace{b_1 + (k-1)d_2}} = 2$$

$$\frac{a_1 + 4b_1}{b_1 + 4a_1} = 2 \quad \Rightarrow \quad a_1 + 4b_1 = 2b_1 + 8a_1$$

$$\therefore \quad \frac{a_1}{b_1} = \frac{2}{7} \quad \ldots(3)$$

From eqns. (1) and (2)

$$(k-1)d_1 = 4b_1 - a_1$$
$$(k-1)d_2 = 4a_1 - b_1$$

On dividing, we get

$$\frac{d_1}{d_2} = \frac{4b_1 - a_1}{4a_1 - b_1} = \frac{4-(2/7)}{4(2/7)-1} = 26 = \alpha$$

Also $\quad \lambda = \frac{a_k}{b_k} = \frac{b_1}{a_1} = \frac{7}{2}$

$$\therefore \quad \alpha + 2\lambda = 26 + 7 = 33$$

Solutions of Paragraph for question nos. 603 → (d) and 604 → (b)

$$\therefore \quad f'(x) = 0 \quad \Rightarrow \quad f(x) = \text{constant} \quad \therefore \quad f(9) = \lambda$$

(i) $\quad \frac{\sum_{k=1}^{9} f(k)}{f(9)} = \frac{9\lambda}{\lambda} = 9$

$$P^2 = P \cdot P = \begin{bmatrix} \cos(2\pi/9) & \sin(2\pi/9) \\ -\sin(2\pi/9) & \cos(2\pi/9) \end{bmatrix}$$

$$\therefore \quad P^n = \begin{bmatrix} \cos(n\pi/9) & \sin(n\pi/9) \\ -\sin(n\pi/9) & \cos(n\pi/9) \end{bmatrix}$$

$$\alpha P^6 + \beta P^3 + \gamma I = \alpha \begin{bmatrix} -1/2 & \sqrt{3}/2 \\ -\sqrt{3}/2 & -1/2 \end{bmatrix} + \beta \begin{bmatrix} 1/2 & \sqrt{3}/2 \\ -\sqrt{3}/2 & 1/2 \end{bmatrix} + \gamma \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$$

$$\frac{-\alpha}{2} + \frac{\beta}{2} + \gamma = 0 \quad \text{and} \quad \frac{\sqrt{3}}{2}(\alpha + \beta) = 0 \quad \Rightarrow \quad \beta = -\alpha$$

$$\therefore \quad \gamma = a$$

$\therefore \quad$ (II) P ; $\quad$ (III) P $\quad$ and $\quad$ (IV) Q

Solutions of Paragraph for question nos. 605 → (a), 606 → (b) and 607 → (c)

a = Number of digits in 2^{50} = 1 + integral part of $50 \log_{10} 2 = 16$

b = Number of zero's in 3^{-50} = | 1 + integral part of $(-50 \log_{10} 3)| = 23$

$3 \le \log_5 N < 4$

$125 \le N < 625 \quad \Rightarrow \quad c = 500$

(i) $\quad c - (a \times b) = 500 - 16 \times 23 = 132$

(ii) $\quad c = 500 \Rightarrow$ sum of digits $= 5$

(iii) $\quad a + b = 39$

Solutions of Paragraph for question nos. 608 → (b) and 609 → (c)

(i) $$\log_{14} 63 = \frac{\log_7 63}{\log_7 14} = \frac{1+\log_7 9}{\log_7 2+1} = \frac{1+\frac{2}{bc}}{1+\frac{1}{abc}} = \frac{abc+2a}{abc+1}$$

(ii) $abc = \log_2 7$

$\Rightarrow \quad \log_2 4 < abc < \log_2 8$

$\Rightarrow \quad 2 < abc < 3$

Solutions of Paragraph for question nos. 610 → (a) and 611 → (c)

From theory it is clear that $\sin 2A + \sin 2B + \sin 2C = 4 \sin A \sin B \sin C$.

Solutions of Paragraph for question nos. 612 → (d) and 613 → (b)

(i) $2x^2 + 3x + a^2 \geq x^2 - ax - b$

$x^2 + (3+a)x + a^2 + b \geq 0 \quad \forall\, x,\ a \in R$

$D \leq 0$

$(3+a)^2 - 4(a^2+b) \leq 0$

$3a^2 - 6a + 4b - 9 \geq 0$

$D \leq 0$

$12 - 4b \leq 0 \qquad \Rightarrow \qquad 3 - b \leq 0$

$b \geq 3$

Given that $b \in [0, 6]$

$b = 3, 4, 5, 6$

Sum of integral values of $b = 3+4+5+6 = 18$

(ii) $|a-1| + |b-2| = 0$

$a = 1,\ b = 2$

$$y = \frac{f(x)}{g(x)} = \frac{x^2 - x - 2}{2x^2 + 3x + 1} = \frac{(x-2)(x+1)}{(2x+1)(x+1)}$$

$$y = \frac{x-2}{2x+1}$$

$$x = \frac{y+2}{1-2y} \qquad \Rightarrow \qquad y \neq \frac{1}{2}$$

Range of $y \in R - \left\{\frac{1}{2}, 3\right\}$

$\alpha + \beta = \frac{1}{2} + 3 = \frac{7}{2}$

Solutions of Paragraph for question nos. 614 → (b) and 615 → (a)

$f(x)=(x-2)(x^2-4x+6)$

$\therefore \quad g(x)=x^2-4x+6$ has roots $=\alpha,\beta$

(i) $\quad \text{L.H.S.}=\dfrac{\alpha^2+\beta^2}{36}=\dfrac{(\alpha+\beta)^2-2\alpha\beta}{36}=\dfrac{16-12}{36}=\dfrac{1}{9}$

(ii) $\quad 8<g(x)\le 18 \;\Rightarrow\; 8<x^2-4x+6\le 18$

$\Rightarrow \quad 6<(x-2)^2\le 16 \qquad \Rightarrow \qquad \sqrt{6}<|x-2|\le 4$

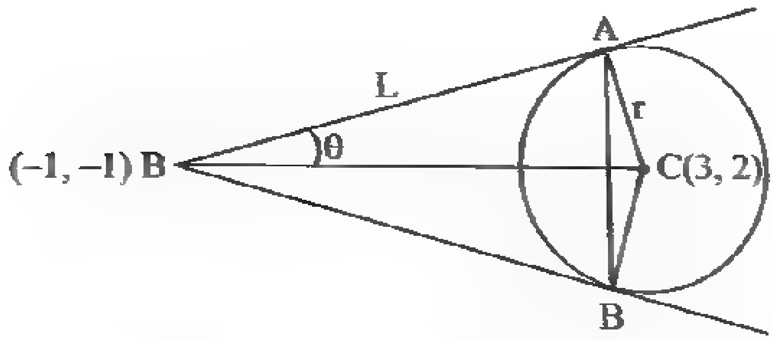
$\therefore \quad |x-2|=3,4$

Solutions of Paragraph for question nos. 616 → (a) and 617 → (d)

(i) Area of quadrilateral $=rL=4\times 3=12$

(ii) Circle circumscribing the triangle PAB is the circle described on CP as diameter

$\therefore \quad \text{radius}=\dfrac{5}{2}$



Solutions of Paragraph for question nos. 618 → (d) and 619 → (c)

$$P=\int_0^1\sqrt{\frac{x}{1-x}}\ln\left(\frac{x}{1-x}\right)dx$$

Put $\dfrac{x}{1-x}=t^2 \quad\Rightarrow\quad x=t^2-t^2x$

$$\Rightarrow\quad x=\frac{t^2}{1+t^2}=1-\frac{1}{1+t^2}$$

$$dx=\frac{2t}{(1+t^2)^2}$$

$$P=\int_0^{\infty}t(2\ln t)\frac{2t}{(1+t^2)^2}\,dt$$

$$P=2\int_0^{\infty}\underbrace{(t\ln t)}_{\text{(i)}}\;\underbrace{\frac{2t}{(1+t^2)^2}}_{\text{(ii)}}\,dt$$

$$P=2\left[(t\ln t)\left(\frac{-1}{1+t^2}\right)\Bigg|_0^{\infty}+\int_0^{\infty}\frac{(1+\ln t)}{1+t^2}dt\right]$$

$$=2\left[0+\underbrace{\tan^{-1}t}_{}\right]_0^{\infty}+\underbrace{\int_0^{\infty}\frac{\ln t}{1+t^2}dt}_{0}$$

$$=\pi$$

Now, $R=\int_0^8\frac{\sqrt{\alpha+1}+1}{2}d_\alpha=\frac{38}{3}$

$\therefore\quad 3R=38$

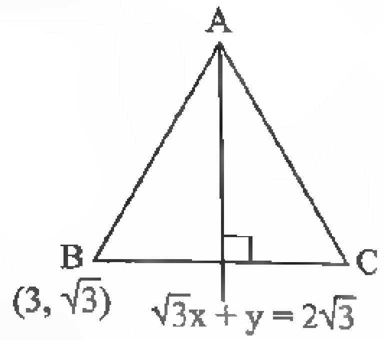
Solutions of Paragraph for question nos. 620 → (c) and 621 → (d)

Let co-ordinate of $c\,(a,b)$

$$\frac{a-3}{\sqrt{3}}=\frac{b-\sqrt{3}}{1}=\frac{-2(2\sqrt{3})}{4}$$

$a=0=b$

$$A\left(\frac{3}{2}\pm3(\cos120^\circ),\ \frac{\sqrt{3}}{2}\pm3(\sin120^\circ)\right)\quad\text{and}\quad H(1,\sqrt{3})$$



Solutions of Paragraph for question nos. 622 → (d) and 623 → (a)

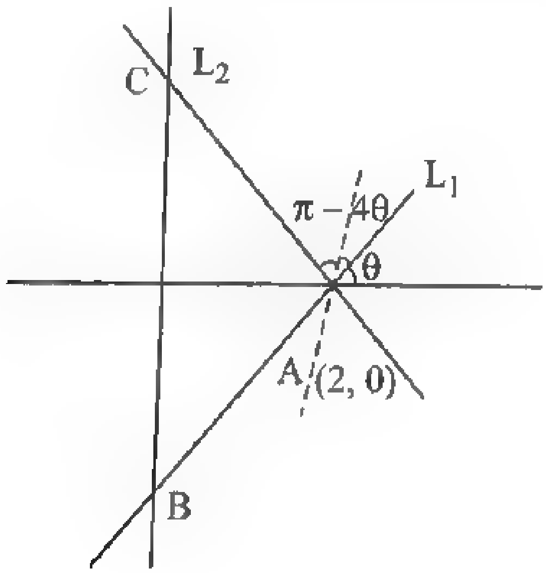
(i) Number of lines $={}^{13}C_2-{}^{3}C_2\cdot10-2\cdot{}^{5}C_2+12=40$

(ii) ${}^{13}C_3-10-{}^{5}C_3\cdot2=256$

Solutions of Paragraph for question nos. 624 → (a) and 625 → (d)

$L_1:\ y-0=\frac{1}{2}(m-2)$

$B\equiv(0,-1)$ Slope of the line $L_2=\tan(\pi-30^\circ)=-\tan30^\circ=\frac{-11}{2}$



$L_2 : y-0=\frac{-11}{2}(x-2)$

$\therefore \quad C \equiv (0,11)$

(i) Slope of angle bisector below L_1 and L_2 is $\tan\left(\frac{\pi}{2}-2\theta+\theta\right)=\cot\theta=2$

(ii) Radius of the largest circle $=\frac{\Delta}{s-a}=\dfrac{\frac{1}{2}\times 2\times 12}{\frac{12+\sqrt{5}+5\sqrt{5}}{2}-12}=4(\sqrt{5}+2)$

Solutions of Paragraph for question nos. 626 → (b) and 627 → (d)

(i) a, b and c are in A.P.

$\therefore \quad \sqrt{ac}<b$

$$f(x)=\lim_{n\to\infty}\frac{-(e^x)^n+x^2+f}{2e^x(e^x)^n+x+d}=\begin{cases}\dfrac{x^2+f}{x+d}, & x<0\\[2ex] \dfrac{-1+f}{2+d}, & x=0\\[2ex] \dfrac{-1}{2e^x}, & x>0\end{cases}$$

For $f(x)$ to be continuous

$$\frac{f}{d}=\frac{-1+f}{2+d}=\frac{-1}{2}$$

$\therefore \quad 2f+d=0 \quad \Rightarrow \quad 2f+d+1=1$

(ii) a, b and c are in G.P.

$$f(x)=\lim_{n\to\infty}\frac{x^2+f}{2e^x(e^x)^n+x+d}=\begin{cases}\dfrac{x^2+f}{x+d}, & x<0\\[2ex] \dfrac{f}{2+d}, & x=0\\[2ex] 0, & x>0\end{cases}$$

For $f(x)$ to be continuous

$$\frac{f}{d}=\frac{f}{2+d}=0 \quad \Rightarrow \quad f=0$$

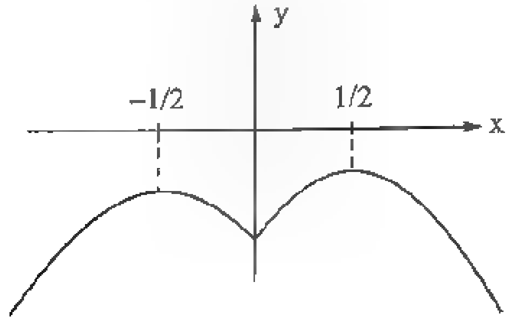
$f(x)=||x-4|-2|-1$

Solutions of Paragraph for question nos. 628 → (b) and 629 → (c)

$f(x) = (x-1)^3$

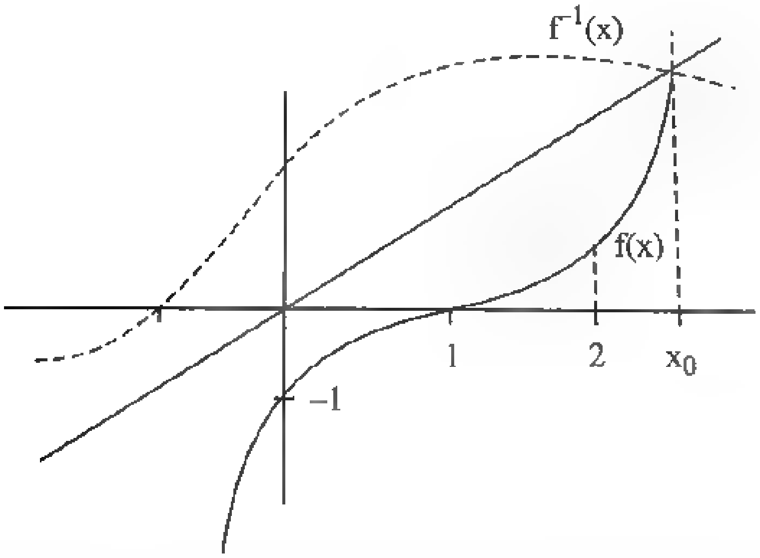
$f(x) - x^3 = -3x^2 + 3x - 1 = h(x)$

(i) Clearly, $y = h(|x|)$ is non-derivable at exactly one point.



(ii) $f(x)$ and $f^{-1}(x)$ meet on the line $y = x$ at $x = x_0$ where $2 < x_0 < 3$.

$\{\because \; f(2) = 1 \;$ and $\; f(3) = 8\}$



$$\therefore \quad \cos^{-1}(\cos 2x_0) + 4\tan^{-1}\left(\tan \frac{x_0}{2}\right) = (2\pi - 2x_0) + 4\frac{x_0}{2} = 2\pi$$

Solutions of Paragraph for question nos. 630 → (c) and 631 → (b)

$$f(x) = \begin{cases} 1, & x = 1 \\ e^{(x^{10}-1)} + (x-1)^2 \sin\left(\dfrac{1}{x-1}\right), & x \neq 1 \end{cases}$$

$$\text{(i)} \quad f'(1) = \lim_{h\to 0} \frac{e^{(1+h)^{10}-1} + h^2 \sin \frac{1}{h} - 1}{h} = \lim_{h\to 0} \frac{e^{(1+h)^{10}-1} - 1}{h} = 10$$

(ii) $\lambda = \lim_{t\to 0}\left(\dfrac{\sum_{k=1}^{100} f(1+tk)-100}{t}\right)$

$= \lim_{t\to 0}\left(\dfrac{f(1+t)-1}{t}+\dfrac{f(1+2t)-1}{t}+......+\dfrac{f(1+100t)-1}{t}\right)$

$= f'(1)+2f'(1)+......+100f'(1) = f'(1)(1+2+......+10) = 10\times 5050$

$\therefore \quad \dfrac{\lambda}{100} = 505$

Solutions of Paragraph for question nos. 632 → (c) and 633 → (d)

$f(x) = (1-x)(1+x^2) = -x^3+x^2-x+1$

If $\quad f^{-1}(x) = g(x) \quad \Rightarrow \quad g'(x) = \dfrac{1}{f'(g(x))}$

(i) Now, $h(x) = g(\ln(f(x))) \Rightarrow h'(x) = \dfrac{f'(x)}{f(x)}\cdot\dfrac{1}{f'(g(\ln f(x)))}$

$\Rightarrow \quad h'(0) = \dfrac{f'(0)}{f(0)}\cdot\dfrac{1}{f'(g(0))} = \dfrac{-1}{1}\cdot\dfrac{1}{f'(1)} = \dfrac{-1}{-2} = \dfrac{1}{2}$

$\Rightarrow \quad 3+\dfrac{1}{h'(0)} = 3+2 = 5$

(ii) $I = \int_0^1 \dfrac{(1-x)\ln(1+x)}{f(x)}dx = \int_0^1 \dfrac{\ln(1+x)}{1+x^2}dx \qquad$ Put $x = \tan\theta$

$I = \int_0^{\frac{\pi}{4}} \ln(1+\tan\theta)d\theta = \dfrac{\pi \ln 2}{8}$

Solutions of Paragraph for question nos. 634 → (d) and 635 → (b)

$S = 0 \quad \Rightarrow \quad (x-2)^2+(y-2)^2 = 1 \quad \Rightarrow \quad x^2+y^2-4x-4y+7 = 0$

$r = L_{\text{medium}} = \dfrac{1}{2}\sqrt{2PA^2+2PB^2-AB^2} = \dfrac{\sqrt{12-8}}{2} = 1$

$S_1 = 0 \quad \Rightarrow \quad (x-3)(x-2)+(y-7)(y-2) = 0$

$\Rightarrow \quad x^2+y^2-5x-9y+20 = 0$

Common chord $\Rightarrow \quad x+5y-13 = 0$

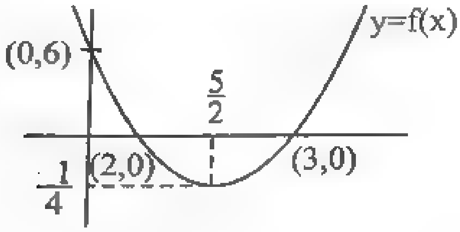
$$\cos\theta = \frac{r_1^2+r_2^2-d^2}{2r_1r_2} = \frac{1+\frac{13}{2}-\frac{13}{2}}{2\cdot 1\times\sqrt{\frac{13}{2}}} = \frac{1}{\sqrt{26}} \qquad \left(r_1 = 1,\ r_2 = \sqrt{\frac{13}{2}} = d\right)$$

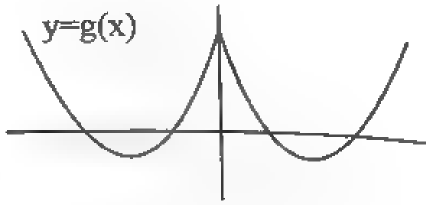
$\tan\theta = 5.$

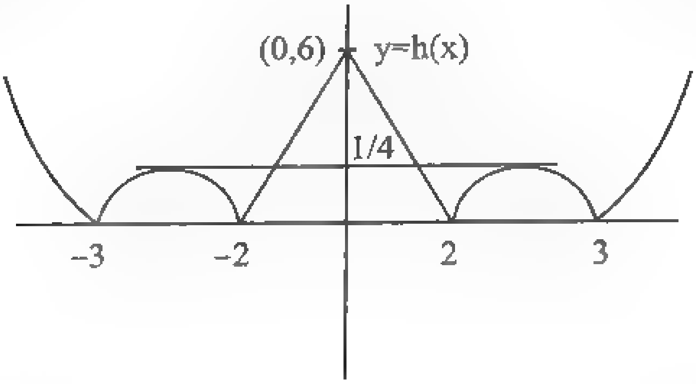
Solutions of Paragraph for question nos. 636 → (b) and 637 → (a)

Hint: $f(x) = \sin x - 3, \quad g(x) = \cos x$

Solutions of Paragraph for question nos. 638 → (a) and 639 → (d)







$\therefore$ $h(x) = k$ will have more than two solutions for $k = 0, 1, 2, 3, 4, 5, 6$

$\therefore$ $h(x) = k$ will have exactly 8 distinct solution for $0 < k < \frac{1}{4}$

$\therefore$ No integral value of k.

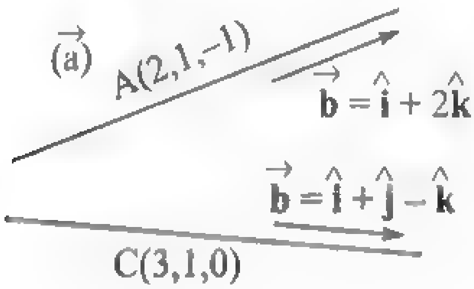
$\therefore$ Probability = 0

Clearly, $|g(x)| = -g(x) \quad \Rightarrow \quad x \in [-3, -2] \cup [2, 3]$

Solutions of Paragraph for question nos. 640 → (d) and 641 → (c)

(i) $\vec{b} \times \vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 2 \\ 1 & 1 & -1 \end{vmatrix} = -2\hat{i} + 3\hat{j} + \hat{k}$

$\because$ $\text{S. D.} = \dfrac{|(\vec{a} - \vec{c}) \cdot (\vec{b} \times \vec{d})|}{|\vec{b} \times \vec{d}|} = \dfrac{|(\hat{i} + \hat{k}) \cdot (-2\hat{i} + 3\hat{j} + \hat{k})|}{\sqrt{14}} = \dfrac{1}{\sqrt{14}}$



(ii) Equation of plane containing L_1 and parallel to L_2 is

$$\begin{vmatrix} x-2 & y-1 & z+1 \\ 1 & 0 & 2 \\ 1 & 1 & -1 \end{vmatrix} = 0$$

$\Rightarrow$ $-2(x-2)+3(y-1)+z+1=0 \Rightarrow -2x+3y+z+2=0 \Rightarrow 2x-3y-z=2$

$\therefore$ Plane Π is $\Rightarrow \dfrac{x}{1}+\dfrac{y}{\left(\frac{-2}{3}\right)}+\dfrac{y}{(-2)}=1$

$\therefore$ $A(1,0,0), B=\left(0,\dfrac{-2}{3},0\right)$ and $C(0,0,-2)$

$\therefore$ Volume $-\dfrac{1}{6}|[\vec{OA}\ \vec{OB}\ \vec{OC}]|=\dfrac{1}{6}\left(1\times\dfrac{2}{3}\times 2\right)=\dfrac{2}{9}$

Solutions of Paragraph for question nos. 642 → (c) and 643 → (c)

For $x\in(-1,1)$

$$g(x)-\int_{-1}^{x} f(t)(x-t)dt+\int_{x}^{1} f(t)(t\quad x)dt = x\int_{1}^{x} f(t)\,dt-\int_{-1}^{x} t\,f(t)\,dt+\int_{x}^{1} t\,f(t)\,dt-x\int_{x}^{1} f(t)dt$$

Now, differentiate

$$g'(x)=\int_{-1}^{x} f(t)\,dt+x\frac{d}{dx}\int_{-1}^{x} f(t)\,dt-\frac{d}{dx}\int_{1}^{x} t\,f(t)\,dt+\frac{d}{dx}\int_{x}^{1} t\,f(t)\,dt-\int_{x}^{1} f(t)\,dt\quad x\frac{d}{dx}\int_{x}^{1} f(t)dt$$

$$g'(x)=\int_{-1}^{x} f(t)\,dt+xf(x)-f(x)x-f(x)x-f(x)x-\int_{x}^{1} f(t)\,dt+x\,f(x)$$

$$g'(x)=\int_{-1}^{x} f(t)\,dt-\int_{x}^{1} f(t)\,dt \qquad \ldots(1)$$

Again differentiate

$$g''(x)=\frac{d}{dx}\int_{-1}^{x} f(t)\,dt-\frac{d}{dx}\int_{x}^{1} f(t)\,dt=f(x)+f(x)=2f(x) \qquad \ldots(2)$$

Now, verify alternatives.

Solutions of Paragraph for question nos. 644 → (c) and 645 → (b)

(i) $\text{tr.}(A)=\displaystyle\sum_{i=1}^{n} a_{ii}=\sum_{i=1}^{n} 2i^2=\frac{2n(n+1)(2n+1)}{6}$

$\therefore$ $\displaystyle\lim_{n\to\infty}\frac{\text{tr.}(A)}{n^3}=\frac{4}{6}=\frac{2}{3}$

(ii) $\displaystyle\lim_{n\to\infty}\sum_{i=1}^{n}\tan^{-1}\left(\frac{1}{a_{ii}}\right)=\lim_{n\to\infty}\sum_{i=1}^{n}\tan^{-1}\left(\frac{1}{2i^2}\right)=\lim_{n\to\infty}\sum_{i=1}^{n}\tan^{-1}\left(\frac{(2i+1)-(2i-1)}{1+(2i+1)(2i-1)}\right)$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} (\tan^{-1}(2i+1) - \tan^{-1}(2i-1))$$

$$= \lim_{n\to\infty} [(\tan^{-1} 3 - \tan^{-1} 1) + (\tan^{-1} 5 - \tan^{-1} 3) \ldots\ldots (\tan^{-1}(2n+1) - \tan^{-1}(2n-1))]$$

$$= \lim_{n\to\infty} (\tan^{-1}(2n+1) - \tan^{-1} 1) = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4} = \cot^{-1}(1)$$

$\therefore \quad \lambda = 1$

Solutions of Paragraph for question nos. 646 → (c), **647** → (a) **and 648** → (d)

(I) $A \cap B \cap C = \{2, 3, 4, 5, 6\}$

Each element of 1, 7, 8, 9, 10 has only two choices

$\therefore$ Number of ways $= 2^5$.

(II) $A \cup B \cup C = \{3, 4, 5\}$

Each element of 3, 4, 5 has 7 choices

$\therefore$ Number of ways $= 7^3 = 343$

(III) $A \cap B \cap C = \{3, 4, 5, 6, 7\}$ and $A = B \neq C$

Each element of 1, 2, 8, 9, 10 has 3 choices

$\therefore$ Number of ways $= 3^5$ but $A = B \neq C$

$\therefore \quad 3^5 - 1 = 242$

(IV) $A \cup B \cup C = \{6, 7, 8, 9, 10\}$ and $A = B \neq C$

Each element of 6, 7, 8, 9, 10 has 3 choices

$\therefore$ Number of ways $= 3^5$ but $A = B \neq C$

$\therefore \quad 3^5 - 1 = 242$

$$P\left(\frac{E}{E_1}\right) = \xrightarrow[\text{(I)}]{\text{According to}} \frac{1}{{}^{10}C_5 \cdot 3^5 \cdot 2^{10}} = \frac{1}{{}^{10}C_5 \cdot 12^5}$$

$$P\left(\frac{E}{E_2}\right) = \xrightarrow[\text{(I)}]{\text{According to}} \frac{1}{{}^{10}C_5 \cdot 3^5 \cdot 2^{10}}$$

$$P\left(\frac{E}{E_1}\right) = \xrightarrow[\text{(II)}]{\text{According to}} 0$$

$$P\left(\frac{E}{E_2}\right) = \xrightarrow[\text{(II)}]{\text{According to}} 0$$

$$P\left(\frac{E}{E_1}\right) = \xrightarrow[\text{(III)}]{\text{According to}} \frac{31}{{}^{10}C_5 \cdot 3^5 \cdot 2^{10}} = \frac{31}{{}^{10}C_5 \cdot 12^5}$$

$$P\left(\frac{E}{E_2}\right) = \xrightarrow[\text{(III)}]{\text{According to}} \frac{31}{{}^{10}C_5 \cdot 3^5 \cdot 2^{10}} = \frac{31}{{}^{10}C_5 \cdot 12^5}$$

$$P\left(\frac{E}{E_1}\right) \xrightarrow[\text{(IV)}]{\text{According to}} \frac{31}{{}^{10}C_5 \cdot 3^5 \cdot 2^{10}} = \frac{31}{{}^{10}C_5 \cdot 12^5}$$

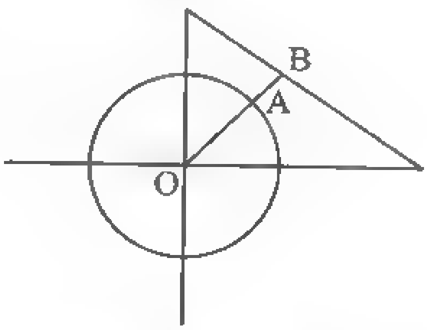
$$P\left(\frac{E}{E_2}\right) \xrightarrow[\text{(IV)}]{\text{According to}} \frac{31}{{}^{10}C_5 \cdot 3^5 \cdot 2^{10}} = \frac{31}{{}^{10}C_5 \cdot 12^5}$$

Solutions of Paragraph for question nos. 649 → (a), **650** → (c) **and 651** → (b)

Do yourself.

Solutions of Paragraph for question nos. 652 → (b) **and 653** → (d)

(I) $|z-2|+|z+2|=6$ represents an ellipse with major axis 6 and focus (2, 0) and (–2 , 0)



$\therefore$ A lies on its auxilliary circle *i.e.*, $x^2+y^2=9$

and $(1-i)z+(1+i)\bar{z}=10\sqrt{2} \quad \Rightarrow \quad (z+\bar{z})-i(z-\bar{z})=10\sqrt{2}$

$\Rightarrow \quad 2x-i(2iy)=10\sqrt{2} \quad \Rightarrow \quad x+y=5\sqrt{2}$

$\therefore$ B lies on straight line $x+y=5\sqrt{2}$

$\therefore$ $AB_{\min}$ = perpendicular distance OB radius $OA = 5 \quad 3 = 2$

(II) If a variable circle touches line and circle, then locus will be the Parabola with focus at centre (0, 0) of given circle and directrix at a line parallel to given line at a distance of 3 units, equal to radius of circle

$\therefore$ Distance between focus and directrix $= 5+3=8$

$\therefore$ Latus rectum = 16

Solutions of Paragraph for question nos. 654 → (b) **and 655** → (c)

Here $P'(x)=0$ at $x=-2,-1,0,\frac{1}{2}$

$\therefore$ $P''(x)=0$ will have roots α, β, γ as $\alpha \in (-2,-1), \beta \in (-1,0)$ and $\gamma \in \left(0,\frac{1}{2}\right)$

$\therefore$ $[\alpha]+[\beta]+[\gamma]=-2-1+0=-3$

and equation $P'(x)\,P''(x)=0$ will have seven real roots as $P'(x)=0$ has four roots and $P''(x)=0$ has three roots.

$\therefore$ Its derivative $(P''(x))^2+P'(x)\cdot P'''(x)=0$ will have 6 real roots.

Solutions of Paragraph for question nos. 656 → (c) and 657 → (d)

$$f(x)=\frac{\pi}{4}+\cos^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)\quad \tan^{-1}x$$

$\therefore \quad f(x)=\frac{\pi}{4}+\cot^{-1}x-\tan^{-1}x$

$=\frac{3\pi}{4}-2\tan^{-1}x \qquad \{\because \ x>0\}$

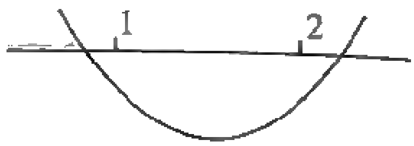
$\text{sgn}(f(x))=1 \Rightarrow f(x)>0$

$\Rightarrow \quad \frac{3\pi}{4}-2\tan^{-1}x>0 \quad \Rightarrow \quad \tan^{-1}x<\frac{3\pi}{8} \quad \rightarrow \quad x<\sqrt{2}+1$

$\therefore$ Possible positive integral values of x are 1, 2

$a_1=1$ and $a_2=2$

(ii) $P(x)=x^2-4kx+3k^2$



$P(x)<0 \quad \forall x\in(a_1,a_2)$

$P(1)\le 0$ and $P(2)\le 0$

$1-4k+3k^2\le 0$ and $4-8k+3k^2\le 0$

$(3k-1)(k-1)\le 0 \qquad (3k-2)(k-2)\le 0$

$k\in\left[\frac{1}{3},1\right] \quad \ldots(1) \qquad k\in\left[\frac{2}{3},2\right] \quad \ldots(2)$

$\therefore$ Intersection of eqns. (1) and (2) is $k\in\left[\frac{2}{3},2\right]$

Solutions of Paragraph for question nos. 658 → (c) and 659 → (a)

Let remainder $g(x)$ be ax^3+bx^2+cx+d

$\because \quad f(x)=x^2(x^2-1)\ Q(x)+ax^3+bx^2+cx+d$, where $Q(x)$ is quotient

$\therefore$ RHS should have common factor x^2.

$\therefore \quad c=d=0$

$f(1)=a+b=3$

and $\underline{f(-1)=-a+b=3}$

$\therefore \quad b=3$ and $a=0$

$\therefore \quad g(x)=3x^2$ which is many one into function.

$\therefore \quad g(x)=0 \quad \Rightarrow \quad x=0$ lies between roots of

$x^2-2(a+1)x+a(a-1)=0$

$\therefore \quad a(a-1)<0 \quad \Rightarrow \quad 0<a<1$

Solutions of Paragraph for question nos. 660 → (b), 661 → (c) and 662 → (d)

(I) $\because \quad \int_0^2 f(x)dx = \int_0^1 f(x)dx + \int_1^2 f(x)dx$

Let $x = 2t \Rightarrow dx = 2dt$

$$\Rightarrow \quad 2\int_0^1 f(2t)dt = I + \int_1^2 f(x)dx$$

$\therefore$ If $f(2x) = 3f(x)$ then $\int_1^2 f(x)dx = 5 \Rightarrow$ (I) (ii) (P)

and also

(II) $\int_0^4 f(x)dx = \int_0^2 f(x)dx + \int_2^4 f(x)dx$

Let $x = 2t$

$$\Rightarrow \quad 2\int_0^2 f(2t)dt = 2 + \int_2^4 f(x)dx$$

$\Rightarrow$ If $f(2x) = 2f(x)$ then $\int_2^4 f(x)dx = 6 \Rightarrow$ (II) (i) (Q)

and

$$\because \quad \int_0^3 f(x)dx = \int_0^1 f(x)dx + \int_1^3 f(x)dx$$

Let $x = 3t$

$$\Rightarrow \quad 3\int_0^1 f(3t)dt = 1 + \int_1^3 f(x)dx$$

$\Rightarrow$ If $f(3t) = 3f(t)$ then $\int_1^3 f(x)dx = 8 \Rightarrow$ (I) (iii) (R)

and

$$\int_0^9 f(x)dx = \int_0^3 f(x)dx + \int_3^9 f(x)dx$$

Let $x = 3t$

$$\Rightarrow \quad 3\int_0^3 f(3t)dt = \int_0^3 f(x)dx + \int_3^9 f(x)dx$$

$\Rightarrow$ If $f(3t) = 3f(t)$ then $24 = \int_3^9 f(x)dx \Rightarrow$ (III) (iii) (S)

Solutions of Paragraph for question nos. 663 → (c), 664 → (b) and 665 → (d)

(I) $\because$ $a^2+b^2+c^2-ab-bc-ca \le 0$

$\Rightarrow$ $(a-b)^2+(b-c)^2+(c-a)^2 \le 0$ $\Rightarrow$ $a=b=c$

$\therefore$ $f(x)=ax^2+ax+a$

$\therefore$ $D<0$

$\therefore$ no real root

$\therefore$ $|f(|x|)|$ is always derivable except one point.

$\therefore$ (I) (i) (Q)

(II) $\because$ $a^2+b^2+c^2+ab+bc+ca \le 0$ $\Rightarrow$ $(a+b)^2+(b+c)^2+(c+a)^2 \le 0$

$\Rightarrow$ $a+b=0$, $b+c=0$, $c+a=0$

$\therefore$ $a=b=c=0$

$\therefore$ $f(x)=ax^2+bx+c$ is an identify

$\therefore$ $f(x)=0$ has infinite roots

$\therefore$ $f(x)$ is always derivable.

$\therefore$ (II) (iv) (P)

(III) $\because$ $(a-1)^2+(b-1)^2+(c-1)^2+(a-b)^2+(b-c)^2+(c-a)^2 \le 0$

$\therefore$ $a=b=c=1$

$\therefore$ $f(x)=0$ has no real root and $|f(|x|)|$ is non-derivable at exactly one point.

$\therefore$ (III) (i) (Q)

(IV) $\because$ $(a-1)^2+(b-3)^2+(c-2)^2 \le 0$

$\Rightarrow$ $a=1, b=3, c=2$

$\therefore$ $f(x)=x^2+3x+2$

$\therefore$ $f(x)=0$ has two distinct real roots

and $y=|f(|x|)|$ is non-derivable at exactly one point.

$\therefore$ (IV) (iii) (Q).

Solutions of Paragraph for question nos. 666 → (b) and 667 → (d)

$f(x)=(ax^2+bx+c)\text{sgn}\,(2\sin x-1)$

$f(x)=-(ax^2+bx+c), 0<x<\dfrac{\pi}{6}$

$f(x)=0, x=\dfrac{\pi}{6}$

$\Rightarrow$ $f(x)=(ax^2+bx+c), \dfrac{\pi}{6}<x<\dfrac{5\pi}{6}$

$\Rightarrow$ $f(x)=0,\ x=\dfrac{5\pi}{6}$

$\Rightarrow \quad f(x) = -(ax^2 + bx + c), \quad \frac{5\pi}{6} < x < 6$

If $y = f(x)$ is continuous, then $ax^2 + bx + c = a\left(x - \frac{\pi}{6}\right)\left(x - \frac{5\pi}{6}\right)$

If $c = 0 \Rightarrow a = b = 0$

Solutions of Paragraph for question nos. 668 → (c) and 669 → (b)

$f(x) = x^3 - ax^2 + bx - 8$

$\Rightarrow \quad \beta = 2 \quad \text{and} \quad b = 2a$

$\Rightarrow \quad \alpha = \frac{2}{p} \quad \text{and} \quad \gamma = 2p$ (p is the common ratio)

$\alpha \in I \quad \Rightarrow \quad p = -1, 1, 2, -2$

$\Rightarrow \quad \alpha = -2, 2, 1, -1; \quad \beta = 2, 2, 2, 2; \quad \gamma = -2, 2, 4, -4$

$\Rightarrow \quad \alpha = 1, \beta = 2, \gamma = 4 \quad \text{and} \quad a = 7; \quad b = 14$

Roots of the equation $f(x - \beta) = 0$ are $\alpha + \beta, 2\beta, \gamma + \beta$ which are in H.P.

Solutions of Paragraph for question nos. 670 → (a) and 671 → (a)

$$f(a,b) = \sqrt{\frac{a^2}{2} + \frac{a^2}{2} - 7\sqrt{2}a + 49} + \sqrt{(b-5)^2 + 5^2} + \sqrt{\frac{a^2}{2} + \frac{a^2}{2} + b^2 - \sqrt{2}\,abc}$$

$$f(a,b) = \underbrace{\sqrt{\left(\frac{a}{\sqrt{2}}\right)^2 + \left(\frac{a}{\sqrt{2}} - 7\right)^2}}_{PQ} + \underbrace{\sqrt{(b-5)^2 + 5^2}}_{RS} + \underbrace{\sqrt{\left(\frac{a}{\sqrt{2}}\right)^2 + \left(\frac{a}{\sqrt{2}} - b\right)^2}}_{RQ}$$

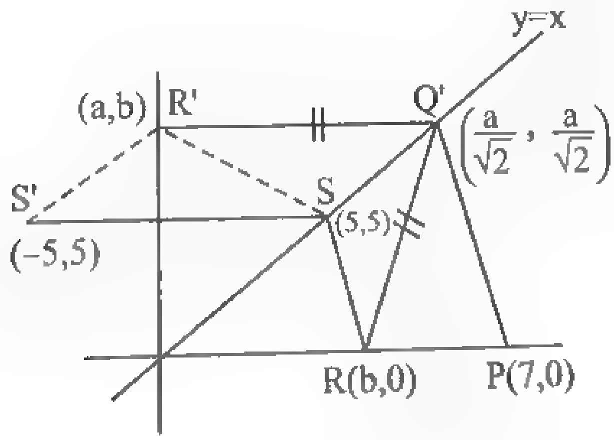


Image of R is R' in $y = x$

$QR = QR'$

Also $SR = SR'$ and image of S is S'

$R'S = R'S'$

$\therefore \quad PQ + QR' + R'S' \geq PS'$

$\therefore \quad SR = SR' = S'R'$

Sum of 3 sides of quad. > 4th side

Min. value if $PS' = \sqrt{144 + 25} = \sqrt{169} = 13$

Solutions of Paragraph for question nos. 672 → (c) and 673 → (c)

$$f(x)=\left|\sin\left((2r_1-1)\frac{\pi}{6}\right)x\right|+\left|\cos\left(\frac{\pi r_2}{6}\right)x\right|$$

$$f(1)=\left|\sin(2r_1-1)\frac{\pi}{6}\right|+\left|\cos\left(\frac{r_2\pi}{6}\right)\right|$$

Possible value of $(2r_1-1)\frac{\pi}{6}=\frac{\pi}{6},\frac{3\pi}{6},\frac{5\pi}{6},\frac{7\pi}{6},\frac{9\pi}{6},\frac{11\pi}{6}$

Possible values of $r_2\frac{\pi}{6}=\frac{\pi}{6},\frac{2\pi}{6},\frac{3\pi}{6},\frac{4\pi}{6},\frac{5\pi}{6},\frac{6\pi}{6}$

(i) For $f(1)$ to be an integer

$$\underbrace{(2r_1=1)\frac{\pi}{6}}_{\frac{3\pi}{6},\frac{9\pi}{6}} \qquad \underbrace{r_2\frac{\pi}{6}}_{\frac{3\pi}{6},\frac{6\pi}{6}}$$

$$\frac{\pi}{6},\frac{5\pi}{6},\frac{7\pi}{6},\frac{11\pi}{6} \qquad \frac{2\pi}{6},\frac{4\pi}{6}$$

$\therefore$ Required probability $=\frac{2}{6}\times\frac{2}{6}+\frac{4}{6}\times\frac{2}{6}=\frac{12}{36}-\frac{1}{3}$

(ii) $f(2)=\left|\sin(2r_1\quad 1)\frac{\pi}{3}\right|+\left|\cos\left(\frac{r_2\pi}{3}\right)\right|$

$$(2r_1-1)\frac{\pi}{3}=\frac{\pi}{3},\frac{3\pi}{3},\frac{5\pi}{3},\frac{7\pi}{3},\frac{9\pi}{3},\frac{11\pi}{3}$$

$$r_2\frac{\pi}{3}-\frac{\pi}{3},\frac{2\pi}{3},\frac{3\pi}{3},\frac{4\pi}{3},\frac{5\pi}{3},\frac{6\pi}{3}$$

For $f(2)$ not to be an irrational number

$$(2r_1-1)\frac{\pi}{3}=\frac{3\pi}{3},\frac{9\pi}{3} \quad \text{and} \quad \frac{r_2\pi}{3}=\frac{\pi}{3},\frac{2\pi}{3},\frac{3\pi}{3},\frac{4\pi}{3},\frac{5\pi}{3},\frac{6\pi}{3}$$

$$P(\bar{E})=\frac{2}{6}\times\frac{6}{6}=\frac{1}{3}$$

$$\therefore\quad P(E)=1-\frac{1}{3}=\frac{2}{3}$$

Note : E denotes event when $f(2)$ is an irrational number.

Solutions of Paragraph for question nos. 674 → (c) and 675 → (b)

(i) Given, $a+b+c=\alpha,\ a+b\omega+c\omega^2=\beta,\ a+b\omega^2+c\omega=\gamma$

$$|\alpha|^2+|\beta|^2+|\gamma|^2=\alpha\bar{\alpha}+\beta\bar{\beta}+\gamma\bar{\gamma} \qquad [\because\ \bar{\omega}=\omega^2]$$

Now, $\alpha\bar{\alpha} = (a+b+c)(\bar{a}+\bar{b}+\bar{c})$ $\quad [\bar{\omega}^2 = \omega]$

$= |a|^2+|b|^2+|c|^2+a\bar{b}+a\bar{c}+b\bar{a}+b\bar{c}+c\bar{a}+c\bar{b}$

$\beta\bar{\beta} = (a+b\omega+c\omega^2)(\bar{a}+\bar{b}\omega^2+\bar{c}\omega)$

$= |a|^2+|b|^2+|c|^2+a\bar{b}\omega^2+a\bar{c}\omega+b\bar{a}\omega+b\bar{c}\omega^2+c\bar{a}\omega^2+c\bar{b}\omega$

$\gamma\bar{\gamma} = (a+b\omega^2+c\omega)(\bar{a}+\bar{b}\omega+\bar{c}\omega^2)$

$= |a|^2+|b|^2+|c|^2+a\bar{b}\omega+a\bar{c}\omega^2+b\bar{a}\omega^2+b\bar{c}\omega+c\bar{a}\omega+c\bar{b}\omega^2$

So, $\alpha\alpha+\beta\bar{\beta}+\gamma\bar{\gamma} = 3(|a|^2+|b|^2+|c|^2)+(a\bar{b}+a\bar{c}+ba+b\bar{c}+c\bar{a}+c\bar{b})(1+\omega+\omega^2)$

$= 3(a|^2+|b^2+|c|^2)$ $\quad [\because 1+\omega+\omega^2=0]$

So, $\lambda = 3$

(ii) $(z+1)\begin{vmatrix} z+\omega^2 & 1 \\ 1 & z+\omega \end{vmatrix} + \omega\begin{vmatrix} 1 & \omega \\ z+\omega & \omega^2 \end{vmatrix} + \omega^2\begin{vmatrix} \omega & z+\omega^2 \\ \omega^2 & 1 \end{vmatrix} = 0$

As $1+\omega+\omega^2=0$, $\quad \omega^4=\omega$

$\Rightarrow$ $(z+1)(z^2+z(\omega+\omega^2)+\omega^3-1)+\omega(\omega^2-z\omega-\omega^2)+\omega^2(\omega \quad z\omega^2-\omega^4)=0$

$\Rightarrow$ $(z+1)(z^2-z)+z(-\omega^2-\omega)=0$

$\Rightarrow$ $z^3-z^2+z^2-z+z=0$

$\Rightarrow$ $z^3=0$

$\rightarrow$ $z=0$ only 1 complex number.

Match the Column Type Questions

676. (a) $\rightarrow$ (P, Q, S); (b) $\rightarrow$ (P, R); (c) $\rightarrow$ (P); (d) $\rightarrow$ (P, R, T)

(A) $x>1$

$(x-1)^{\log_2 x^2 - 2\log_x 4} = (x-1)^7$

$\Rightarrow$ $x-1=1$ or $\log_2 x^2 - 2\log_x 4 = 7$, let $\log_2 x = a$

$2a - \dfrac{4}{a} = 7 \Rightarrow 2a^2-7a-4=0 \Rightarrow (a-4)(2a+1)=0$

$\Rightarrow$ $x=16,\ x=2^{\frac{-1}{2}}$ (rejected)

(B) $2\log_{\sqrt{6}} 3 + \log_{\sqrt{6}} 4 = \log_{\sqrt{6}} 36 = 4$

(C) $3^{\sqrt{\log_3 5}} = 5^{\sqrt{\log_5 3}}$

(D) $\log_2 N = 5+m_1 \Rightarrow 5 \le \log_2 N < 6 \Rightarrow 32 \le N < 64$

$\log_5 N = 2+m_2$

$2 \le \log_5 N < 3 \Rightarrow 25 \le N < 125$

$\Rightarrow$ $32 \le N < 64 \Rightarrow x = 32$

677. (a) $\to$ (S); (b) $\to$ (R); (c) $\to$ (P)

(A) $P = 2^{\log_2(\log_2 6)} = \log_2 6$

$(A^P)^P = 4^{\log_2 6} = 36$

(B) $\dfrac{a}{1-r} = 7$; $\dfrac{a^2}{1-r^2} = \dfrac{147}{11}$

$\Rightarrow$ $7(a+r) = 25$

(C) $E_7(101)! = \left[\dfrac{101}{7}\right] + \left[\dfrac{101}{7^2}\right] = 14 + 2 = 16$

678. (a) $\to$ (P, Q, R); (b) $\to$ (Q); (c) $\Rightarrow$ (P)

(A) Let $z = x + iy$

$\therefore$ $2x = 2\sqrt{(x-1)^2 + y^2}$

$\Rightarrow$ $x^2 = x^2 - 2x + 1 + y^2$ $\Rightarrow$ $y^2 = 2x - 1$

If $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$, then

$y_1^2 - y_2^2 = 2(x_1 - x_2)$ $\Rightarrow$ $\dfrac{y_1 + y_2}{2} = \dfrac{x_1 - x_2}{y_1 - y_2}$

$\because$ $\arg(z_1 - z_2) = \dfrac{\pi}{4}$ $\Rightarrow$ $\dfrac{y_1 - y_2}{x_1 - x_2} = 1$ $\Rightarrow$ $y_1 + y_2 = 2$

$\therefore$ $\text{Im}(z_1 + z_2) = 2$

(B) Let mid-point of PQ be (h, k)

$\therefore$ Equation of PQ is $\dfrac{xk + yh}{2} - c^2 = hk - c^2$ $\Rightarrow$ $xk + yh = 2hk$

$\Rightarrow$ $$y = \left(\frac{-k}{h}\right)x + 2k \quad \ldots(1)$$

and equation of PQ as tangent on $x^2 = 8y$

$$y = mx - 2m^2 \quad \ldots(2)$$

$\therefore$ $m = \dfrac{-k}{h}$ and $k = -m^2$ $\Rightarrow$ $-k = \dfrac{k^2}{h^2}$

$\Rightarrow$ $x^2 = -y$

$\therefore$ Latus Rectum $= 1$

(C) $\because$ $D \le 0$ $\Rightarrow$ $16 - 4\log_{\left(\frac{1}{2}\right)} a \le 0$ $\Rightarrow$ $\log_{\left(\frac{1}{2}\right)} a \ge 4$ $\Rightarrow$ $a \le \dfrac{1}{16}$ and $a > 0$

$\therefore$ Number of integral values of $a = 0$.

679. (d) Do yourself.

680. (b) (P) $m \geq 9 \cdot (3^9)^{1/9} \quad \Rightarrow \quad m \geq 27$

(Q) $\frac{n}{9} \geq \frac{1}{3} \quad \Rightarrow \quad n \geq 3$

(R) $\sum_{i=1}^{9} t_i = 3 \times 7 + 2 = 23$

(S) $\sum_{i=1}^{9} t_i = 3 \times 7 + 2 = 23$

681. (a) Do yourself.

682. (c) $f(x) = (k+2)(x^2 - kx + 2k - 3)$

683. (d) $x^3 - 3x - 1 = 0 \begin{matrix} \alpha+\beta \\ \beta+\gamma \\ \gamma+\alpha \end{matrix}$

$$\alpha+\beta+\beta+\gamma+\gamma+\alpha = 0 \quad \Rightarrow \quad \alpha+\beta+\gamma = 0$$

$\therefore$ Roots of the given equation are $-\alpha, -\beta, -\gamma$

Equation whose roots are α, β, γ is $x^3 - 3x + 1 = 0$

(P) $\alpha^2 + \beta^2 + \gamma^2 = (\Sigma a)^2 - 2\Sigma\alpha\beta = 0 - 2(-3) = 6$

(Q) $\alpha^3 + \beta^3 + \gamma^3 = 3\alpha\beta\gamma = 3(-1) = -3$

(R) $(\alpha+\beta+\gamma)(\beta+\gamma-\alpha)(g+a-b) = (-2\alpha)(-2\beta)(-2\gamma) = -8(-1) = 8$

(S) $(\alpha^3 - 3\alpha + 1)(\beta^3 - 3\beta + 1)(\gamma^3 - 3\gamma + 1) = 8$

684. (a) $xy^3 = 81,\ x, y > 0$

(P) $$\frac{x + 3 \cdot \left(\frac{y}{x}\right)}{4} \geq \left(x \cdot \frac{y^3}{27}\right)^{\frac{1}{4}}$$

$$\Rightarrow \quad (x+y)^4 \geq 4^4 \cdot \frac{81}{27} = 3 \cdot 2^8$$

(Q) $$\frac{x+3y}{4} \geq (xy^3)^{\frac{1}{4}}$$

$$\Rightarrow \quad (x+3y)^4 \geq 4^4 \cdot 81 = (12)^4$$

(R) $$\frac{3x + \frac{3y}{3}}{4} \geq \left(3x \cdot \frac{y^3}{27}\right)^{\frac{1}{4}}$$

$$\Rightarrow \quad (3x+y)^4 \geq 4^4 \cdot 9 = 9 \cdot 2^8$$

(S) $\frac{2x+3y}{4} \geq (2xy^3)^{\frac{1}{4}}$

$\Rightarrow \quad (2x+3y)^4 \geq 4^4 \cdot 2 \cdot 81 = 2(12)^4$

685. (b) $\cos(\theta+70^\circ) = \frac{-1}{3}$ where $\theta \in (0^\circ, 110^\circ)$

(P) $\theta + 70^\circ \in (70^\circ, 180^\circ)$

$\therefore \quad \tan(\theta+70^\circ) = -2\sqrt{2}$

(Q) $\cos(160^\circ+\theta) = \cos(90^\circ+\theta+70^\circ) = -\sin(\theta+70^\circ) = \frac{-2\sqrt{2}}{3}$

(R) $\sin(20^\circ-\theta) = \sin(90^\circ-(\theta+70^\circ)) = \cos(\theta+70^\circ) = \frac{-1}{3}$

(S) $\tan(25^\circ+\theta) = \tan(70^\circ+\theta-45^\circ) = \frac{\tan(70^\circ+\theta)-1}{1+\tan(70^\circ+\theta)} = \frac{-2\sqrt{2}-1}{1-2\sqrt{2}} = \frac{9+4\sqrt{2}}{7}$

686. (c) $x^4 - px^3 + qx^2 - rx + \frac{15}{32} = 0$

$$\text{A.M.} = \frac{\frac{a}{2}+\frac{b}{3}+\frac{c}{4}+\frac{d}{5}}{4} = \frac{1}{4}$$

$$\text{G.M.} = \left(\frac{a \cdot b \cdot c \cdot d}{2 \cdot 3 \cdot 4 \cdot 5}\right)^{\frac{1}{4}} = \left(\frac{15}{32 \cdot 2 \cdot 3 \cdot 4 \cdot 5}\right)^{\frac{1}{4}} = \frac{1}{4}$$

$\because \quad \text{A.M.} = \text{G.M.} \quad \Rightarrow \quad \frac{a}{2} = \frac{b}{3} = \frac{c}{4} = \frac{d}{5} = \frac{1}{4}$

$\Rightarrow \quad a = \frac{1}{2}, \; b = \frac{3}{4}, \; c = 1, \; d = \frac{5}{4}$

687. (b) , **688.** (c) and **689.** (d)

Sol. (I) $(\sqrt{2})^2 = 2^{\sin\theta} \cdot 2^{\cos\theta} \quad \Rightarrow \quad \sin\theta + \cos\theta = 1$

$\Rightarrow \quad \cos\left(\theta - \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} \quad \Rightarrow \quad \theta - \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{4}$

$\Rightarrow \quad \theta = 2n\pi + \frac{\pi}{4}$ or $2n\pi$ (rejected)

(II) $\sec\theta, \cos\theta, \tan\theta$ A.P.

$2\cos\theta = \sec\theta + \tan\theta \quad \Rightarrow \quad 2\cos^2\theta = 1 + \sin\theta$

$\Rightarrow \quad (1+\sin\theta)(2\sin\theta - 1) = 0$

$\Rightarrow \quad \sin\theta = \frac{1}{2} \quad \Rightarrow \quad \theta = 2n\pi + \frac{\pi}{6}$

(III) $2\log\sec\theta,\ \log 2,\ 2\log \operatorname{cosec}\theta$ A. P.

$\Rightarrow\quad \sec^2\theta,\ 2,\ \operatorname{cosec}^2\theta$ G. P.

$\Rightarrow\quad \sec^2\theta\cdot\operatorname{cosec}^2\theta = 4$

$\Rightarrow\quad (1+\tan^2\theta)(1+\cot^2\theta)=4 \quad\Rightarrow\quad \tan^2\theta+\cot^2\theta=2$

$\Rightarrow\quad \tan^2\theta = 1 \quad\Rightarrow\quad \theta = 2n\pi+\dfrac{\pi}{4}$

(IV) $(2+\sin\theta)(3+\sin\theta)(4+\sin\theta)=6$

$\Rightarrow\quad \sin\theta=-1 \qquad\Rightarrow\qquad \theta=2n\pi-\dfrac{\pi}{2}$

690. (d), **691.** (b) **and** **692.** (a)

Sol. (I) $\dfrac{\alpha}{\beta}+\dfrac{\beta}{\alpha}=2 \;\Rightarrow\; \alpha=\beta$

$D=0 \;\Rightarrow\; 64-4(k^2-6k)-0$

$k^2-6k-16=0 \;\Rightarrow\; (k-8)(k+2)=0$

(II) $(k-2)(3k+8)<0$

$\dfrac{-8}{3}<k<2$

(III) $|\alpha-\beta|<\sqrt{3}$

$\dfrac{\sqrt{4k^2-16}}{4}<\sqrt{3} \;\Rightarrow\; \sqrt{k^2-4}<2\sqrt{3}$

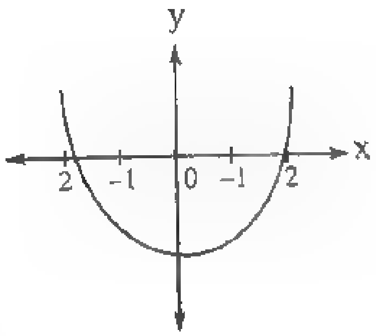
$\Rightarrow\quad 0\le k^2-4<12$

$\Rightarrow\quad k\in(-\sqrt{12},-2)\cup(2,\sqrt{12})$

(IV) $(x-2)(2kx+5)=0$ where $k>0$

$-2\le\dfrac{-5}{2k}<-1 \;\Rightarrow\; 2\ge\dfrac{5}{2k}>1$

$\Rightarrow\quad \dfrac{5}{4}\le k<\dfrac{5}{2}$



693. (c) $\dfrac{a+b}{c}+\dfrac{c}{a}+\dfrac{c}{b}\le 4$

$\left(\dfrac{a}{c}+\dfrac{c}{a}\right)+\left(\dfrac{b}{c}+\dfrac{c}{b}\right)\le 4 \;\Rightarrow\; a=b=c$

$\therefore\quad \Delta ABC$ is equilateral triangle.

$$r = (s-a)\tan\frac{A}{2}$$

$$\sqrt{3} = \left(\frac{3a}{2} - a\right)\frac{1}{\sqrt{3}} \quad \Rightarrow \quad a = 6$$

(P) $2(\cos A + 2\cos B) = 2\left(\frac{1}{2}+1\right) = 3$

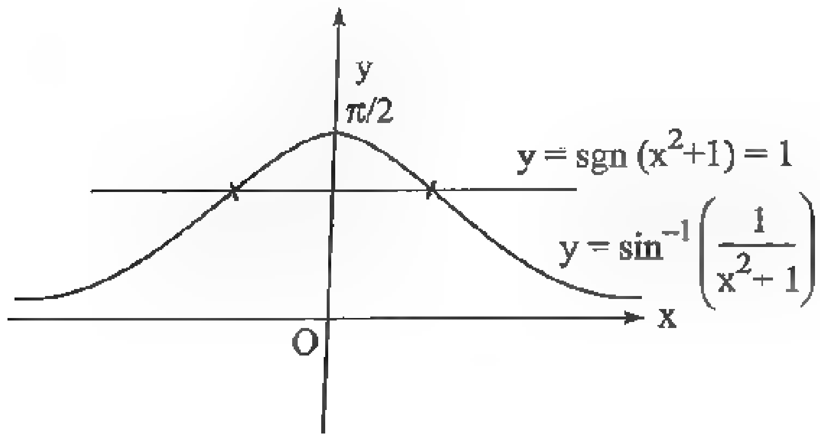
(Q) $R = 2r = 2\sqrt{3}$

(R) $\tan A + \text{Ar.}(\Delta ABC) = \sqrt{3} + \frac{\sqrt{3}}{4} \times 36 = 10\sqrt{3}$

(S) $r_1 = r_2 = r_3 = \frac{\Delta}{s-a} = \frac{9\sqrt{3}}{\frac{a}{2}} = 3\sqrt{3}$

$\therefore \quad r_1 + r_2 + r_3 - 9\sqrt{3}$

694. (d)



(P)

Clearly, number of solutions is 2.

(Q) $f(x) = \cos^{-1}\left(\frac{x^2}{1+x^2}\right) = \cos^{-1}\left(1 \quad \frac{1}{1+x^2}\right)$

$$t = 1 - \frac{1}{1+x^2} \quad \Rightarrow \quad t \in [0, 1)$$

$$\therefore \quad \cos^{-1} t \in \left(0, \frac{\pi}{2}\right] = (a, b] \quad \Rightarrow \quad [a+b] = 1$$

(R) $12(\tan^2(\tan^{-1}\alpha) + \tan^2(\tan^{-1}\beta)) = 12(\alpha^2 + \beta^2)$

$= 12((\alpha+\beta)^2 - 2\alpha\beta)$

$= 12\left(\frac{9}{4} - 2\cdot 1\right) = 3$

(S) $\sin(\cos^{-1}x)=\sin\left(\frac{\pi}{2}-\sin^{-1}x\right)=\cos(\sin^{-1}x)=t$

$f(x)=\sin(\cos^{-1}t+\sin^{-1}t)=\sin\frac{\pi}{2}=1$

$\sum_{x=1}^{n} f\left(\frac{3x}{16}\right)=1+1+1+1=4$

695. (b) $L: 3x-2y-4+\lambda(x\quad 2y+4)=0$

$P(a,b)\equiv(4,4)$

$S: x^2+y^2=8$

(P) $a+b=8$

(Q) $L_T=\sqrt{S_1}=\sqrt{16+16-8}=2\sqrt{6}$

(R) Least distance $= OP-r=4\sqrt{2}-2\sqrt{2}=2\sqrt{2}$

(S) Least radius of the circle containing the given circle is

$=OP+r=4\sqrt{2}+2\sqrt{2}=6\sqrt{2}$

696. (a) $\int\phi(x)\sin x\,dx$

$=\cos(-\phi(x)+\phi''(x)-\phi''''(x)+\ldots\ldots)+\sin x(\phi'(x)-\phi'''(x)+\phi'''''(x)-\ldots\ldots)$

$g(x)=-x^4-2x^2-1+12x^2+4-24=-x^4+10x^2\quad 21$

$f(x)=4x^3+4x-24x=4x^3-20x$

(P) $f(x)$ is many-one onto function.

(Q) $g(x)$ is many-one into function.

(R) number of points where $|f(x)$ is non-derivable is 3.

(S) number of points where $|g(x)|$ is non-derivable is 4.

697. (d) (P) $\int_1^3 g(x)dx+\int_3^1 g^{-1}(x)dx=0$

(Q) $f(x)|_{max.}=f(\pi)=\frac{5+3}{3-1}=4$

(R) $f'(x)=3x^2+2px+q \begin{matrix} \nearrow 1 \\ \searrow 3 \end{matrix}$

$\frac{q}{3}=3 \quad\Rightarrow\quad q=p$

$\frac{-2p}{3}=4 \quad\Rightarrow\quad p=-6$

$\Rightarrow\quad p+q=3$

(S) $L=\int_0^1 \frac{dx}{(1+x)(2+x)}$

$$=\int_0^1\left(\frac{1}{1+x}-\frac{1}{2+x}\right)dx=\left(\ln\left|\frac{1+x}{2+x}\right|\right)_0^1$$

$\Rightarrow \quad \ln\left(\frac{2}{3}\right)-\ln\left(\frac{1}{2}\right)$

$\Rightarrow \quad \ln\left(\frac{4}{3}\right)=\ln\left(\frac{a}{b}\right)$

$\therefore \quad |a-b|=1$

698. (a) $3x+y-z=0$ ...(1)

$x-\frac{py}{4}+z=0$ ...(2)

$2x-y+2z=q$ ...(3)

Eqn. (2) × (2) – eqn. (3)

$\Rightarrow \quad \left(1-\frac{p}{2}\right)y=4-q$

For unique solution, $p\neq 2$, $q\in N$ $\Rightarrow$ Number of ordered pairs (p, q) in [1, 10] are 90.

For infinite solution, $p=2$ and $q=4 \Rightarrow$ exactly one ordered pair.

For no solution, $p=2$ and $q\neq 4 \Rightarrow$ Number of ordered pairs (p, q) in [1, 10] are 9.

699. (b) $f(x)=\sin^{-1}(2x-1)+\cos^{-1}(2\sqrt{x-x^2})+\tan^{-1}\left(\frac{1}{1+[x^2]}\right)$

Domain of $f(x)=[0, 1]$

$$f(x)=\begin{cases} 0+\frac{\pi}{4}=\frac{\pi}{4} & , \quad x\in\left[0,\frac{1}{2}\right] \\ 2\sin^{-1}(2x-1)+\frac{\pi}{4} & , \quad x\in\left(\frac{1}{2},1\right) \\ \pi+\tan^{-1}\left(\frac{1}{2}\right) & , \quad x=1 \end{cases}$$

Now, verify the options.

700. (c) AEEI→ 4V, DRJLNG→ 6 C

– × × – × × – × × –

$$N=\frac{4!}{2!}\times 6!=12\times 720=2^6\times 3^3\times 5^1$$

Integer Type Questions

701. (100) $f(0)=\pm 10$

Differentiating both sides, we get

$$\underbrace{(f(x)+1)}_{\text{rejected}}(f'(x)+1)=0$$

$\therefore$ $f(x)=10-x$ or $-10-x$

Hence, $\frac{1}{|F|}\sum_{f(x)\in F}|f(100)|=\frac{1}{2}(90+110)=100$

702. (3)

$$L=\lim_{n\to\infty}\frac{\sum_{k=1}^{n}S_kS_{n-k+1}}{\sum_{k=1}^{n}S_k^2}$$

Now, $S_n=\frac{n}{1-\frac{1}{n+1}}=\frac{n(n+1)}{n}=n+1$

$$L=\lim_{n\to\infty}\frac{\sum_{k=1}^{n}(k+1)(n-k+2)}{\sum_{k=1}^{n}(k+1)^2}=\lim_{n\to\infty}\frac{\sum_{k=1}^{n}(k+1)(n-(k+1)+3)}{\sum_{k=1}^{n}(k+1)^2}=\frac{1}{2}$$

703. (0) Range in null set

$\therefore$ Answer is zero.

704. (8)

$$p=\lim_{n\to\infty}\left(\frac{\binom{5n}{3n}}{\binom{3n}{2n}}\right)^{\frac{1}{n}}$$

$$\ln p=\lim_{n\to\infty}\frac{1}{n}\sum_{r=0}^{2n-1}\ln\left(\frac{5n-r}{3n-r}\right)-\int_0^2\ln\left(\frac{5-x}{3-x}\right)dx$$

$\therefore$ $p=\frac{5^5}{3^6}$

$\therefore$ $a+b=5+3=8$

705. (2)

$$I(n)=\int_0^{\pi}\ln(1-2n\cos x+n^2)\,dx$$

Using King and add

$$I(n)=\frac{1}{2}\int_0^{\pi}\ln(1+n^4-2n^2\cos 2x)dx;$$ [Put $2x=t$]

$$I(n)=\frac{1}{4}\int_0^{2\pi}\ln(1+n^4-2n^2\cos t)\,dt$$

Using Queen

$$I(n)=\frac{1}{2}\int_0^{\pi}\ln(1+n^4-2n^2\cos t)\,dt=\frac{1}{2}I(n^2)$$

$\therefore \qquad \dfrac{I(n^2)}{I(n)}=2;$

Hence, $\dfrac{I(100)}{I(10)}=2$

706. (16) $\qquad f'(x)=\tan^2 x+K$ where $K=\displaystyle\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} f(x)\,dx$

$$f(x)=\tan x-x+Kx+C$$

$$f\left(\frac{\pi}{4}\right)=1-\frac{\pi}{4}+\frac{K\pi}{4}+C=\frac{-\pi}{4}$$

$$C+1=\frac{-K\pi}{4}$$

$$f(x)=\tan x-x+Kx-\frac{K\pi}{4}-1$$

Now, $\qquad K=\displaystyle\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\left(\underbrace{\tan x-x+Kx}_{\text{odd function}}-\frac{K\pi}{4}-1\right)dx=\frac{-\pi}{2}-\frac{K\pi^2}{8}$

Hence, $\qquad K=\dfrac{-4\pi}{8+\pi^2}$

$$\therefore\ \frac{8+\pi^2}{\pi}\cdot\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} f(x)\,dx=-4\equiv m\Rightarrow m^2=16$$

707. (100) $\qquad P(x)=100(x-\alpha_1)(x-\alpha_2)......(x-\alpha_n)$

$$L=\lim_{x\to\alpha_1}\left(\underbrace{1+100\prod_{i=1}^{n}(x-\alpha_i)}_{t}\right)^{\frac{100\prod_{i=2}^{n}(x-\alpha_i)}{100\prod_{i=1}^{n}(x-\alpha_i)}}=\lim_{x\to\alpha_1}\left((1+t)^{\frac{1}{t}}\right)^{100\prod_{i=2}^{n}(x-\alpha_i)}$$

$$= e^{\lim_{x\to\alpha_1} 100\prod_{i=2}^{n}(x-\alpha_i)} = e^{100K}$$

$$\therefore \qquad \frac{\ln L}{K} = 100$$

708\. (3) Given, $f(f(1)) = 0 \Rightarrow f(1+\alpha+\beta) = 0$

$f(f(2)) = 0 \Rightarrow f(4+2\alpha+\beta) = 0$

Hence, roots of $f(x)$ are $\alpha+\beta+1$ and $2\alpha+\beta+4$

Now, sum of roots $= 3\alpha+2\beta+5 = -\alpha$

$$\Rightarrow \qquad 4\alpha+2\beta = -5 \qquad ...(1)$$

Now, product of roots $= (\alpha+\beta+1)(2\alpha+\beta+4) = \beta$

$$(\alpha+\beta+1)\frac{3}{2} = \beta \qquad ...(2)$$

From Eqns. (1) and (2), $b = f(0) = \frac{-3}{2}$

Hence, $2|f(0)| = 3$

709\. (8) Replace $x \to \frac{2}{x}$

$$\left(\frac{8}{x^2}+\frac{6}{x}+4\right)^{10} - \sum_{r=0}^{20} a_r\left(\frac{2}{x}\right)^r$$

$$2^{10}(2x^2+3x+4)^{10} = \sum_{r=0}^{20} a_r \cdot 2^r \cdot x^{20-r}$$

$$2^{10}\cdot\sum_{r=0}^{20} a_r \cdot x^r = \sum_{r=0}^{20} a_r \cdot 2^r \cdot x^{20-r}$$

$\therefore$ Coefficient of x^7.

$$2^{10}a_7 = a_{13}2^{13}$$

$$\frac{a_7}{a_{13}} = 2^3 = 8$$

710\. (3) Let $f(x) = x^5 - x^3 + x - 2$

$$f'(x) = 5x^4 - 3x^2 + 1 > 0 \;\forall\; x \in R$$

$\therefore$ $f(x)$ is increasing $\Rightarrow$ only one real root.

$$f(1) = -1,\; f(2) = 24 \Rightarrow 1 < \alpha < 2$$

Since, α is a root of $x^5 - x^3 + x - 2 = 0$

$$\Rightarrow \qquad \alpha^5 - \alpha^3 + \alpha = 2$$

$$\alpha^4 - \alpha^2 + 1 = \frac{2}{\alpha}$$

$$(\alpha^2+1)(\alpha^4-\alpha^2+1) = \frac{2}{\alpha}(\alpha^2+1)$$

$$\alpha^6+1=2\alpha+\frac{2}{\alpha} \Rightarrow \alpha^6=2\alpha+\frac{2}{\alpha}-1$$

$$g(\alpha)=2\alpha+\frac{2}{\alpha}-1$$

$$g'(\alpha)=2-\frac{2}{\alpha^2}=\frac{2}{\alpha^2}(\alpha^2-1)=\frac{2}{\alpha^2}(\alpha-1)(\alpha+1)$$

g is increasing for $\alpha>1$.

$$g(1)<g(\alpha)<g(2)$$

$$3<g(\alpha)<4$$

$$3<\alpha^6<4$$

$$\therefore \quad [\alpha^6]=3$$

711. (125) Using King

$$I=\int_0^{\pi}\frac{\sin x(1+\sin x)e^{\sin x-\cos x}}{e^{-\cos x}+1}dx$$

$$I=\int_0^{\pi}\frac{\sin x(1+\sin x)e^{\sin x}}{e^{\cos x}+1}dx \qquad \text{...(1)}$$

Add $$2I=\int_0^{\pi}\frac{\sin x(1+\sin x)e^{\sin x}(e^{\cos x}+1)}{e^{\cos x}+1}dx$$

$$I=\frac{1}{2}\int_0^{\pi}\sin x(1+\sin x)e^{\sin x}dx$$

$$I=\frac{1}{2}\int_0^{\pi}e^{\sin x}(\sin x+1-\cos^2 x)dx$$

$$I=\frac{1}{2}\int_0^{\pi}e^{\sin x}dx+\frac{1}{2}\int_0^{\pi}e^{\sin x}(\sin x-\cos^2 x)dx$$

$$I=\frac{1}{2}\int_0^{\pi}e^{\sin x}dx+\frac{1}{2}[e^{\sin x}(-\cos x)]_0^{\pi}$$

$$I=\frac{1}{2}\int_0^{\pi}e^{\sin x}dx+\frac{1}{2}[1+1]$$

$$I=1+\frac{1}{2}\int_0^{\pi}e^{\sin x}dx$$

$$\therefore \quad 100\left(1+\frac{1}{4}\right)=125$$

712. (6)

$$S_n=1!+2!+3!+4!+5!+6!+7!+\ldots\ldots$$

$$S_n=1+2+6+24+120+720+7I$$

$$S_n=873+7I \qquad \text{...(1)}$$

$$\frac{S_n}{7}=\frac{873}{7}+I$$

$$\left[\frac{S_n}{7}\right]=124+I$$

$$7\left[\frac{S_n}{7}\right]=868+7I \qquad ...(2)$$

Eqn. (1) – Eqn. (2) = 5

$$T=\sin^{-1}(\sin 5)=5-2\pi$$

$$\int_0^1\frac{5-2\pi}{\sqrt{1-x^2}}\,dx=\frac{(5-2\pi)\pi}{2}=\frac{5\pi}{2}-\pi^2$$

$$\left(\frac{b}{c}+a\right)=\frac{2}{2}+5=6$$

713. (2)

$$\cos^2\theta+\sin^2\theta+2\beta=-4$$

$$\beta=\frac{-5}{2}$$

$$(\cos^2\theta+\beta)(\sin^2\theta+\beta)=2 \;\Rightarrow\; \sin^2 2\theta=-7$$

Now,

$$\cos^4\theta+\sin^4\theta+2\alpha=-2b$$

$$1-\frac{\sin^2 2\theta}{2}+2\alpha=-2b$$

$$9+4\alpha=-4b \qquad ...(1)$$

$$(\cos^4\theta+\alpha)(\sin^4\theta+\alpha)=b$$

$$\frac{\sin^4 2\theta}{10}+\frac{9\alpha}{2}+\alpha^2=b$$

$$(4\alpha+5)(4\alpha+17)=0$$

If $\alpha=\frac{-5}{4} \Rightarrow b=-1$ (rejected)

If $\alpha=\frac{-17}{4} \Rightarrow b=2$

714. (3)

$$6\cot\theta=\cot(\theta-\alpha)+\cot(\theta+\alpha)$$

$$\therefore \quad 6\cot\theta=\frac{\cos(\theta-\alpha)}{\sin(\theta-\alpha)}+\frac{\cos(\theta+\alpha)}{\sin(\theta+\alpha)}$$

$$6\cot\theta=\frac{\sin 2\theta}{\sin^2\theta-\sin^2\alpha}=\frac{2\sin\theta\cos\theta}{\sin^2\theta-\sin^2\alpha}$$

$$\frac{3}{\sin\theta}=\frac{\sin\theta}{\sin^2\theta-\sin^2\alpha}$$

$$3\sin^2\theta-3\sin^2\alpha=\sin^2\theta \Rightarrow \frac{2\sin^2\theta}{\sin^2\alpha}=3$$

715. (3)
$$x^2+y^2=9 \quad \ldots(1)$$
$$z^2+t^2=4 \quad \ldots(2)$$
Put
$$x=3\cos\theta \text{ and } z=2\cos\alpha$$
$$y=3\sin\theta \text{ and } t=2\sin\alpha$$
$$xt-yz=6$$
$$6(\sin\alpha\cos\theta-\cos\alpha\sin\theta)=6$$
$$\sin(\alpha-\theta)=1$$
$$p=xz=6\cos\theta\cos\theta$$
$$p=3[\cos(\theta+\alpha)+\underbrace{\cos(\theta-\alpha)}_{0}]$$
$$p-3\cos(\theta+\alpha)$$
$$p_{\max}=3$$

716. (1) $(2+h,3h-1)$ lies on the line $y=3x-7$

This must be the line at point of inflection
$$y'=3x^2-12x+b$$
$$y''=6x-12=0$$
$$x=2$$
$(2,-1)$ lies on the curve
$$\Rightarrow \quad 2b-a=15 \quad \ldots(1)$$
Also $$\left.\frac{dy}{dx}\right|_{x=2}-12-24+b=b\ \ 12=3$$
$$\Rightarrow \quad a-15 \text{ and } b=15 \Rightarrow \frac{a}{b}=1$$

717. (6) T: $$\frac{x}{a}\sec\theta-\frac{y}{b}\tan\theta-1$$
$$x=0 \Rightarrow y=-b\cot\theta=-4$$
$$\therefore \quad b\cot\theta=4 \quad \ldots(1)$$
N: $$\frac{ax}{\sec\theta}+\frac{by}{\tan\theta}=a^2+b^2$$
$$x=0;\ y=\frac{a^2+b^2}{b}\tan\theta=9 \quad \ldots(2)$$
Eqn. (1)× Eqn. (2) $$a^2+b^2=36=a^2e^2$$
$$ae=\pm 6$$
$\therefore$ x-coordinate of c is $ae=6$

718. (743) $$(EM)^T=20I$$
Take transpose on both sides
$$EM=20I \quad \ldots(1)$$

$$(E+M)^T = 17(E-M)^T$$
$$E^T + M^T = 17(E^T - M^T)$$
$$16E^T = 18M^T$$

Take transpose on both sides

$$16E - 18M \qquad \ldots(2)$$

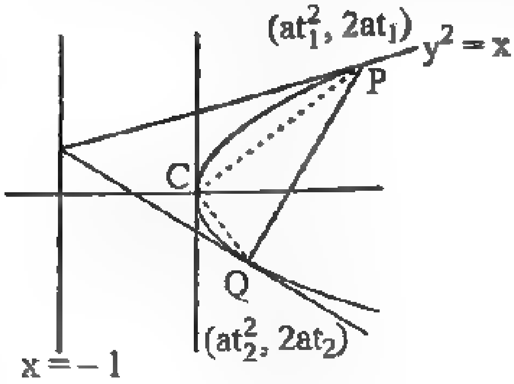
From Eqns. (1) and (2), we get

$$E = \pm\frac{3\sqrt{10}}{2}I; \qquad M = \pm\frac{4\sqrt{10}}{3}I$$
$$E^2 + M^2 = \frac{725}{18}I \Rightarrow a+b = 743$$

719. (37)
$$2((\cos x+\sin x)^2 - 1) = \sqrt{2}(\cos x+\sin x)$$
$$C+S = t$$
$$2t^2 - 1 = \sqrt{2}t \Rightarrow 2t^2 - \sqrt{2}t - 1 = 0$$
$$t = \frac{\sqrt{2}\pm\sqrt{2+8}}{4} - \frac{\sqrt{2}\pm\sqrt{10}}{4}$$
$$\frac{1}{\sqrt{2}}(\cos x+\sin x) = \frac{\sqrt{2}(1-\sqrt{5})}{4}$$
$$\sin\left(x+\frac{\pi}{4}\right) = -\left(\frac{\sqrt{5}-1}{4}\right)$$
$$x+\frac{\pi}{4} = \frac{-\pi}{10} \text{ or } \frac{11\pi}{10}$$
$$x = \frac{11\pi}{10} - \frac{\pi}{4} = \frac{17\pi}{20} \equiv \frac{a\pi}{b} \rightarrow a+b = 37$$

720. (4) Put
$$x - y = 1$$
$$(1)f(2x-1) - (2x-1)f(1) = 4(x)(x-1)(2x-1)$$
$$f(2x-1) + 2(2x-1) = 4x(x-1)(2x-1)$$
$$f(2x-1) = (2x-1)(4x^2 - 4x - 2)$$
$$f(t) = t(t^2-3) = t^3 - 3t$$
$$f'(t) = 3t^2 - 3 = 0$$
$$f(1) = -2 \rightarrow \text{min.}$$
$$f(\sqrt{3}) = 0 = f(-\sqrt{3})$$
$$f(-1) = 2 \rightarrow \text{max.}$$
$$\therefore \quad |2-(-2)| = 4$$

721. (4)



$$at_1t_2 = -1 \Rightarrow t_1t_2 = \frac{-1}{a} = -4$$

$$\Delta = \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ at_1^2 & 2at_1 & 1 \\ at_2^2 & 2at_2 & 1 \end{vmatrix} = \frac{1}{2}(2a^2t_1^2t_2 - 2a^2t_1t_2^2)$$

$$\Delta = |a^2t_1t_2(t_1 - t_2)| = \frac{1}{4}|t_1 - t_2|$$

$$\Delta = \frac{1}{4}\left(t_1 + \frac{4}{t_1}\right) = \frac{1}{4}\left[\left(\sqrt{t_1} - \frac{2}{\sqrt{t_1}}\right)^2 + 4\right]$$

$$\Delta_{\min} = 1 = M$$

$$4M = 4$$

722. (4)

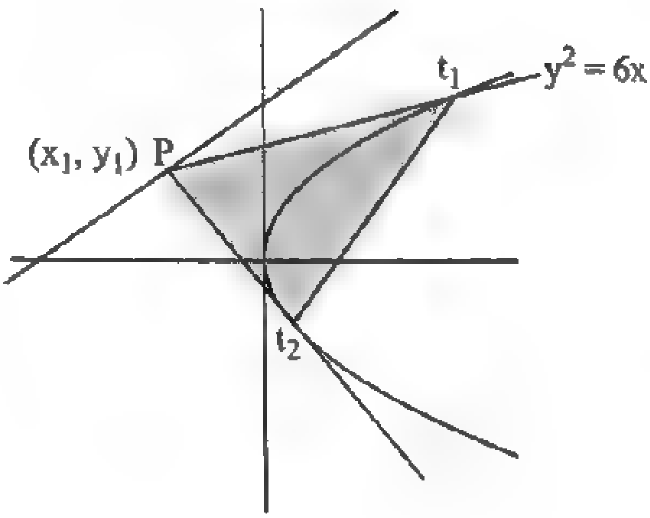
$$x_1 = at_1t_2$$

$$y_1 = a(t_1 + t_2)$$

$$x_1 - y_1 + 3 = 0$$

$$3 = a(t_1 + t_2 - t_1t_2)$$

$$\Rightarrow t_1 + t_2 - t_1t_2 = 2 \Rightarrow a = \frac{3}{2}$$



$$t_1 - 2 = t_2(t_1 - 1)$$

$$\Delta = \frac{1}{2}\begin{vmatrix} at_1^2 & 2at_1 & 1 \\ at_2^2 & 2at_2 & 1 \\ at_3^2 & 2at_3 & 1 \end{vmatrix}$$

$$\Delta = \frac{1}{2a}\left|(y_1^2 - 4ax_1)^{\frac{3}{2}}\right| = \frac{1}{3}\left|((x_1+3)^2 - 6x_1)^{\frac{3}{2}}\right| = \frac{1}{3}|x_1^2 + 9| = \frac{1}{3}\left|9 + \frac{3}{2}\cdot\frac{t(t-2)}{t-1}\right|$$

$$= 3 + \frac{1}{2}\left(\frac{t^2 - 2t}{t-1}\right) = 3 + \frac{1}{2}\left(\frac{(t-1)^2 + 1}{t-1}\right)$$

723. (6)

$$T_{r+1} = {}^nC_r 2^r x^r$$

$$a_k = {}^nC_k 2^k$$

$$\sum_{k=0}^{n} (3k+1)\, {}^nC_k \cdot a_k = 3\sum_{k=0}^{n} k \cdot {}^nC_k \cdot 2^k + \sum_{k=0}^{n} {}^nC_k \cdot 2^k = 3\sum_{k=0}^{n} n \cdot {}^{n-1}C_{k-1} \cdot 2^k + (1+2)^n$$

$$= 3n\sum_{k=0}^{n} {}^{n-1}C_{k-1} \cdot 2^{k+1} + 3^n = 2\cdot 3n(1+2)^{n-1} + 3^n = 2n\cdot 3^n + 3^n = (2n+1)3^n$$

$\therefore\ p = 2,\ q = 1,\ r = 3$

724. (4)

$$f(x) = \begin{bmatrix} \sin x, & a < x < b \\ c - x - d, & x \in (-\infty, a] \\ x - c - d, & x \in (b, \infty) \end{bmatrix}$$

[**Note:** C must lies between a and b.]

Now, $\left.\begin{matrix} f'(a^+) = \cos a \\ f'(a^-) = -1 \end{matrix}\right]$ $\therefore\ \cos a = -1 \Rightarrow a = (2n+1)\pi,\ n \in I$

$\left.\begin{matrix} f'(b^-) = \cos b \\ f'(b^+) = 1 \end{matrix}\right]$ $\therefore\ \cos b = 1 \Rightarrow b = 2m\pi,\ m \in I$

$\therefore\quad \sin a = 0 = \sin b$

$\therefore\quad f(a) = 0 = f(b)$

Coordinate $\Rightarrow f(a^+) = f(a^-) \Rightarrow 0 = c - a - d$

and $\quad \Rightarrow f(b^+) = f(b^-) \Rightarrow b - c - d = 0 \Rightarrow b = c + d$

$\therefore\ |[a+b+c+d]| = [a+2b]| = [(2n+1)\pi + 4m\pi] = |[(2n+1+4m)\pi]|_{\min} = 4$

Which occur at $n = -1$ and $m = 0$ since $a < b$ (note this point).

725. (4)

$$f(x) = x(2x^2 + ax + b)$$

$$D > 0$$

$$a^2 - 8b > 0$$

$$a^2 > 8b$$

$$(a, b)|_{\min} = (3, 1)$$

Note that b can not be zero. (think!)

$\therefore\quad a + b|_{\min} = 4$

726. (9) Given, $\quad \vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{c} = \vec{c}\cdot\vec{a} = \cos\alpha$

$\therefore\quad$ Volume of tetrahedron $= \frac{1}{6}[\vec{a}\ \vec{b}\ \vec{c}]$

Now, $\quad [\vec{a}\ \vec{b}\ \vec{c}]^2 = \begin{vmatrix} 1 & \cos\alpha & \cos\alpha \\ \cos\alpha & 1 & \cos\alpha \\ \cos\alpha & \cos\alpha & 1 \end{vmatrix}$

$$36V^2 = (1+2\cos\alpha)(1-\cos\alpha)^2$$

$$36\times\frac{1}{360} = (1+2\cos\alpha)(1-\cos\alpha)^2 = \frac{1}{10}$$

$$\therefore \qquad 3\cos^2\alpha - 2\cos^3\alpha = \frac{9}{10}$$

727. (7)
$$\int_1^4 x(5-f^{-1}(x))dx = \int_3^5 g(y)(5-y)g'(y)dy = \left((5-y)\frac{g^2(y)}{2}\right)_3^5 + \int_3^5 \frac{g^2(y)}{2}dy$$

$$-2\cdot\frac{g^2(3)}{2} + \frac{9}{2} = \frac{7}{2}$$

Hence $\quad 2\int_1^4 x(5-f^{-1}(x))dx = 7$

728. (22) Let
$$P(x) = \sum_{k=1}^{n} kx^k \equiv n(x-a_1)(x-a_2)\ldots\ldots(x-a_n)$$

$$\ln P(x) = \ln n + \sum_{i=1}^{n} \ln(x-a_i)$$

Differentiate two times and put $x = 1$, we get
$$\frac{P(1)P''(1)-(P'(1))^2}{(P(1))^2} = \sum_{k=1}^{n}\frac{1}{(1-a_k)^2} = 13$$

Now, $\quad P(x) = \sum_{k=1}^{n} kx^k$

$$\Rightarrow \qquad P(1) = \sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$P'(1) = \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$P''(1) = \sum_{k=1}^{n} k^2(k-1) = \sum n^3 - \sum n^2 = \left(\frac{n(n+1)}{2}\right)^2 - \frac{n(n+1)(2n+1)}{6}$$

On putting these values we get $n = 22$.

729. (9)
$$y = f(x) \text{ in } [0, 10]$$

Use L.M.V.T.
$$f'(x) = \frac{f(10)-f(0)}{10} = \frac{19-f(0)}{10}$$

$$-4 \le \frac{19-f(0)}{10} - 5 \le 4$$

$$1 \le \frac{19-f(0)}{10} \le 9$$

$$10 \le 19 - f(0) \le 90$$
$$-71 \le f(0) \le 9$$
$$\therefore \quad f(0)|_{max.} = 9$$

730. (720) Let the roots be $a - ib$, $a + ib$, then $a^2 + b^2 = 1$ (given $|z| = 1$)

Also, $\quad$ H.M. $= 2$

$$a^2 + b^2 = 2a$$
$$\therefore \quad a = \frac{1}{2}$$

$\therefore$ Quadratic equation $x^2 - x(2a) + a^2 + b^2 = 0$

$$x^2 - x + 1 = 0 \quad \text{...(1)}$$
$$x^2 - \log_2\left(\frac{\alpha}{\beta}\right) + \cos\alpha - \sin\beta = 0 \quad \text{...(2)}$$

On comparing equations (1) and (2), we get

$$\therefore \quad \log_2\left(\frac{\alpha}{\beta}\right) = 1 \quad \text{and} \quad \cos\alpha - \sin\beta = 1$$
$$\alpha = 2\beta \text{ in } (\cos\alpha - \sin\beta = 1)$$
$$\cos 2\beta \ \sin\beta = 1$$
$$2\sin^2\beta + \sin\beta = 0$$
$$\sin\beta = \frac{-1}{2} \quad \text{or} \quad \sin\beta = 0$$
$$\beta = \frac{7\pi}{6} = 210^\circ \quad \text{or} \quad 330^\circ \quad \text{or} \quad 180^\circ$$
$$\therefore \quad \text{Sum} = 720$$

731. (4) $(2xy\,dx + x^2 dy) + x^2 y\,dx + \left(\frac{y^3}{3} dx + y^2 dy\right) = 0$

$$x^2 y = t \quad \text{and} \quad \frac{y^3}{3} = u$$
$$(dt + t\,dx) + (u\,dx + du) = 0$$
$$dt + du = -(t+u)dx$$
$$\int \frac{dt + du}{t + u} = -\int dx$$
$$\ln(t + u) = -x + C \Rightarrow x^2 y + \frac{y^3}{3} = k'e^{-x} \quad \text{at } x = 1,\ y = 1$$
$$\frac{4}{3} = \frac{k'}{e} \Rightarrow k' = \frac{4e}{3}$$

$$x^2y+\frac{y^3}{3}=\frac{4e}{3};\ \text{put } x=0$$

$$\frac{y^3}{3}=\frac{4e}{3}$$

$$\therefore \qquad y^3(0)=4e \ \Rightarrow\ k=4$$

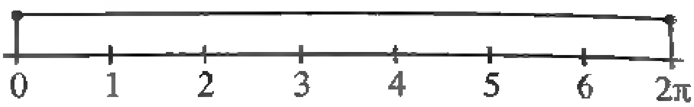
732. (2) $\underbrace{\int\left(\frac{1}{(\cos x)^{2019}}\right)}_{\text{I}}\cdot\underbrace{\operatorname{cosec}^2 x}_{\text{II}}\,dx-2019\int\frac{dx}{(\cos x)^{2019}}$

$$\frac{1}{(\cos x)^{2019}}\cdot(-\cot x)+2019\int\frac{\sin x}{(\cos x)^{2020}}\cdot\cot x\,dx$$

$\therefore \quad f(x)=\cot x,\quad g(x)=\cos x$

$$\Rightarrow \qquad \left|f\left(\frac{\pi}{4}\right)+g(0)\right|=2$$

733. (15) $x=I+\frac{1}{4}\left\{\frac{1}{4},\frac{5}{4},\frac{9}{4},\frac{13}{4},\frac{17}{4},\frac{21}{4},\frac{25}{4}\right\}$



$[x]\to\{1,2,3,4,5,6\}$

$(2x-1)^{1/3}\to\left\{\frac{1}{2}\right\}$

$\sin x\to\pi$

$\Rightarrow$ Total 15 points

734. (11) $\frac{1}{3}\lim_{x\to 0}\frac{3\int_0^x e^{-t^2}\,dt-3x+x^3}{x^5}\quad\left(\frac{\infty}{0}\right)$

$$=\frac{1}{3}\lim_{x\to 0}\frac{3e^{-x^2}-3+3x^2}{5x^4}=\frac{1}{5}\lim_{t\to 0}\frac{e^{-t}-1+t}{t^2}=\frac{1}{10}$$

735. (1) For $x=2$,

$$\lim_{x\to 2^-}f(x)=f(2);\qquad 2+a=bf(1)+c;\qquad 2+a=b(a+1)+c \qquad \ldots(1)$$

For $x=-2$,

$$\lim_{x\to 2^+}f(x)=f(-2);\quad -2+a=bf(-1)+c;\quad -2+a=b(a-1)+c \qquad \ldots(2)$$

Eqn. (1) – Eqn. (2), we get

$$b=2 \text{ and } a+c=0$$

$$\frac{a}{c}+b=1$$

736. (4) $$f(x)=(x-1)^{100}(x-2)^{2(99)}(x-3)^{3(98)}\ldots\ldots(x-100)^{100}=\sum_{n=1}^{100}(x-n)^{n(101-n)}$$

$$f'(x)=f(x)\sum_{n=1}^{100}\frac{n(101-n)}{x-n};\qquad \frac{f'(x)}{f(x)}=\sum_{n=1}^{100}\frac{n(101-n)}{x-n}$$

$$\Rightarrow\quad k=\sum_{n=1}^{100}\frac{n(101-n)}{x-n}=\sum_{n=1}^{100}n=\frac{100(101)}{2}=5050$$

$$\Rightarrow\quad \frac{k}{50}-97=\frac{5050}{50}-97=4$$

737. (3) Let $S=\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{2}{9}\right)+\tan^{-1}\left(\frac{1}{8}\right)+\tan^{-1}\left(\frac{2}{25}\right)+\tan^{-1}\left(\frac{1}{18}\right)+\ldots\ldots\infty$

$$=\tan^{-1}\left(\frac{2}{4}\right)+\tan^{-1}\left(\frac{2}{9}\right)+\tan^{-1}\left(\frac{2}{16}\right)+\tan^{-1}\left(\frac{2}{25}\right)+\tan^{-1}\left(\frac{2}{36}\right)+\ldots\ldots\infty$$

$$T_n=\tan^{-1}\left(\frac{2}{(n+1)^2}\right)$$

$$S=\sum_{r=1}^{\infty}\tan^{-1}\left(\frac{2}{r^2+2r+1}\right)=\sum_{r=1}^{\infty}\tan^{-1}\left(\frac{(r+2)-r}{1+r(r+2)}\right)=\sum_{r=1}^{\infty}\tan^{-1}(r+2)-\tan^{-1}(r)$$

$$S=\lim_{n\to\infty}\tan^{-1}\left(\frac{3n^2+7n}{n^2+9n+10}\right)=\tan^{-1}(3)$$

Hence, $\tan S=\tan(\tan^{-1}3)=3$

738. (18) $$AA^T=I\ \Rightarrow\ A^T=A^{-1}$$

$$A^T=\frac{adj.\,A}{|A|}=\pm\,adj.\,A$$

$$mq-np=\pm(0.3)$$

$$\therefore\quad 10(mq-np)=2,\ 3\ \text{or}\ -3$$

$$\text{Sum}=9+9=18$$

739. (6) $$f(x)=x^4-4x^3-8x^2+a$$

$$f'(x)=4(x^3-3x^2-4x)$$

$$=4x(x^2-3x-4)$$

$$=4x(x-4)(x+1)=0\ \text{at}\ x=-1,0,4$$

$$f(-1)=a-3\le 0,\ a\le 3$$

$$f(0)\ge 0\Rightarrow a\ge 0$$

$$a\in[0,3]$$

$$\text{Sum}=0+1+2+3=6$$

740. (4) $\sin x+\cos x=\sqrt{2}\cos\left(x-\frac{\pi}{4}\right)\in[1,\sqrt{2}]\ \forall\ x\in[0,1]$

Hence, as $x\in[0,1]$

$$10x^{\sqrt{2}}\le f(x)\le 10x$$

With equality holding if and only if $x=0$ or 1

As equality holds only for finitely many points, the inequalities become on strict integrating on all sides.

Hence, $4<10(\sqrt{2}-1)<\int_0^1 f(x)dx<5 \Rightarrow \left[\int_0^1 f(x)dx\right]=4$

741. (7) Let any point on the curve be (h,h^3).

The equation of tangent at this point is

$$y-h^3=3h^2(x-h)$$

$$y=3h^2x-3h^3$$

Equating it with the curve again,

$$x^3=3h^2x-2h^3$$

$$x^3-3h^2x+3h^3=0$$

$$(x-h)^2(x+2h)=0$$

This forms a relation,

$$x_{r+1}=-2x_r \to y_{r+1}=-8y_r$$

$$\frac{\lim\limits_{n\to\infty}\sum\limits_{r=1}^{n}\frac{1}{x_r}}{\lim\limits_{n\to\infty}\sum\limits_{r=1}^{n}\frac{1}{y_r}}=\frac{\dfrac{1}{1-\left(\frac{-1}{2}\right)}}{\dfrac{1}{1-\left(\frac{-1}{8}\right)}}=\frac{3}{4}$$

$$m+n=7$$

742. (7) $S_k=\int_0^1 x^2(1-x)^k dx=\int_0^1(1-x)^2x^k dx$ (use $=\int_a^b f(x)dx=\int_a^b f(a+b-x)dx$)

Hence $\sum\limits_{k=1}^{\infty}\int_0^1(1-x)^2x^k dx=\int_0^1(1-x)^2\underbrace{\sum\limits_{k=1}^{\infty}x^k}_{\text{infinite G.P.}}dx$

$$=\int_0^1(1-x)^2\left(\frac{x}{1-x}\right)dx=\int_0^1(x-x^2)dx=\frac{1}{2}-\frac{1}{3}=\frac{1}{6} \Rightarrow p+q=7$$

743. (6) $\ln A=\lim\limits_{x\to0}\frac{1}{\sin^{-1}nx}\ln(1+arc\tan(arc\sin x)+arc\tan(arc\sin 2x)+\ldots+arc\tan(arc\sin nx))$

$$= \lim_{x\to 0} \frac{1}{nx}(arc\tan(arc\sin x) + arc\tan(arc\sin 2x) + \ldots\ldots + arc\tan(arc\sin nx))$$

$$= \frac{1}{n}(1+2+3+\ldots\ldots+n) = \frac{n+1}{2}$$

Put $n = 11 \Rightarrow \ln A = 6$

744. (4) $$h(x) = x^2 \text{ for } x \geq \frac{1+\sqrt{5}}{2}$$

$$h(x) = 1+x \text{ for } 0 \leq x \leq \frac{1+\sqrt{5}}{2}$$

$$h(x) = 1-x \text{ for } \frac{-81}{100} \leq x < 0$$

$$h(x) = 1-x^2 \text{ for } x < \frac{-81}{100}$$

h is non-continuous at $\frac{-81}{100}$

h is non-differentiable at $\frac{1+\sqrt{5}}{2}$, 0 and $\frac{-81}{100}$

$m = 3$ and $n = 1$

Hence, $m + n = 4$

745. (2) $$\int_0^x g'(x)dx = \int_0^x f(x)dx$$

$$g(x) = \int_0^x f(t)dt$$

$\lim_{x\to 0} xg\left(\frac{1}{x}\right)$ Put $x = \frac{1}{t}$

$$\lim_{t\to\infty} \frac{g(t)}{t}$$

$$\lim_{t\to\infty} \frac{\int_0^t f(x)dx}{t}$$

Let $t = na$ where $n \in I$ a is period $= 3$

$$\lim_{t\to\infty} \frac{\int_0^{na} f(t)dt}{na} = \frac{n\int_0^3 f(x)dx}{3n} = \frac{6}{3} = 2$$

746. (2) $$f(x) + e^{f(x)} = \frac{2}{x} - \ln x - 1 \qquad \ldots(1)$$

Differentiate both sides

$$f'(x)+e^{f(x)}f'(x)=\frac{-2}{x^2}-\frac{1}{x}<0\ \forall\ x>0$$

Hence, $\quad f'(x)<0\ \forall\ x>0$

$\Rightarrow f$ is decreasing

Put $x=1$ in equation (1) $\Rightarrow f(1)+e^{f(1)}=1$

$\Rightarrow \qquad f(1)=0$

$f(2x^2+1)-f(x^2+5)\geq f(1),\ f(1)=0$

$f(2x^2+1)\geq f(x^2+5)$, f is decreasing

$2x^2+1\leq x^2+5$

$x^2\leq 4$

$-2\leq x\leq 2$, but $f:(0,\infty)\to R$

so the x that satisfies the inequality belongs to $0<x\leq 2$

747. (21) Given α,β are the roots of the quadratic equation $2x^2-5x+1=0$

Let us find an equation with roots α^2 and β^2, let $y=x^2$, so $x=\sqrt{y}$

$2y-5\sqrt{y}+1=0\Rightarrow 2y+1=5\sqrt{y}\Rightarrow 4y^2+4y+1=25y\Rightarrow 4y^2-21y+1=0\left\langle{c \atop d}\right.$

Put $\qquad \alpha^2=c$ and $\beta^2=d$

Now, $\qquad S_n=(c)^n+(d)^n$

Consider

$$\begin{aligned}4S_{2021}+S_{2019}&=4(c^{2021}+d^{2021})+c^{2019}+d^{2019}\\&=c^{2019}(4c^2+1)+d^{2019}(4d^2+1)\\&=c^{2019}(21c)+d^{2019}(21d)\\&=21S_{2020}\end{aligned}$$

Hence, $$\frac{4S_{2021}+S_{2019}}{S_{2020}}=21$$

748. (5) $$L=e^{\lim\limits_{x\to0}\frac{\int_0^{\sqrt{a^x-1}}(\sin 2\arctan t)(1+t^2)^{\ln a}\,dt}{x}}=e^{\lim\limits_{x\to0}\left(2\tan^{-1}\left(\sqrt{a^x-1}\right)\right)(1+a^x-1)^{\ln a}\ \frac{1}{2\sqrt{a^x-1}}\cdot a^x\ln a}$$

$$L=e^{\lim\limits_{x\to0}\frac{\left(2\sqrt{a^x-1}\right)(1)a^x\ln a}{25}}=e^{\ln a}=a=5$$

749. (12) For $p=0$ unique solution

For $p\neq0$, $\quad pt+\frac{1}{t}=5\ \Rightarrow\ pt^2-5t+1=0$

$D=0,\ 25=4p$

$$p = \frac{25}{4}$$

$$\therefore \quad a = 2$$

Now, $2, \alpha_1, \alpha_2, \ldots\ldots, \alpha_{20}, 6 \to$ H.P.

$2, \beta_1, \beta_2, \ldots\ldots, \beta_{20}, 6 \to$ A.P.

Now, $\alpha_{18}\beta_3 = 2 \times 6 = 12$

750. (1) $[f(y) - f(x)]y^y = x^x f\left(\frac{y^y}{x^x}\right)$

Differentiate w.r.t. x keeping y constant.

$$-f'(x) \cdot y^y = x^x f\left(\frac{y^y}{x^x}\right)(1 + \ln x) - x^x f'\left(\frac{y^y}{x^x}\right) \cdot \frac{y^y(1 + \ln x)}{x^x}$$

Given $\quad f'(1) = 1$

Put $\quad y^y = x^x$

$$-f'(x) \cdot x^x = x^x f(1)(1 + \ln x) - x^x f'(1)(1 + \ln x)$$

Now, $\quad f(1) = 0$

Hence, $\quad f'(x) = 1 + \ln x$

Integrate $\quad f(x) = C + x \ln x$

$$f(1) = 0 \Rightarrow C = 0$$

$$f(x) = x \ln x$$

So, $f(e) = e$ and $f(1/e) = -1/e$. So the answer is 1.

751. (6) $$x^2 + 2ax + a = \sqrt{a^2 + x - \frac{1}{16} - \frac{1}{16}}$$

$$(x + a)^2 + a - a^2 + \frac{1}{16} = \sqrt{(x + a) - \left(a - a^2 + \frac{1}{16}\right)}$$

Let $\quad x + a = y$ and $a - a^2 + \frac{1}{16} = p$

Then $\quad y^2 + p = \sqrt{y - p} \equiv f(x) = f^{-1}(x)$

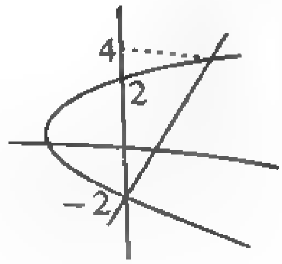
$\Rightarrow \quad f(x) = x$ has no real root $\Rightarrow y^2 + p = y$ has no real root

$$D < 0 \Rightarrow p > \frac{1}{4} \Rightarrow a - a^2 + \frac{1}{16} > \frac{1}{4} \Rightarrow a \in \left(\frac{1}{4}, \frac{3}{4}\right)$$

$$\therefore \quad \frac{c}{d} = \frac{5}{8} \Rightarrow \sqrt{4c^2 - d^2} = \sqrt{100 - 64} = 6$$

752. (9) $y' = \dfrac{2}{y-2} \Rightarrow y'y - 2y' = 2$

Integrating on both sides gives $\dfrac{y^2}{2} - 2y = 2x + C$



$f(1) = 2 \Rightarrow$ curve is parabola $P: y^2 = 4(x+y) - 8$

$$(y-2)^2 = 4(x-2) + 4$$

$$Y^2 = 4(X+1)$$

Where $X = x - 2$ and $Y = y - 2$

Line become $L: Y = 2X - 2$

Hence, required area $A = \displaystyle\int_{-2}^{4} \left(\left(\frac{Y+2}{2} \right) - \left(\frac{Y^2}{4} - 1 \right) \right) dy = 9$

753. (6) $A \Rightarrow B$ is true

$B \Rightarrow C$ is true

$C \Rightarrow A$ is true

$A \Rightarrow C$ is true

$B \Rightarrow A$ is true

$C \Rightarrow B$ is true

754. (108) $\vec{l} = x\vec{m} + y\vec{n} + z\vec{k}$

$\Rightarrow \quad 1 = \dfrac{-1}{11}(x+y+z)$ (dot product with $\vec{l}$)

$x + y + z = -11$ ...(1)

$\dfrac{-1}{11} = x - \dfrac{1}{11}(y+z)$ (dot product with $\vec{m}$)

$1 + 11m = y + z$ ...(2)

$\Rightarrow \quad x = -1$

$\Rightarrow \quad \vec{l} + \vec{m} = y\vec{n} + z\vec{k}$

$\Rightarrow \quad y(\vec{n} \cdot \vec{k}) + z = y + (\vec{n} \cdot \vec{k})z$ (dot product with $\vec{n}$ and $\vec{k}$)

$\Rightarrow \quad y = z \Rightarrow \vec{l} + \vec{m} = y(\vec{n} + \vec{k})$

$\Rightarrow \quad y = -5; \quad |\vec{l} + \vec{m}| = 5|\vec{n} + \vec{k}|$

$\Rightarrow \quad 2 - \dfrac{2}{11} = 25(2 + 2\vec{n} \cdot \vec{k}) \Rightarrow 2 + 2\vec{n} \cdot \vec{k} = \dfrac{4}{55}$

$\Rightarrow \quad \vec{n} \cdot \vec{k} = \dfrac{-53}{55}$

Hence, $A + B = 53 + 55 = 108$

755. (40) Taking tan of both the sides and using

$$\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$$

$$\tan(arc\tan\theta)=\theta \quad \text{and} \quad \tan\left(\frac{\pi}{2}+\theta\right)=-\cot\theta$$

$$\frac{(3+a)+(3+b)}{1-(3+a)(3+b)}=-\cot\left(arc\cot\frac{1}{3}\right)=\frac{-1}{3}$$

$$\Rightarrow \quad 18+3a+3b=ab+3a+3b+8 \quad\Rightarrow\quad ab=10$$

$$a+b\ge 2\sqrt{ab} \quad (\text{A.M.}-\text{G.M.})$$

$$=2\sqrt{10}$$

For equality $a=b=\sqrt{10}$

756. (6) Let $\tan x=a, \tan y=b, \tan z=c$

Given system of equation is equal to

$$a+b+c=6-\frac{1}{a}-\frac{1}{b}-\frac{1}{c}$$

$$a^2+b^2+c^2=6-\frac{1}{a^2}-\frac{1}{b^2}-\frac{1}{c^2}$$

$$a^3+b^3+c^3=6-\frac{1}{a^3}-\frac{1}{b^3}-\frac{1}{c^3}$$

From second equation we complete squares to get

$$\left(a-\frac{1}{a}\right)^2+\left(b-\frac{1}{b}\right)^2+\left(c-\frac{1}{c}\right)^2=0 \quad\Rightarrow\quad a=b=c=\pm 1$$

Now rearranging 3rd equation and adding 3 time first equation, we get

$$a^3+b^3+c^3+\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}+3\left(a+\frac{1}{a}+b+\frac{1}{b}+c+\frac{1}{c}\right)=6+18$$

$$\left(a+\frac{1}{a}\right)^3+\left(b+\frac{1}{b}\right)^3+\left(c+\frac{1}{c}\right)^3=24$$

Therefore $a=b=c=1$

Hence, $\left[\frac{\tan(x)}{\tan(y)}+\frac{\tan(y)}{\tan(z)}+\frac{\tan(z)}{\tan(x)}+3\tan(x)\tan(y)\tan(z)\right]=6$

757. (96)

$$T_1=(\vec{\mathbf{a}}\times\vec{\mathbf{b}})\times(\vec{\mathbf{c}}\times\vec{\mathbf{d}})$$

$$T_2=(\vec{\mathbf{a}}\times\vec{\mathbf{c}})\times(\vec{\mathbf{d}}\times\vec{\mathbf{b}})$$

$$T_3=(\vec{\mathbf{a}}\times\vec{\mathbf{d}})\times(\vec{\mathbf{b}}\times\vec{\mathbf{c}})$$

Then, $T_1 = [\vec{c}\ \vec{d}\ \vec{a}]\vec{b} - [\vec{c}\ \vec{d}\ \vec{b}]\vec{a}$

$T_2 = [\vec{d}\ \vec{b}\ \vec{a}]\vec{c} - [\vec{d}\ \vec{b}\ \vec{c}]\vec{a}$

$T_3 = [\vec{a}\ \vec{d}\ \vec{c}]\vec{b} - [\vec{a}\ \vec{d}\ \vec{b}]\vec{c}$

$\text{Sum} = -2[\vec{b}\ \vec{c}\ \vec{d}]\vec{a} + k\vec{a} = 0$ [Given $[\vec{b}\ \vec{c}\ \vec{d}] = 48$]

Hence, $k = 96$

758. (14) The answer is $\frac{4}{3^{10}}$, here is the solution.

Since tangents are at right angles, the line joining the point of contacts is the focal chord.

Here $a = 1$

Let $A(t_1^2, 2t_1)$ and $B(t_2^2, 2t_2)$

Since, these are end points of focal chord $t_1 t_2 = -1$

Let $t_1 = t$

Point $A(t^2, 2t)$ and $B\left(\frac{1}{t^2}, -\frac{2}{t}\right)$

Let centroid $C(h, k)$ and given $K(0, 0)$

$$h = \frac{t^2 + \frac{1}{t^2} + 0}{3} \quad \text{...(1)}$$

and $$k = \frac{2t - \frac{2}{t} + 0}{3} \Rightarrow t - \frac{1}{t} = \frac{3k}{2}$$

From Eqn. (1), we have $3h - 2 = \left(t - \frac{1}{t}\right)^2$

$$P_1 = y^2 = \frac{4}{3}\left(x - \frac{2}{3}\right)$$

On repeating similar procedure

$$P_2 = y^2 = \frac{4}{3^2}\left(x - \frac{4}{9}\right) \text{ thus}$$

$$P_{10} = y^2 = \frac{4}{3^{10}}\left(x - \frac{2^{10}}{3^{10}}\right)$$

$$a + b = 4 + \log_3 3^{10} = 4 + 10 = 14$$

759. (5) Let $$A = \int_0^2 y\,dx$$

$$\frac{dy}{y+A} = dx$$

Its solution is

$$\ln(y+A) = x+c$$

Given $y(0)=1$, so $c = \ln(1+A)$

$$Y = (1+A)e^x - A$$

$$A = \int_0^2 y\,dx = \int_0^2 ((1+A)e^x - A)\,dx = \frac{e^2-1}{4-e^2}$$

Hence solution is

$$y(x) = \frac{3e^x - e^2 + 1}{4-e^2}$$

760. (144) $\det.(A^2 + A) = \det.[(P^{-1}DP)(P^{-1}DP) + P^{-1}DP]$

or $\det.(P^{-1}D^2P + P^{-1}DP)$

or $\det.(P^{-1}(D^2+D)P)$

or $\det.(P^{-1}P)\cdot\det.(D^2+D)$

or $\det.(I)\cdot\det.(D)\cdot\det.(D+I)$

or $1^3\cdot(1\cdot2\cdot3)\cdot(2\cdot3\cdot4) = 6\cdot24 = 144$

761. (72) $\mu = 3$; $\lambda = 12$; $\gamma = 2$

762. (3) $$\int_{x_1}^{x_2} \frac{f(x)f'(x)}{\sqrt{1-(f(x))^4}}\,dx \geq \int_{x_1}^{x_2} x\,dx$$

$$\frac{1}{2}\sin^{-1} f^2(x)\Big]_{x_1}^{x_2} \geq \frac{1}{2}(x_2^2 - x_1^2)$$

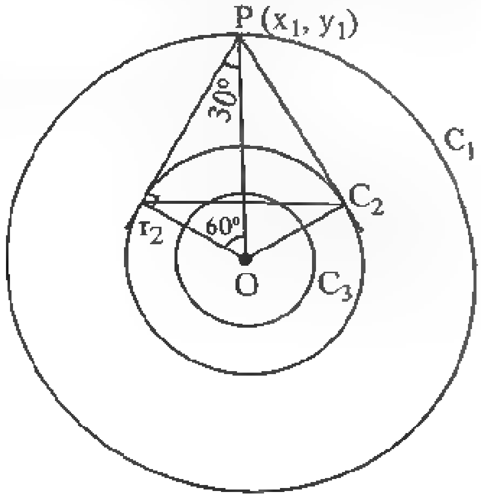
$$\sin^{-1} f^2(x_2^-) - \sin^{-1} f^2(x_1^+) \geq (x_2^2 - x_1^2)$$

$$\frac{\pi}{6} - \frac{\pi}{2} \geq (x_2^2 - x_1^2)$$

$$\frac{\pi}{3} \leq (x_1^2 - x_2^2)$$

$\therefore$ $$x_1^2 - x_2^2 \geq \frac{\pi}{3}$$

Hence minimum value of $x_1^2 - x_2^2$ is $\frac{\pi}{3}$ $\Rightarrow$ $k = 3$



763. (20) $x^2 + y^2 = r_1^2$

Chord of contact : $xx_1 + yy_1 = r_2^2$

$p = r$

$$\left|\frac{r_2^2}{\sqrt{x_1^2 + y_1^2}}\right| = r_3 \Rightarrow \left|\frac{r_2^2}{r_1}\right| = r_3$$

$$\rightarrow \qquad r_2^2 = r_1 r_3$$

Hence, $r_1, r_2, r_3, \ldots\ldots$ G.P.

$$\cos 60^\circ = \frac{r_2}{10} = \frac{1}{2} \Rightarrow r_2 = 5$$

Hence, G.P. is $10, 5, \frac{5}{2}, \ldots\ldots$

$$\therefore \qquad \lim_{n\to\infty} \sum_{r=1}^{n} r_i = \frac{10}{1-\frac{1}{2}} = 20$$

764. (20) $$a_n = \int_0^{\frac{\pi}{2}} \frac{\sin^2 nx}{\sin^2 x}\, dx$$

$a_1 = \frac{\pi}{2}, a_2 = \pi$; $< a_n >$ is an A.P. with common difference $= a_2 - a_1 = \frac{\pi}{2} = a_1$

$\therefore\ a_n = na_1$

$a_r = ra_1 \Rightarrow a_r a_{r+1} a_{r+2} a_{r+3} = r(r+1)(r+2)(r+3) \times a_1^4$

$$\therefore \sum_{r=1}^{n} a_r a_{r+1} a_{r+2} a_{r+3} = a_1^4 \sum_{r=1}^{n} r(r+1)(r+2)(r+3) = \frac{a_1^4}{5} \times n(n+1)(n+2)(n+3)(n+4)$$

Thus, $$S = \frac{5}{a_1^4} \sum_{n=1}^{\infty} \frac{1}{n(n+1)(n+2)(n+3)(n+4)}$$

$$S = \frac{5}{4 \times 4!}\left(\frac{2}{\pi}\right)^4$$

$$\Rightarrow \quad 24\pi^4 \cdot S = 20$$

765. (8) $\pi(f'(x))^2 \cos(\pi(f(x))) + \sin(\pi(f(x)))f''(x) = \frac{d}{dx}(\sin(\pi f(x)) \times f'(x))$

Let $g(x) = \sin(\pi f(x)) \times f'(x)$ in $[\alpha, \beta]$

$f'(x) = 0$ has one root in $[\alpha, \beta]$

Now, $\sin(\pi f(x)) = 0 \Rightarrow \pi f(x) = k\pi\ (k \in I) \Rightarrow f(x) = k$

Now, $f(x) = 3, 2, 1, 0$ for each value we will get 2 values of x.

Hence minimum number of roots of $g(x)$ in $[\alpha, \beta]$ is 9.

Using Rolle's Theorem minimum number of $g'(x)$ is 8.

766. (75) $(a+b+c)^2 = a^2+b^2+c^2+2(ab+ac+bc)$

$(\log_{10}x)(\log_{10}yz) = (\log_{10}x)(\log_{10}y+\log_{10}z)$

$= \log_{10}x\log_{10}y+\log_{10}x\log_{10}z$

and $xyz = 10^{81}$ implies $\log_{10}x+\log_{10}y+\log_{10}z = 81$

Using the property mentioned first, we get

$(\log_{10}x)^2+(\log_{10}y)^2+(\log_{10}z)^2$

$=(\log_{10}x+\log_{10}y+\log_{10}z)^2-2(\log_{10}x\cdot\log_{10}y+\log_{10}x\cdot\log_{10}z+\log_{10}y\cdot\log_{10}z)$

$\sqrt{(\log_{10}x)^2+(\log_{10}y)^2+(\log_{10}z)^2} = \sqrt{(81)^2-2\times 468}$

$\sqrt{(\log_{10}x)^2+(\log_{10}y)^2+(\log_{10}z)^2} = 75$

767. (150) If $f(x)-l(x)$ has four zeroes, where $l(x)$ is linear and $f(x)=x^4+2x^3+cx^2+9x+4$, then the second derivative has at least two zeroes (by two application of Rolle's theorem) but the second derivative is just $f''(x)=6x^2+6x+c=0$. This has two zeroes if and only if the discriminant $36-24c>0$, which happens if and only if $c<3/2$.

$\therefore \quad b=3/2$

Hence, $\quad 100(b)=150$

768. (46) Since the coefficient of x and x^0 are 0 and 6 respectively, let $f(x)=ax^3+bx^2+6$. Then

$$f(x)=ax^3+bx^2+6$$

$$f'(x)=3ax^2+2bx$$

$$f''(x)=6ax+2b$$

$$f'''(x)=6a=24$$

Since $\quad f'''(x)=6a=24 \Rightarrow a=4$

Since an extreme of $f'(x)$ occurs when $x=\frac{1}{6}$

$$\Rightarrow \quad f''\left(\frac{1}{6}\right)=6a\left(\frac{-1}{6}\right)+2b=-4+2b=0 \Rightarrow b=2$$

Therefore $f(x)=4x^3+2x^2+6$ and $f'(x)=12x^2+4x>0$ for $x>0$

Therefore $f(x)$ is an increasing function for $x>0$ and it is maximum when $x=2$

$\Rightarrow \quad \max(f(x)) = f(2)=4(2^3)+2(2^2)+6=46$

769. (2) Do yourself.

770. (5) $$\log_5(5x^2+5)\geq\log_5(ax^2+4x+a)\,\forall\, x\in R$$

$$\therefore \quad 5x^2+5\geq ax^2+4x+a\,\forall\, x$$

$$(5-a)x^2-4x+5-a\geq 0\,\forall\, x\in R$$

$$\therefore \quad 5-a>0$$

and $$D \le 0 \quad \text{...(2)}$$

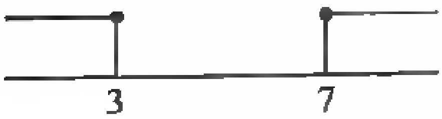
$$a < 5$$

and $$16-4(5-a)^2 \le 0$$

$$4-(5-a)^2 \le 0$$

$$(a-5)^2-4 \ge 0$$

$$(a-3)(a-7) \ge 0$$



From Eqns. (1) and (2) $\Rightarrow$ $a \in (2, 3]$ ...(3)

$\therefore$ $p+q=5$

771. (1) $b=0$ and $c=1$

Note that both x^3+2x^2+x+c and e^x are differentiable in their domain.

So make the function differentiable at $x=b$.

Since f is differentiable at $x=b$.

L.H.D. = R.H.D. (at $x=b$)

$$3b^2+4b+1=e^b$$

b is an integer, so LHS of the equation will always be an integer. However RHS will be an integer only if $b-0$.

If $b=0$, LHS = RHS, so $b=0$ is the solution to this equation.

Since f is differentiable at $x-b$, it is implied that it is also continuous at $x=b$.

$$\lim_{x \to b^-} f(x) = f(b) = \lim_{x \to b^+} f(x)$$

$$b^3+2b^2+b+c=e^b$$

when $b=0$, we get $c=e^0=1$

so the answer is $0+1=1$

772. (2) $P_1 : x+y+z=1$; $P_2 : x+y+z=\frac{9}{2}$ and $P_3 : 2x-5y+z=-5$

Vector along the line of intersection of the planes P_1 / P_3 or P_2 / P_3 is

$$=\vec{n}=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 2 & -5 & 1 \end{vmatrix} = 6\hat{i}+\hat{j}-7\hat{k}$$

Point on line L_1 which is the line of intersection of P_1 and P_3 say $(x_1, y_1, 0)$

Hence, $x_1 + y_1 = 1$ and $2x_1 - 5y_1 = -5$

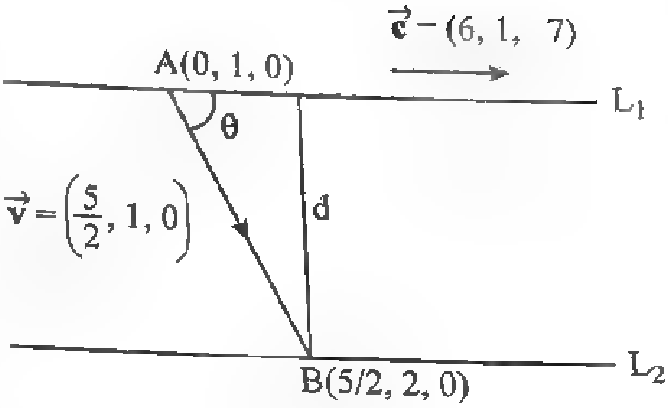
Solving, we get $y_1 = 1$ and $x_1 = 0$

Hence, point on L_1 is (0, 1, 0) ......(A)

|||ly point on line L_2 which is line of intersection of P_2 and P_3

say $(x_2, y_2, 0)$

Hence $x_2 + y_2 = \frac{9}{2}$ and $2x_2 - 5y_2 = -5$



Solving, we get $y_2 = 2$ and $x_2 = \frac{5}{2}$

Hence, point on L_2 is (5/2, 2, 0) ......(B)

$$\text{Now } d = |\vec{v}|\sin\theta = \frac{|\vec{v}\times\vec{c}|}{|\vec{c}|} = \frac{\sqrt{|\vec{v}|^2|\vec{c}|^2-(\vec{v}\cdot\vec{c})^2}}{|\vec{c}|} \quad ...(1)$$

$$\text{Now } |\vec{v}| = \sqrt{\frac{25}{4}+1} = \sqrt{\frac{29}{4}}; \quad |\vec{c}| = \sqrt{86}$$

$$\vec{v}\cdot\vec{c} = 16$$

$$\text{Now from equation (1), } d = \sqrt{\frac{\frac{29}{4}\times 86 - 256}{86}} = \sqrt{\frac{\frac{29}{4}\times 86 - 256}{86}}$$

$$= \sqrt{\frac{735}{2\times 86}} = \sqrt{4.27} = 2.06$$

$$\therefore \quad [d] = 2$$

773. (1801)

Since $e = \frac{7}{25} = \frac{c}{a}$, $c = 7k$ and $a = 25k$ and since $a^2 - b^2 = c^2$ in an ellipse, $b = 24k$.

The area of an ellipse is $A = \pi ab$, so $A_E = 600\pi k^2$.

The set of points for which two tangents of any curve meet at a right angle is an orthopetic, and an orthopetic for any ellipse is a circle with a radius of $\sqrt{a^2+b^2}$, and therefore an area of

$$A = \pi(a^2+b^2), \text{ so } A_F = 1201\,pk^2.$$

$$\text{Therefore, } \frac{A_E}{A_F} = \frac{600\pi k^2}{1201\pi k^3} = \frac{600}{1201}, \text{ so } p = 600, q = 1201, \text{ and } p + q = 1801$$

774. (2) Using AP-GP

$$\sqrt{(5\sqrt{5}+5)\sqrt{(5\sqrt{5}+5)^2\sqrt{(5\sqrt{5}+5)^3\sqrt{\ldots}}}} = (5\sqrt{5}+5)^{\frac{1}{2}+\frac{2}{4}+\frac{3}{8}+\ldots} = (5(\sqrt{5}+1))^2$$

Let $\frac{1}{2}+\frac{2}{4}+\frac{3}{8}+\ldots=2$

$$S=\frac{1}{2}+\frac{2}{4}+\frac{3}{8}+\ldots\ldots \quad \ldots(1)$$

$$2S=\frac{1}{1}+\frac{2}{2}+\frac{3}{4}+\ldots\ldots \quad \ldots(2)$$

$\therefore$ $$S=1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots\ldots$$ [subtracting Eqn. (1) from Eqn. (2)]

Now geometric progression, with $a=1$ and $r=\frac{1}{2}$, and thus evaluates to $\frac{1}{1-(1/2)}=2$.

Now note that $6+2\sqrt{5}=(\sqrt{5}+1)^2$

As a last step, recall the identity

$$(a-b)^3=a^3-b^3-3ab(a-b)$$

$$\therefore \quad (a-b)=\sqrt[3]{a^3-b^3-3ab(a-b)}$$

$$=\sqrt[3]{a^3-b^3-3ab\sqrt[3]{a^3-b^3-3ab(a-b)}}$$

$$=\sqrt[3]{a^3-b^3-3ab\sqrt[3]{a^3-b^3-3ab\sqrt[3]{a^3-b^3-3ab(a-b)}}}$$

Replacing a with 6 and b with 1, we get

$$6-1=\sqrt[3]{6^3-1-3\cdot6\sqrt[3]{6^3-1-3\cdot6\sqrt[3]{6^3-1-3\cdot6\sqrt[3]{\ldots}}}}$$

$$5=\sqrt[3]{215-18\sqrt[3]{215-18\sqrt[3]{215-18\sqrt[3]{\ldots}}}}$$

Replacing a with 6 and b with 1, we get

$$\therefore \log_{\sqrt{5}}\left[\frac{\sqrt{(5\sqrt{5}+5)\sqrt{(5\sqrt{5}+5)^2\sqrt{(5\sqrt{5}+5)^3\sqrt{\ldots}}}}}{(6+2\sqrt{5})\sqrt[3]{215-18\sqrt[3]{215-18\sqrt[3]{215-18\sqrt[3]{\ldots}}}}}\right]=\log_{\sqrt{5}}\left[\frac{25(\sqrt{5}+1)^2}{(\sqrt{5}+1)^2\cdot5}\right]$$

$$=\log_{\sqrt{5}}5=2$$

775. (8) $\because$ $$\lim_{x\to0}\frac{f(x)}{\sin^2x}=\lim_{x\to0}\frac{\frac{f(x)}{x^2}}{\frac{\sin^2x}{x^2}}=\lim_{x\to0}\frac{f(x)}{x^2}=8 \text{ and } \lim_{x\to0}f(x)=0$$

and $$\lim_{x\to0}\frac{g(x)}{2\left(1-\frac{x^2}{2!}+\ldots\ldots\right)-x\left(1+x+\frac{x^2}{2!}+\ldots\ldots\right)+x^3+x-2}=\lambda$$

$$= \lim_{x\to 0}\frac{g(x)}{x^2\left(-2+\frac{x}{2}+\ldots\ldots\right)} = 1 \;\Rightarrow\; \lim_{x\to 0}\frac{g(x)}{-2x^2} = \lambda$$

$$\therefore \quad \lim_{x\to 0}(1+2f(x))^{1/g(x)} = e^{\lim\limits_{x\to 0}\frac{2f(x)/x^2}{g(x)/x^2}} = e^{-8/\lambda} = \frac{1}{e} \;\Rightarrow\; \lambda = 8$$

776. (7)
$$I = \frac{1}{\sqrt{2}}\int_0^{\frac{\pi}{4}} e^{\sec x}\frac{\sin x+\cos x}{(1-\sin x)\cos x}\,dx$$

Dividing numerator and denominator by $\cos^2 x$, we have

$$I = \frac{1}{\sqrt{2}}\int_0^{\frac{\pi}{4}} e^{\sec x}\frac{\tan x\sec x+\sec x}{(\sin x-\tan x)}\,dx$$

$$\Rightarrow \quad I = \frac{1}{\sqrt{2}}\int_0^{\frac{\pi}{4}} e^{\sec x}\left(\frac{\tan x\sec x}{(\sec x-\tan x)}+\frac{\sec x}{(\sec x-\tan x)}\right)dx$$

$$\Rightarrow \quad I = \frac{1}{\sqrt{2}}\int_0^{\frac{\pi}{4}} e^{\sec x}\left(\tan x\sec x\left(\frac{1}{(\sec x-\tan x)}\right)+\left(\frac{\sec x(\sec x-\tan x)}{(\sec x-\tan x)^2}\right)\right)dx$$

$$f(x) = \frac{1}{\sec x-\tan x}$$

$$\Rightarrow \quad I = \frac{1}{\sqrt{2}}\left.\frac{e^{\sec x}}{(\sec x-\tan x)}\right|_0^{\frac{\pi}{4}}$$

Putting the limits, we have

$$I = \frac{1}{\sqrt{2}}\left(\frac{e^{\sqrt{2}}}{\sqrt{2}-1}-e\right) = \frac{(1+\sqrt{2})e^{\sqrt{2}}-e}{\sqrt{2}}$$

which gives, $\quad a=1;\ b=2;\ c=2;\ d=2$

$\Rightarrow \quad a+b+c+d = 1+2+2+2 = 7$

777. (2) Put $\sin x = t$

$$J_n = \int_0^1 t(1-t)^n dt = 2\left(\int_0^1 (1-t)^n\,dt - \int_0^1 \underbrace{(1-t)(1-t)^n}_{(1-t)^{n+1}}\,dt\right)$$

$$= 2\left(\int_0^1 t^n\,dt - \int_0^1 t^{n+1}\,dt\right) = 2\left(\frac{1}{n+1}-\frac{1}{n+2}\right)$$

$$\therefore \quad \sum_{n=0}^{\infty} J_n = 2\sum_{n=0}^{\infty}\left(\frac{1}{n+1}-\frac{1}{n+2}\right)$$
$$= 2(1) = 2$$

778. (4) Apply fundamental theorem of calculus

$$\lim_{x\to 0}\frac{x^2\int_0^{x^2} e^{-y^2}dy}{x^6} = \lim_{x\to 0}\frac{2x-2xe^{-x^4}}{6x^5} = \lim_{x\to 0}\frac{1}{3}\left(\frac{1-e^{-x^4}}{x^4}\right) = \lim_{y\to 0}\frac{1}{3}\left(\frac{1-e^{-y}}{y}\right)$$
$$= \lim_{y\to 0}\frac{1}{3}e^{-y} = \frac{1}{3}e^{-0} = \frac{1}{3}$$

Hence, $a = 1,\ b = 3 \Rightarrow a+b = 4$

779. (200) $S = \frac{2+6}{4^{100}}+\frac{2+2(6)}{4^{99}}+\frac{2+3(6)}{4^{98}}+\ldots\ldots+\frac{2+99(6)}{4^2}+\frac{2+100(6)}{4}$

$$= \frac{1}{4^{100}}((2+6)+(2+2(6))4+(2+3(6))4^2+\ldots\ldots+(2+100(6))4^{99})$$
$$= \frac{1}{4^{100}}\left(2\sum_{n=0}^{99}4^n+\sum_{n=1}^{100}n\cdot 4^{n-1}\right) \qquad \text{G.P. and A.G.P.}$$
$$= \frac{1}{4^{100}}\left(2\cdot\frac{4^{100}-1}{4-1}+6\left(\frac{1-100(4^{100})}{1-4}+\frac{4(1-4^{99})}{(1-4)^2}\right)\right) - 200$$

780. (30) $\frac{1}{d}\left(\frac{a_2-a_1}{a_2a_1}+\frac{a_3-a_2}{a_3a_2}+\ldots\ldots+\frac{a_{4001}-a_{4000}}{a_{4001}a_{4000}}\right) = 10$

$$= \frac{1}{d}\left(\frac{1}{a_1}-\frac{1}{a_2}+\frac{1}{a_2}-\frac{1}{a_3}+\ldots\ldots+\frac{1}{a_{4000}}-\frac{1}{a_{4001}}\right) = 10$$
$$= \frac{1}{d}\left(\frac{1}{a_1}-\frac{1}{a_{4001}}\right) = 10$$
$$= \frac{1}{d}\left(\frac{a_{4001}-a_1}{a_1\cdot a_{4001}}\right) = 10$$
$$= \frac{4000}{a_1\cdot a_{4001}} = 10 \quad (\text{as } a_{4001} = a_1+4000d)$$

$$a_1a_{4001} = 400;\ a_2+a_{4000} = 50 \Rightarrow (a_1+d)+(a_1+3999d) = 50$$
$$\Rightarrow \quad a_1+a_{4001} = 50$$
$$(a_1-a_{4001})^2 = (a_1+a_{4001})^2-4a_1a_{4001}$$
$$|a_1-a_{4001}| = 30$$

781. (6)

$$a+b = (k-2)c \qquad \ldots(1)$$
$$ab = -(k-1)d \qquad \ldots(2)$$
$$c+d = (k-2)a \qquad \ldots(3)$$

$$cd = -(k-1)b \qquad \ldots(4)$$

Substitute Eqn. (1), (3) to (2)

$$a((k-2)c-a) = -(k-1)((k-2)a-c)$$
$$(k-2)ac-a^2 = -(k-1)(k-2)a+(k-1)c \qquad \ldots(5)$$

Substitute Eqn. (1), (3) to (4)

$$c((k-2)a-c) = -(k-1)((k-2)c-a)$$
$$(k-2)ac-c^2 = -(k-1)(k-2)c+(k-1)a \qquad \ldots(6)$$

Eqn. (6) – Eqn. (5)

$$a^2-c^2 = (k-1)(k-2)(a-c)+(k-1)(a-c)$$
$$a+c = (k-1)^2$$
$$(k-2)(k-1)^2 = 100$$
$$k^3-4k^2+5k-102 = 0$$
$$(k-6)(k^2+2k+17) = 0$$
$$k = 6$$

782. (5) $$x!-(x-1)!>0 \qquad \text{(think!)}$$

$$\therefore \quad \underbrace{\left(2^{\frac{\pi}{\tan^{-1}x}}-4\right)}_{\substack{\text{Minimum} > 0 \\ \text{when } x\to\infty}}(x-4)(x-10) < 0$$

$\therefore$ $$x \in (4, 10)$$

$\therefore$ 5 integral values *i.e.*, 5, 6, 7, 8, 9

783. (13) $$f(x) \in (1, 2]$$

$\therefore$ $$[f(x)] = 1, 2$$

$$\frac{2(k+1)}{3} = 3 \ \Rightarrow\ k = \frac{7}{2} \ \text{ and } \ \frac{\mu}{3}-2 \ \Rightarrow\ \mu = 6$$

$\therefore$ $$2k+\mu = 13$$

784. (17) $$x_0 = \tan^{-1}(2)$$

$$b = \lim_{x\to\tan^{-1}2} \frac{(\tan^2 x-a)(1+\tan x)}{e^{(\tan x-2)}-1}$$
$$= \lim_{x\to\tan^{-1}2} \frac{(\tan x+\sqrt{a})(\tan x-\sqrt{a})(1+\tan x)}{(\tan x-2)}$$

For the existence of limit $\sqrt{a} = 2 \ \Rightarrow\ a = 4$

$$b = 12$$

$\therefore$ $$[a+b+x_0] = [4+12+\tan^{-1}(2)] = 17$$

785. (20) $\alpha=\sin\frac{\pi}{9}+\sin\frac{2\pi}{9}+\sin\frac{3\pi}{9}+\ldots\ldots+\sin\frac{20\pi}{9}=\dfrac{\sin\left(20\cdot\frac{\pi}{18}\right)}{\sin\frac{\pi}{18}}\cdot\sin\left(\frac{21\pi}{18}\right)=\cos\frac{\pi}{18}=\cos 10^{\circ}$

|||ly $\qquad \beta=\sqrt{3}\cos\frac{\pi}{18}=\sqrt{3}\cos 10^{\circ}$

$$\beta^2-\alpha^2=2\cos^2 10^{\circ}=1+\cos 20^{\circ}\equiv 1+\cos\lambda^{\circ}$$

$\therefore \qquad \lambda=20$

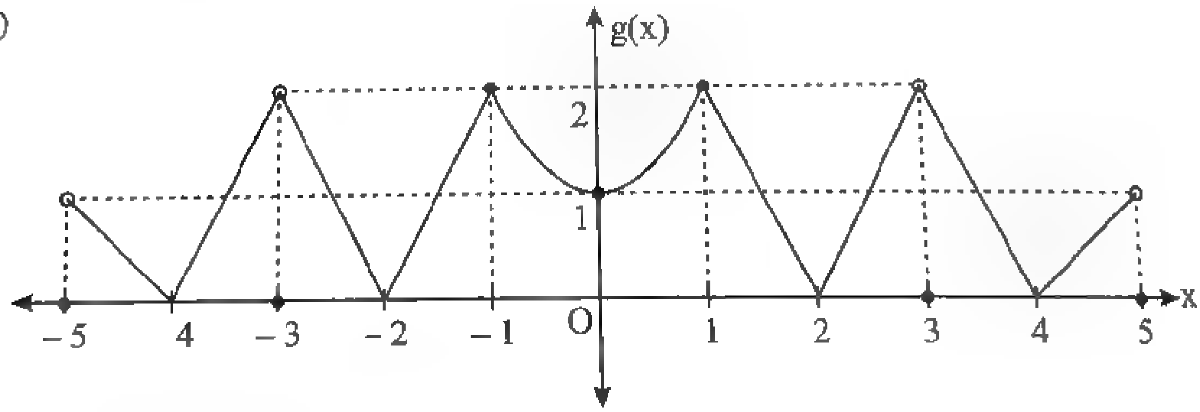
786. (5) $\qquad \displaystyle\sum_{n=2}^{m} f(n)=\ln\left(\frac{3}{m^2+2m}\right)$

$\therefore \qquad e^{\sum_{n=2}^{m} f(n)}=\dfrac{3}{m^2+2m}$

$$L=\lim_{m\to\infty}\left(e^{\frac{3}{m^2+2m}}-1\right)m^{\alpha}=\lim_{m\to\infty}\frac{3m^{\alpha}}{m^2+2m}\Rightarrow \alpha=2 \text{ and } L=3$$

$\therefore \qquad \alpha+L=5$

787. (16)



$$g(g(x))=1, \text{ let } g(x)=t$$

$$g(t)=1 \Rightarrow t=0, t_1, t_2, t_3, t_4, t_5, t_6$$

Where $\qquad 1<t_1<2, 2<t_2<3, 3<t_3<4, t_4, t_5, t_6<0$

Now, $\qquad \left.\begin{array}{l} g(x)=0 \Rightarrow 8 \text{ solutions} \\ g(x)=t_1 \Rightarrow 8 \text{ solutions}\end{array}\right\} \Rightarrow 16 \text{ solutions}$

788. (36) $\qquad T_r=\sqrt{r}\cdot\sqrt{r+1}(r+2)-\sqrt{r-1}\cdot\sqrt{r}(r+1)$

$$\sum_{r=1}^{16} T_r=4\sqrt{17}\cdot 18 \Rightarrow \frac{1}{\sqrt{68}}\sum_{r=1}^{16} T_r=36$$

789. (20) $\qquad f(x)=a(x-1)(x-2)(x-3)+2x+1$

$$(f(x))^2+4xf(x)+3x^2=0$$

Product of the roots, $\qquad \dfrac{(1-6a)^2}{a^2}=4 \Rightarrow 1-6a=\pm 2a\left\langle\begin{array}{l}a=1/8\\ a=1/4\end{array}\right.$

$$\therefore \qquad f(4) = 6a + 9 \begin{cases} \xrightarrow{a=1/8} \dfrac{3}{4}+9 \\ \xrightarrow{a=1/4} \dfrac{3}{2}+9 \end{cases}$$

$$k = 20 + \frac{1}{4} \quad \Rightarrow \quad [k] = 20$$

790. (9) $$f(x) = x + \sin x - [x + \sin x] + [x - \sin x] + [x]$$

$$x + \sin x = 0, 1, 2, 3 \Rightarrow x = 0, \alpha_1, \alpha_2, \alpha_3$$

$$x - \sin x = 0, 1, 2, 3 \Rightarrow x = 0, \beta_1, \beta_2, \beta_3$$

f is continuous at $x = 0$, but discontinuous at $\alpha_1, \alpha_2, \alpha_3, \beta_1, \beta_2, \beta_3, 1, 2, 3$

$\therefore$ Number of points of discontinuity $= 9$.

791. (3) Do yourself.

792. (8) $$\lim_{x\to 0} \frac{\ln(1+\sin^3 x\cos^2 x)\cot(\ln^3(1+x))\tan^4 x}{\sin(\sqrt{x^2+2}-\sqrt{2})\cdot\ln(1+x^2)} = \lim_{x\to 0}\frac{(\sin^3 x\cos^2 x)x}{\left(\sqrt{x^2+2}-\sqrt{2}\right)x^2}$$

$$= \lim_{x\to 0}\frac{2\sqrt{2}\sin^2 x}{x^2} = 2\sqrt{2} = \sqrt{8} \quad \Rightarrow \quad n = 8$$

793. (7) Do yourself.

794. (5) $$S = \sum_{r=1}^{\infty} \frac{r^3 + (r^2+1)^2}{(r^4+r^2+1)(r^2+r)} = \sum_{r=1}^{\infty} \frac{r^4+r^3+2r^2+1}{(r^4+r^2+1)(r^2+r)}$$

$$= \sum_{r=1}^{\infty}\left(\frac{1}{r(r+1)} + \frac{r}{r^4+r^2+1}\right) = \sum_{r=1}^{\infty}\left(\frac{1}{r} - \frac{1}{r+1} + \frac{r}{(r^2-r+1)(r^2+r+1)}\right)$$

$$= \frac{1}{1} + \sum_{r=1}^{\infty}\frac{1}{2}\left(\frac{1}{r^2-r+1} - \frac{1}{r^2+r+1}\right)$$

Let $\qquad s = r + 1$

$$= 1 + \frac{1}{2}\left(\sum_{r=1}^{\infty}\frac{1}{r^2-r+1} - \sum_{s=2}^{\infty}\frac{1}{s^2-s+1}\right)$$

$\Rightarrow \qquad r^2 + r + 1 = s^2 - s + 1$

$$= 1 + \frac{1}{2}(1) = \frac{3}{2}$$

Therefore, $\quad a + b = 3 + 2 = 5$

795. (3) $$\overrightarrow{OC} = m\overrightarrow{OA} + n\overrightarrow{OB}$$

$$\vec{c} = m\vec{a} + n\vec{b} \qquad \ldots(1)$$

Given $\qquad |\vec{a}| = 1,\ |\vec{b}| = 1,\ |\vec{c}| = \sqrt{2},\ \tan\alpha = 7$

Now take dot of equation (1) with $\vec{a}$ and $\vec{c}$ to get

$$m = 5/4; \quad n = 7/4$$

$$\therefore \quad m+n = 3$$

796. (2) Multiply N^r and D^r by $(\sec x)^6$ and proceed.

797. (3)

$$h(x) = 2 - |x-1| = \begin{cases} -x+3, & x \ge 1 \\ x+1, & x < 1 \end{cases}$$

$$g(x) = h(|x|) + |h(x)| = \begin{cases} 0, & x \ge 3 \\ 2(3-x), & 1 \le x < 3 \\ 2(x+1), & 0 \le x < 1 \\ 2, & x < 0 \end{cases}$$

Clearly, $g(x)$ is N.D. at $x = 0, 1, 3$.

798. (14) (i) If $x \ge 1, 2\tan^{-1} x - \frac{\pi}{2} - 2(\pi - 2\tan^{-1} x) \Rightarrow 2\tan^{-1} x = \frac{-3\pi}{2} \Rightarrow$ no solution.

(ii) If $0 \le x < 1, 2\tan^{-1} x = \frac{\pi}{2} - 2(2\tan^{-1} x) \Rightarrow \tan^{-1} x = \frac{\pi}{12} \rightarrow x = 2-\sqrt{3}$

(iii) If $-1 \le x < 0, -2\tan^{-1} x = \frac{\pi}{2} - 2(2\tan^{-1} x) \Rightarrow 2\tan^{-1} x = \frac{\pi}{2} \Rightarrow x = 1$ (not possible)

(iv) If $x < -1, -2\tan^{-1} x = \frac{\pi}{2} - 2(-\pi - 2\tan^{-1} x) \Rightarrow 6\tan^{-1} x = \frac{-5\pi}{2} \Rightarrow x = -2-\sqrt{3}$

$\therefore$ Sum of square = 14.

799. (5) Consider

$$I_2 - I_1 = \int_1^3 (3x^2 - 4x - 5) f(x^3 - 2x^2 - 5x + 2020)\,dx$$

Put $\quad x^3 - 2x^2 - 5x + 2020 = t$

$$\therefore \quad (3x^2 - 4x - 5)\,dx = dt$$

$$\therefore \quad I_2 - I_1 = \int_{2014}^{2014} f(t)\,dt = 0$$

Hence, $\quad I_2 = I_1$

$$\therefore \quad \frac{2I_1}{3I_2} = \frac{2}{3} = \frac{a}{b} \Rightarrow a+b = 5$$

800. (14) $I = \int_0^{\pi/2} \sin^2 t \ln(\sin t)\,dt = \int_0^{\pi/2} \frac{1}{2}(1-\cos 2t)\ln(\sin t)\,dt$ (Using I.B.P.)

$$= \frac{1}{2}\left(t - \frac{\sin 2t}{2}\right)\ln(\sin t)\Bigg]_0^{\pi/2} - \frac{1}{2}\int_0^{\pi/2} t\cot t\,dt + \frac{1}{4}\int_0^{\pi/2} \sin 2t \cot t\,dt \quad \text{(Using I.B.P.)}$$

$$= 0 - \frac{1}{2} t \ln \sin t\Bigg]_0^{\pi/2} + \frac{1}{2}\int_0^{\pi/2} \ln \sin t\,dt + \frac{1}{2}\int_0^{\pi/2} \cos^2 t\,dt$$

$$= 0 - 0 - \frac{\pi}{4}\ln 2 + \frac{1}{4}\int_0^{\pi/2}(1+\cos 2t)\,dt$$

[Note: $\int_0^{\pi/2}\ln(\sin x)dx = \frac{-\pi}{2}\ln 2$]

$$= -\frac{\pi}{4}\ln 2 + \frac{1}{4}\left[t + \frac{\sin 2t}{2}\right]_0^{\pi/2} = -\frac{\pi}{4}\ln 2 + \frac{\pi}{8} = \frac{\pi}{8}(1-\ln 4)$$

Therefore, $a+b+c+d = 1+8+1+4 = 14$

801. (4) Do yourself.

802. (4) $$f(x) = x^3 + ax^2 + bx + c = 0$$

Put $x = 1/t$, we get

$$ct^3 + bt^2 + at + 1 = 0 = g(t)$$

Hence, roots of $f(x)$ and $g(x)$ are reciprocal to each other.

Also, $g(1/3) = 0$ hence for $\lim\limits_{x\to p}\frac{f(x)}{g(x)}$ exists for all $p \in R - \{4\}$

$f(1/3)$ is also 0.

$\therefore$ Roots of $f(x)$ are $x^3 + ax^2 + bx + c = 0 \begin{cases} 3 \\ 1/4 \\ 1/3 \end{cases}$

Now, $$\lim_{x\to -1}\frac{f(x)+g(x)}{x+1} = 3(c+1) - (a+b)$$

$$x^3 + ax^2 + bx + c = 0 \begin{cases} 3 \\ 1/4 \\ 1/3 \end{cases}$$

$$-a = 3 + \frac{1}{4} + \frac{1}{3} = \frac{43}{12} \Rightarrow a = \frac{-43}{12}$$

$$b = \frac{3}{4} + \frac{1}{12} + 1 = \frac{11}{6} = \frac{22}{12}$$

$$c = \frac{-1}{4}$$

Now, $$\lim_{x\to -1}\frac{f(x)+g(x)}{x+1} = 3(c+1) - (a+b) = 3\left(\frac{-1}{4}+1\right) - \left(\frac{-43}{12} + \frac{22}{12}\right) = \frac{9}{4} + \frac{7}{4} = 4$$

803. (20) $$g(x) = \frac{2}{e^4}\int_1^x \underbrace{2te^{t^2}}_{\text{II}}\ \underbrace{f(t)}_{\text{I}}\,dt = \frac{2}{e^4}\left(\frac{f(t)}{t}\cdot e^{t^2}\Big|_1^x - \int_1^x\left(\frac{f(t)}{t}\right)' e^{t^2}\,dt\right)$$

$$= \frac{2}{e^4}\left(\frac{f(x)}{x}\cdot e^{x^2} - 1 - \int_1^x t^2\,dt\right) = \frac{2}{e^4}\left(\frac{f(x)}{x}\cdot e^{x^2} - 1 - \frac{1}{3}(x^3-1)\right)$$

$$g(x)=\frac{2}{e^4}\left(\frac{f(2)}{2}\cdot e^4-1-\frac{8}{3}+\frac{1}{3}\right)=f(2)-\frac{20}{3e^4}$$

$\Rightarrow\ (f(2)-g(2))3e^4=20$

804. (73) Given $|\vec{b}\times\vec{c}|=2$

$$|\vec{b}||\vec{c}|\sin\theta=2 \Rightarrow \sin\theta=\frac{1}{2}$$

$\therefore$ $\theta=30°$

Hence, angle between $\vec{b}$ and $\vec{c}$ is 30°.

Now, $2\vec{b}-\vec{c}=\lambda\vec{a} \Rightarrow |2\vec{b}-\vec{c}|=|\lambda\vec{a}| \Rightarrow |2\vec{b}-\vec{c}|^2=\lambda^2|\vec{a}|^2$

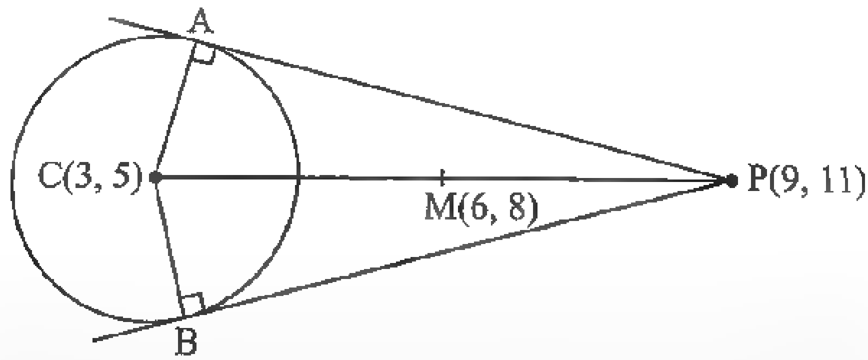
Hence, $\lambda^2=65-8\sqrt{3}$

$$\lambda=\sqrt{65-8\sqrt{3}}=\sqrt{\alpha-\beta\sqrt{3}}$$

Hence, $\alpha-65$ and $\beta=8$

$\therefore$ $\alpha+\beta=73$

805. (10)



The point inside the quadrilateral $ACBP$ which is equidistant from all the four vertices is the centre $M(6, 8)$ of the circle described on PC as diameter.

Hence, distance from origin to the point M is $\sqrt{36+64}=\sqrt{100}=10$

806. (132) If $m=2$

$$A=\begin{bmatrix}1&1\\1&1\end{bmatrix}, \text{ then } A^2=\begin{bmatrix}2&2\\2&2\end{bmatrix}, A^3=\begin{bmatrix}4&4\\4&4\end{bmatrix},\dots\dots, A^n=\begin{bmatrix}2^{n-1}&2^{n-1}\\2^{n-1}&2^{n-1}\end{bmatrix}$$

$$\text{If } m=3, \text{ then } A^n=\begin{bmatrix}3^{n-1}&3^{n-1}&3^{n-1}\\3^{n-1}&3^{n-1}&3^{n-1}\\3^{n-1}&3^{n-1}&3^{n-1}\end{bmatrix}$$

$$\therefore \text{ If } m=m, \text{ then } A^n=\begin{bmatrix}m^{n-1}&m^{n-1}\dots\dots&m^{n-1}\\m^{n-1}&m^{n-1}\dots\dots&m^{n-1}\\m^{n-1}&m^{n-1}\dots\dots&m^{n-1}\end{bmatrix}$$

$\therefore A^n=m^{n-1}A=16^{17}A=2^{68}A$

Factors of 68 are 1, 2, 4, 17, 34, 68.

If $n-1=1 \Rightarrow n=2$, then $m=2^{68}$

If $n-1=2 \Rightarrow n=3$, then $m=2^{34}$

If $n-1=4 \Rightarrow n=5$, then $m=2^{17}$

If $n-1=34 \Rightarrow n=35$, then $m=4$

If $n-1=68 \Rightarrow n=69$, then $m=2$

$\therefore$ Sum of all $n=132$.

807. (26)
$$f(x)=x+\frac{2}{3}x^3+\frac{2}{3}\cdot\frac{4}{5}x^5+\frac{2}{3}\cdot\frac{4}{5}\cdot\frac{6}{7}x^7+\ldots\ldots\infty$$
$$f'(x)=1+x\left(2x+\frac{2}{3}4x^3+\frac{2}{3}\cdot\frac{4}{5}6x^5+\ldots\ldots\right)$$
$$=1+x\frac{d}{dx}(xf(x))=1+xf(x)+x^2f'(x)$$

$(1-x^2)f'(x)=1+xf(x)$, which is a linear differential equation.

Also, $f(0)=0$

$\therefore$ $f(x)=\dfrac{\sin^{-1}x}{\sqrt{1-x^2}}$

Hence, Area $(A)=\displaystyle\int_{1/2}^{\sqrt{3}/2}\frac{\sin^{-1}x}{\sqrt{1-x^2}}dx=\frac{\pi^2}{24}$ $\Rightarrow$ $a+b=2+24=26$

808. (1) $|2x+\sin^2 a|+|2x+3+2\sin a|=0$

$\Rightarrow$ $2x+\sin^2 a=0$ and $2x+3+2\sin a=0$

Hence, $\sin^2 a-2\sin a-3=0$ $\Rightarrow$ $\sin a=-1$

$\therefore$ $2x=-1$ $\Rightarrow$ $x=\lambda=-1/2$

$\therefore$ $4\lambda^2=1$

809. (9) Given $\dfrac{\binom{a}{b}}{\left(\frac{b-1}{b}\right)}=4$

$\therefore$ $a=4(b-1)$ ...(1)

To find $S=\dfrac{\left(\frac{a}{a+b}\right)}{\left(1-\frac{1}{a+b}\right)}=\dfrac{a}{a+b-1}$ ...(2)

From Eqns. (1) and (2)

$S=\dfrac{4}{5}$

810. (72) $r = \frac{\Delta}{s} = 1 \Rightarrow \Delta = s = 7$

Now, $\Delta^2 = s(s-a)(s-b)(s-c)$

$7 = (7-a)(7-b)(7-c)$

$7 = 343 - 49\sum a + 7\sum ab - abc$ ...(1)

Now, $R = \frac{abc}{4\Delta} = 3 \Rightarrow abc = 12\Delta = 84$

$2s = 14 = (a+b+c)$

From equation (1)

$7 = 343 - (49 \times 14) + 7\sum ab - 84$

$\sum ab = 62$

Now, $a^2+b^2+c^2 = (a+b+c)^2 - 2\sum ab = 196 - 124 = 72$

811. (2) $\sin A \sin B \sin C + \cos A \cos B = 1$

$$\sin C = \frac{1-\cos A \cos B}{\sin A \sin B} \leq 1$$

$\cos(A-B) \geq 1$

Hence, $\cos(A-B) = 1 \Rightarrow A = B$

and $\sin C = 1 \Rightarrow C = 90^\circ$

and $A = B = 45^\circ$

Hence, $\cos^2 A + \sin^2 B + 2\sin^2 \frac{C}{2} = \frac{1}{2} + \frac{1}{2} + 2 \times \frac{1}{2} = 2$

812. (18) Let A be the event that the letter is from TATANAGAR and B be the event that letter is from CALCUTTA.

Also, let E be the event that on the letter, two consecutive letters TA are visible.

$P(A) = \frac{1}{2}$; $P(B) = \frac{1}{2}$

And $P(E/A) = \frac{2}{8}$ and $P(E/B) = \frac{1}{7}$

[If the letter is TATANAGAR, we see that the events of two consecutive letters visible are TA, AT, TA, AN, NA, AG, GA, AR]

So, $P(E/A) = \frac{2}{8}$ and same in case of CALCUTTA, so $P(E/B) = \frac{1}{7}$

Therefore, $P(A/E) = \frac{P(A)P(E/A)}{P(A)P(E/A) + P(B)P(E/B)}$

$$= \frac{(1/2)(2/8)}{[(1/2)(2/8)] + [(1/2)(1/7)]} = \frac{(1/8)}{(1/8)+(1/14)} = \frac{(1/8)}{(11/56)} = \frac{7}{11}$$

$p = 7, q = 11, p+q = 11+7 = 18$

813. (4) $\lim_{x\to 0}\frac{\ln((\cos x)^a)}{x^b}=\lim_{x\to 0}\frac{a\ln(\cos x)}{x^b}$

Now applying L' Hospital rule

$$\lim_{x\to 0}\frac{-a\tan x}{bx^{b-1}}=\lim_{x\to 0}\frac{\tan x}{x}\cdot\frac{-a}{b}\frac{1}{x^{b-2}}=\lim_{x\to 0}\frac{-a}{b}\frac{1}{x^{b-2}}$$

Now for limit to be finite

$$b-2=\{0,-1,-2,-3,-4,\,.........\}$$
$$b=\{2,1,0,-1,-2,-3,........\}$$

But b can only be $b=\{2,1\}$ as it is an outcome of a dice.

Now probability is

$$P=\frac{\text{No. of ways to select '}a\text{'}}{\text{Total no. of ways to select '}a\text{'}}\cdot\frac{\text{No. of ways to select '}b\text{'}}{\text{Total no. of ways to select '}b\text{'}}$$

$$P=\frac{6}{6}\cdot\frac{2}{6}=\frac{1}{3}\quad\Rightarrow\quad p+q=1+3=4$$

814. (0) Since point of minima is negative therefore point of maxima is also negative.

Hence, both roots of $f'(x)$ must be negative and distinct.

Sum of the roots < 0 and $D > 0$

Their intersection is ϕ, hence no values of a.

815. (3) Since the ellipse contains the circle

$\therefore$ Solving circle with ellipse, we get

$$b^2x^2+a^2(1-(x-1)^2)=a^2b^2$$
$$(b^2-a^2)x^2+2a^2x-a^2b^2=0$$
$$D=0$$
$$4a^4+4(a^2b^2)(b^2-a^2)=0$$
$$a^2+b^2(b^2-a^2)=0$$
$$a^2-b^2(a^2e^2)=0$$
$$1=b^2e^2\quad\Rightarrow\quad be=1$$

Now, area of ellipse $A=\pi ab$

$\therefore$ $A^2=\pi^2a^2b^2$

Now, $e^2=1-\frac{b^2}{a^2}=\frac{1}{b^2}$

$$e^2=1-\frac{1}{b^2}=\frac{b^2}{a^2}$$

$$a^2=\frac{b^4}{b^2-1}$$

Now, $$A^2 = f(b) = \frac{b^6}{b^2-1}$$

For maxima and minima $f'(b) = 0$

$$(b^2-1)6b^5 - b^6(2b) = 0 \quad \Rightarrow \quad 3(b^2-1) = b^2$$

$$b^2 = \frac{3}{2}; \quad \therefore \quad a^2 = \frac{9}{2}$$

$$\therefore \quad a^2 + b^2 = \frac{9}{2} + \frac{3}{2} = 6 = 2n \quad \Rightarrow \quad n = 3$$

816. (3) $$P(A) = ({}^nC_1 + {}^nC_2 + {}^nC_3 + \ldots\ldots + {}^nC_{n-1})\left(\frac{1}{2}\right)^n = \frac{2^n - 2}{2^n}$$

$$P(B) = P\text{ (0 girl or 1 girl)} = ({}^nC_0 + {}^nC_1)\left(\frac{1}{2}\right)^n = \frac{n+1}{2^n}$$

$$P(A \cap B) = P\text{ (exactly one girl)} = {}^nC_1 \times \left(\frac{1}{2}\right)^n$$

Now, $P(A \cap B) = P(A)P(B)$

$$\frac{n}{2^n} = \frac{2^n - 2}{2^n}\left(\frac{n+1}{2^n}\right) \quad \Rightarrow \quad n = \frac{(2^n - 2)(n+1)}{2^n}$$

Hence, $$\frac{n+1}{n} = \frac{2^n}{2^n - 2} = 1 + \frac{2}{2^n - 2}$$

Hence, $$\frac{1}{n} = \frac{2}{2^n - 2}$$

$$\therefore \quad 2^n - 2 = 2n \quad \rightarrow \quad n - 3$$

817. (0) Differentiate both sides

$$x\sin(f(x)) + \int_0^x \sin(f(t))dt = (x+2)\sin(f(x)) + \int_0^x t\sin(f(t))dt$$

$$\therefore \quad x^2\sin(f(x)) + x\sin(f(x)) = \int_0^x \sin(f(t))dt - \int_0^x t\sin(f(t))dt$$

Again differentiate

$$x^2\cos(f(x))f'(x) + 2x\sin(f(x)) + \sin(f(x)) + x\cos(f(x))f'(x) = \sin(f(x)) - x\sin(f(x))$$

$$= x(x+1)\cos(f(x)) + 3x\sin(f(x)) = 0$$

$$\Rightarrow \quad f'(x)\cot(f(x)) + \frac{3}{1+x} = 0$$

818. (25) A : Mr. A reaches late

B_1 : A goes to school by walking

B_2 : A takes but to school

E : A will be on time for atleast one out of 2 consecutive days.

$$P(B_1) = 3/4$$
$$P(B_2) = 1/4$$
$$P(A/B_1) = 1/3$$
$$P(A/B_2) = 2/3$$

$\therefore \quad P(A) = P(B_1 \cap A) + P(B_2 \cap A)$

$$= \frac{3}{4} \times \frac{1}{3} + \frac{1}{4} \times \frac{2}{3} = \frac{5}{12}$$

$\therefore \quad P(E) = 1 - P(A \cap A) = 1 - \frac{5}{12} \times \frac{5}{12} = \frac{119}{144} = \frac{p}{q}$

$\Rightarrow \quad q - p = 144 - 119 = 25$

819. (74) The probability of drawing one white balls and one green ball from the first urn is $\frac{1}{5}$.

The probability of drawing one white ball and one green ball from the second urn is $\frac{1}{3}$.

The probability of drawing one white ball and one green ball from the third urn is $\frac{2}{11}$.

Therefore, the probability that the third urn was chosen is $\dfrac{\frac{2}{11}}{\frac{1}{5} + \frac{1}{3} + \frac{2}{11}} = \frac{15}{59} = \frac{a}{b}$

Hence $15 + 59 = 74$

820. (4) $I = \displaystyle\int_0^{\frac{\pi}{2}} \frac{\cos^4 x + \sin x \cos^3 x + \sin^2 x \cos^2 x + \sin^3 x \cos x}{\sin^4 x + \cos^4 x + 2\sin x \cos^3 x + 2\sin^2 x \cos^2 x + 2\sin^3 x \cos x} dx$

$$I = \int_0^{\frac{\pi}{2}} \frac{(\sin x \cos^3 x + \sin^3 x \cos x) + (\cos^4 x + \sin^2 x \cos^2 x)}{\sin^4 x + \cos^4 x + 2\sin^2 x \cos^2 x + (2\sin x \cos^3 x + 2\sin^3 x \cos x)} dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sin x \cos x + (\cos^4 x + \sin^2 x \cos^2 x)}{\sin^4 x + \cos^4 x + 2\sin^2 x \cos^2 x + 2\sin x \cos x} dx$$

Use King and add

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sin^4 x + \cos^4 x + 2\sin^2 x \cos^2 x + 2\sin x \cos x}{\sin^4 x + \cos^4 x + 2\sin^2 x \cos^2 x + 2\sin x \cos x} dx = \frac{\pi}{2}$$

$\therefore \quad I = \frac{\pi}{4}$

Hence, the given value of definite integral is 4.

821. (6) $a^{(\log_5 11)^2} = (a^{\log_5 11})^{\log_5 11} = 25^{\log_5 11} = 121$

$a^{(\log_{11} 25)^2} = (b^{\log_{11} 25})^{\log_{11} 25} = (\sqrt{11})^{\log_{11} 25} = 5$

822. (8) $x = 8\log_3 2,$

$y = \log_2(\log_3 9) = 1,$

$z = \log_5 3 \cdot \log_7 5 \cdot \log_2 7 = \log_2 3$

$\Rightarrow \quad xyz = 8$

823. (6) $\left|\log_{\sqrt{2}} 30 - \left|\log_2 9 + \left|\log_2 3 - \log_2 5\right|\right|\right| = \left|\log_{\sqrt{2}} 30 - \left|\log_2 9 + \log_2 5 - \log_2 3\right|\right|$

$= \left|\log_{\sqrt{2}} 30 - \left|\log_2 3 + \log_2 5\right|\right| = \left|\log_{\sqrt{2}} 30 - \log_2 15\right| = \log_2 60$

$5 < \log_2 60 < 6$

824. (3) $\sqrt{x} - \dfrac{1}{\sqrt{x}} = 3 \quad \Rightarrow \quad x + \dfrac{1}{x} - 2 = 9$

$\Rightarrow \quad x + \dfrac{1}{x} = 11$

$\Rightarrow \quad x^3 + \dfrac{1}{x^3} + 3\left(x + \dfrac{1}{x}\right) = 1331 \quad \Rightarrow \quad x^3 + \dfrac{1}{x^3} = 1298$

$\Rightarrow \quad 3 < \log_{10} 1298 < 4$

825. (1) $5^{(\log_5 x)^2} = x^{\log_5 x} = a$

$a + a = 1250 \quad \Rightarrow \quad a = 625 = x^{\log_5 x}$

$(\log_5 x)^2 = \log_5 625 = 4$

$\log_5 x = 2 \quad \text{or} \quad -2 \quad \Rightarrow \quad x = -25 \quad \text{or} \quad \dfrac{1}{25}$

826. (0) $\log_{(x^2+2)}(5+\sqrt{x}) > 2$ and $\log_{(2+\sqrt{x})}(5+x^2) > 0$

827. (2) $x^2 - 2x = 2x^2 + 2x + 3$

$\Rightarrow \quad x^2 + 4x + 3 = 0$

$\therefore \quad x = -1, -3 \quad \Rightarrow \quad$ 2 real values

828. (32) $\sin^2 18° + \sin^2 36° + \sin^2 54° + \sin^2 72° = \sin^2 18° + \sin^2 36° + \cos^2 36° + \cos^2 18° = 2$

$\Rightarrow 16(2) = 32.$

829. (3) $\therefore \quad \tan A = \tan B = \tan C$

$\therefore \quad a = b = c = 2$

$\therefore \quad \text{Area} = \dfrac{\sqrt{3}}{4} a^2 = \sqrt{3}$

830. (3) $\log_{27} x = t$

$\therefore \quad 1 - 2(2t)^2 = t - 2t^2$

$\therefore \quad 1-8t^2 = t-2t^2 \quad \Rightarrow \quad 6t^2+t-1=0$

$\therefore \quad t = \log_{27} x = \frac{-1}{2}, \frac{1}{3}$

$\Rightarrow \quad x = \frac{1}{\sqrt{27}}, 3$

831. (12) $$\frac{4\sin^4\theta}{4\cos^2\theta - 4\sin^2\theta\cdot\cos^2\theta} = \frac{\sin^4\theta}{\cos^2\theta\cdot\cos^2\theta} = (\tan\theta)^4$$

$\therefore \quad (\tan\theta)^4 = \left(\frac{2-\sqrt{3}}{2+\sqrt{3}}\right)^2 = (2-\sqrt{3})^4$

$\therefore \quad \tan\theta = 2-\sqrt{3} \quad \Rightarrow \quad \theta = \frac{\pi}{12}$

832. (1) Denominator $= \sin 20^\circ + 2\sin 60^\circ\cdot\cos 10^\circ = \sin 20^\circ + \sqrt{3}\cos 10^\circ$

and Numerator $= 2\sin 80^\circ(\sin 120^\circ + \sin 10^\circ)$

$$= 2\sin 80^\circ\frac{\sqrt{3}}{2} + \cos 70^\circ - \cos 90^\circ = \sqrt{3}\cos 10^\circ + \sin 20^\circ$$

$\therefore \quad x = 1$

833. (3) LHS $= (\sqrt{3}\sin 55^\circ + \sqrt{3}\sin 5^\circ)^2 = 3\cos^2 25^\circ = \frac{3}{2}(1+\cos 50^\circ)$

$\therefore \quad a = b = \frac{3}{2}$

834. (45) $x = (\cos\theta+1)-(\cos\theta-1)-(\cos\theta-2)-(\cos\theta-3)$

$\therefore \quad x = 7-2\cos\theta$

$x_{max} = 7-2(-1) = 9$

$x_{min} = 7-2(1) = 5$

835. (1) $\lambda^2-4\lambda+3=0 \quad \Rightarrow \quad \lambda = 1, 3$

$\lambda^2-5\lambda+6=0 \quad \Rightarrow \quad \lambda = 2, 3$

$\lambda^2-9=0 \quad \Rightarrow \quad \lambda = 3, -3$

$\therefore \quad \lambda = 3$

836. (5) $3^{2+2\log_3 x} = 3^{1+\log_3 x} + 210$

$\Rightarrow \quad 9x^2 = 3x+210$

$\therefore \quad 9x^2-3x-210=0 \quad \Rightarrow \quad x=5$

837. (1) $(x+1)(x^2-2x+5)=0$ and $(ax^2+bx+5)=0$

$\therefore \quad \frac{a}{1} = \frac{b}{-2} = 1 \quad \Rightarrow \quad a=1;\ b=-2$

838. (5) $x = 2\sin\theta$ and $y = 2\cos\theta$

$\therefore \quad x^2-xy+y^2 = 4\sin^2\theta + 4\cos^2\theta - 4\sin\theta\cos\theta = 4-2\sin 2\theta \in [2, 6]$

839. (4) $x = 2+\sqrt{3} \Rightarrow (x-2)^2 = 3 \Rightarrow x^2 - 4x + 1 = 0$

$$\therefore \quad f(x) = (x^2 - 4x + 1)^2 + 2x - 2\sqrt{3}$$

$$f(x = 2+\sqrt{3}) = 4 + 2\sqrt{3} - 2\sqrt{3} = 4$$

840. (3) $\lambda = \left(\dfrac{\cos 65^\circ + \sqrt{3}\cos 85^\circ + \sin 85^\circ}{\sin 65^\circ}\right)^2 = \left(\dfrac{\cos 65^\circ + 2\sin 145^\circ}{\sin 65^\circ}\right)^2 = \left(\dfrac{\sin 25^\circ + \sin 35^\circ + \sin 35^\circ}{\sin 65^\circ}\right)^2$

$$\lambda = \left(\frac{\sin 85^\circ + \sin 35^\circ}{\sin 65^\circ}\right)^2 = \left(\frac{2\sin 60^\circ \cos 25^\circ}{\sin 65^\circ}\right)^2 = 3$$

841. (5) $\ln(4^x - 2)^2 = \ln\left(8\left(4^x - \dfrac{31}{8}\right)\right)$

$$(4^x - 2)^2 = 8 \cdot 4^x - 31$$

Let $4^x = t$

$$(t-2)^2 = 8t - 31$$

$$t^2 - 12t + 35 = 0 \quad \Rightarrow \quad t = 5, 7$$

$$4^x = 5, 7 \Rightarrow \quad x = \log_4 5,\ \log_4 7$$

$\therefore$ Sum of the roots $= \log_4 5 + \log_4 7 = \log_4 35 \in (2, 3) \equiv (a, b)$

$\therefore \quad a + b = 5$

842. (5) $|\log_2^2 x - 5\log_2 x + 6| + |2\log_2 x - 6| = |\log_2^2 x - 7\log_2 x + 12|$

$|x| + |y| = |x - y| \Rightarrow xy \leq 0$

$\therefore (\log_2^2 x - 5\log_2 x + 6)(2\log_2 x - 6) \leq 0$

$(\log_2 x - 2)(\log_2 x - 3)(\log_2 x - 3) \leq 0$

$(\log_2 x - 2)(\log_2 x - 3)^2 \leq 0$

$\Rightarrow \log_2 x \leq 2$ or $\log_2 x = 3$

$$x \in (0, 4] \cup \{8\}$$

843. (70) $\dfrac{1}{16}(\cos 36^\circ \sin 54^\circ)^2 - \left(\dfrac{1}{4}\sin 36^\circ \sin 36^\circ\right)^2$

$$\frac{1}{16}(\cos^2 36^\circ - \sin^2 36^\circ) = \frac{1}{16}\cos 72^\circ = \frac{\sqrt{5}-1}{64} \equiv \frac{\sqrt{a}-b}{c}$$

$\therefore (a+b+c) = 5 + 1 + 64 = 70$

844. (1) $S_n = \displaystyle\sum_{r=1}^{n} \frac{6r+9}{(r+1)^2(r+2)^2} = \sum_{r=1}^{n} \frac{3(2r+3)}{(r+1)^2(r+2)^2} = 3\sum_{r=1}^{n}\left(\frac{1}{(r+1)^2} - \frac{1}{(r+2)^2}\right)$

$$S_n = 3\left(\frac{1}{4} - \frac{1}{9} + \frac{1}{9} - \frac{1}{16} + \ldots\ldots + \frac{1}{(n+2)^2}\right) = 3\left(\frac{1}{4} - \frac{1}{(n+2)^2}\right)$$

$$\lim_{n\to\infty} S_n = \frac{3}{4} \equiv \frac{p}{q} \quad \Rightarrow \quad |p - q|_{\text{least}} = 1$$

845. (2) $$x^2-2px+3p^2-5=-x^2+2px+2p-3q$$

$$2x^2-4px+3p^2-2p+3q-5=0$$

$D\leq 0$

$$2+6p^2-4\cdot 2(3p^2-2p+3q-5)\leq 0$$

$$-p^2+2p-3q+5\leq 0$$

$$p^2-2p+3q-5\geq 0\ \forall\ p\in R$$

$D\leq 0$

$$4-4(3q-5)\leq 0 \Rightarrow q\geq 2$$

846. (25) $$m=(\cos^2\theta-2\cos\theta+1)\sec^2\phi+9\text{cosec}^2\phi+4\sec^2\phi$$

$$m=(\cos\theta-1)^2\sec^2\phi+9\text{cosec}^2\phi+4\sec^2\phi$$

$$M_{\text{least}}=0+(3+2)^2=25$$

847. (4) $$(2\alpha)^3+\beta^3+(-\gamma)^3=3(2\alpha)(\beta)(-\gamma)$$

$\Rightarrow$ $$2\alpha+\beta-\gamma=0 \rightarrow \beta=\gamma-2\alpha$$

or $$2\alpha=\beta=-\gamma \Rightarrow \beta+\gamma=0$$ {But $\beta+\gamma\neq 0$}

$$\alpha^2+3\gamma=2(\gamma-2\alpha)$$

$\Rightarrow$ $$\alpha^2+4\alpha+\gamma=0$$

For α to be real, $D\geq 0$

$$16-4\gamma\geq 0 \Rightarrow \gamma\leq 4$$

848. (4) $3, 7-b, \dfrac{3a^2+10}{a+2}\rightarrow$ A.P.

$$\frac{-3(a^2-4+4)+10}{a+2}=-3(a-2)-\frac{2}{a+2}$$

$a+2=\pm 1, \pm 2, \Rightarrow a=-1,-3,0,-4$

$a=-1\Rightarrow 3,7-b,7\Rightarrow b=2$

$a=-3\Rightarrow 3,7-b,17\Rightarrow b=-3$

$a=0\Rightarrow 3,7-b,5\Rightarrow b=3$

$a=-4\Rightarrow 3,7-b,19\Rightarrow b=-4$

$\therefore$ Number of A.P.'s are 4.

849. (7) $$2\log(2^x+2)=\log 4+\log(2^{x+2}+1)$$

(Let $2^x=t$)

$\Rightarrow$ $$(t+2)^2=4(4t+1) \Rightarrow t^2-12t=0$$

$\Rightarrow t=0$ (rejected); $t=12$

$\Rightarrow$ $$2^x=12$$

$$x=\log_2(12)$$

$$x \in (3,4) = (p,q)$$

$$\therefore \quad p+q = 7$$

850. (2)

$$E = \log_2\left(\frac{P}{\sqrt{3}}\right)$$

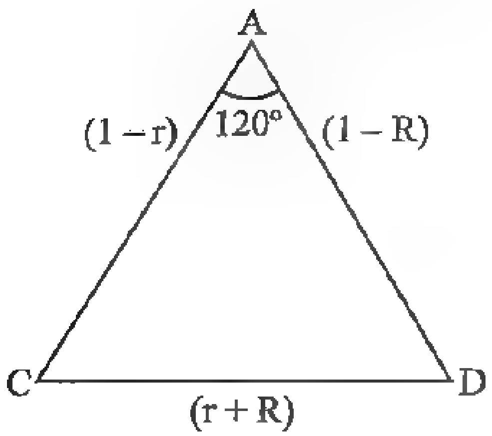
$$\therefore \quad P = 8\cos(30-2\alpha) - \frac{8\sin\alpha\cdot\sin(30-\alpha)}{\cos\alpha\sin(30-\alpha)+\cos(30-\alpha)\sin\alpha}$$

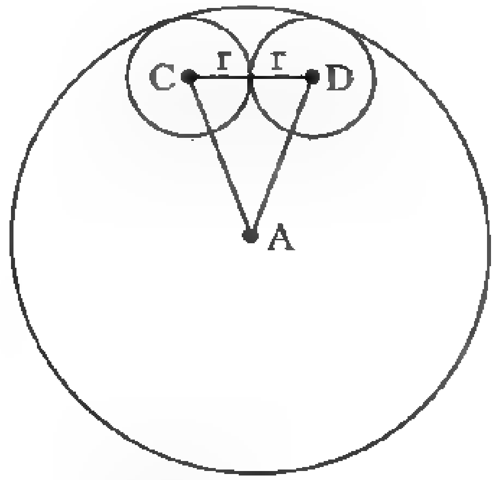
$$= 8\cos(30-2\alpha) - \frac{4(\cos(30-2\alpha)-\cos 30^\circ)}{\sin 30^\circ}$$

$$= 8\cos(30-2\alpha) - 8\cos(30-2\alpha) + 4\sqrt{3} = 4\sqrt{3}$$

$$E = \log_2\left(\frac{P}{\sqrt{3}}\right) = \log_2 4 = 2$$

851. (3)





$$\cos 120^\circ = \frac{(1-r)^2+(1-R)^2-(r+R)^2}{2(1-r)(1-R)} = -\frac{1}{2}$$

$$\Rightarrow \quad 3-3R-3r-rR = 0$$

$$\Rightarrow \quad Rr+3R+3r = 3$$

852. (7)

$$2\sin 2\theta\cos 2\theta + \cos^2\theta = \frac{1-\cos 2\theta}{2}$$

$$\Rightarrow \quad 2\cos 2\theta\sin 2\theta + \cos^2\theta = \sin^2\theta \quad \Rightarrow \quad 2\cos 2\theta\sin 2\theta + \cos 2\theta = 0$$

$$\Rightarrow \quad \cos 2\theta = 0, \quad \sin 2\theta = -\frac{1}{2}$$

$$2\theta = \frac{\pi}{2}, \frac{3\pi}{2}; \quad 2\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$\Rightarrow \quad \theta = \frac{\pi}{4}, \frac{3\pi}{4}; \quad \theta = \frac{7\pi}{12}, \frac{11\pi}{12}$$

$$\text{Sum} = \frac{5\pi}{2}$$

853. (4) Given $\quad R : r = 3 : 1.5 = 2 \quad \Rightarrow \quad \Delta ABC$ must be equilateral.

So $$a = b = c = 2R\sin\frac{\pi}{3} = R\sqrt{3}$$ (By sine rule)

Now $$a\cot^2 A + b^2\cot^3 B + c^3\cot^4 C = R\sqrt{3}\left(\frac{1}{\sqrt{3}}\right)^2 + (R\sqrt{3})^2\left(\frac{1}{\sqrt{3}}\right)^3 + (R\sqrt{3})^3\left(\frac{1}{\sqrt{3}}\right)^4$$

$$= \frac{R}{\sqrt{3}} + \frac{R^2}{\sqrt{3}} + \frac{R^3}{\sqrt{3}} = \frac{3+3^2+3^3}{\sqrt{3}} = \frac{39}{\sqrt{3}} = 13\sqrt{3} = m\sqrt{n}$$

Hence, $$\left(\frac{m-1}{n}\right) = 4$$

854. (202)

$$S = 1\cdot 2^1 + 2\cdot 2^2 + 3\cdot 2^3 + \ldots\ldots + 100\cdot 2^{100}$$

$$2S = \quad 1\cdot 2^2 + 2\cdot 2^3 + 3\cdot 2^4 + \ldots\ldots + 99\cdot 2^{100} + 100\cdot 2^{101}$$

Sub

$$-S = (2 + 2^2 + 2^3 + \ldots\ldots + 2^{100}) - 100\cdot 2^{101}$$

$$-S = 2(2^{100} - 1) - 100\cdot 2^{101}$$

$$S = 100\cdot 2^{101} - 2^{101} + 2$$

$$S = 99\cdot 2^{101} + 2$$

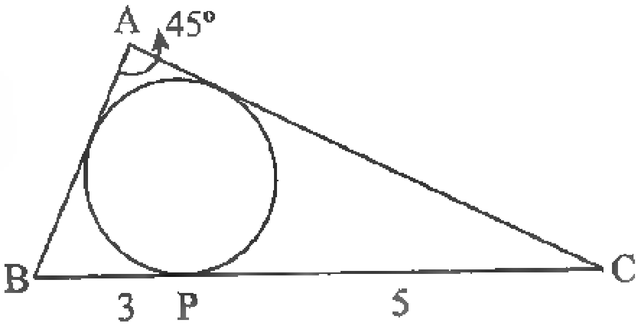
$\therefore$ $$S - 2 = 99\cdot 2^{101}$$

$\therefore$ $$\log_2(99\cdot 2^{101}) = 101 + \log_2 99 \equiv a + \log_c b$$

$\therefore$ $$a = 101,\ b = 99,\ c = 2$$

855. (60)

$$\frac{8}{\sin A} = 2R$$



$\therefore$ $$R = 4\sqrt{2}$$

$$PB = s - b = 3;$$

$$PC = s - c = 5$$

Now, $$\Delta^2 = s(s-a)(s-b)(s-c)$$

$$= 15 \times s \times (s-a)$$

$$\Delta \times \frac{\Delta}{s(s-a)} = 15$$

$$\Delta\tan\frac{A}{2} = 15 \quad \Rightarrow \quad \Delta = 15(\sqrt{2}+1)$$

Now, $$\frac{\Delta R}{2+\sqrt{2}} = \frac{15(\sqrt{2}+1)\times 4\sqrt{2}}{2+\sqrt{2}} = 60$$

856. (150)

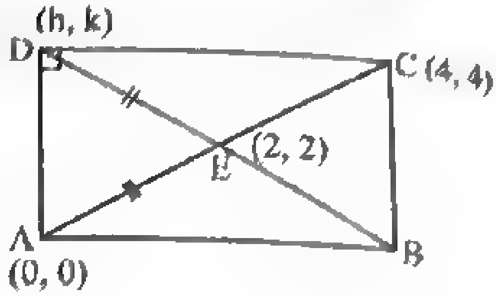
$$p_1 + p_2 = 6$$

$$\sqrt{r^2 - 25} + \sqrt{r^2 - 49} = 6$$

$$r^2 = 50$$

$$A = n\left(\frac{1}{2}r^2\sin\frac{2\pi}{n}\right) = 12 \times \frac{50}{2}\sin\frac{\pi}{6} = 150$$

857. (8) The locus of the extremities of the other diagonal is the circle with given diagonal as diameter.



$\Rightarrow$ $(x-0)(x-4)+(y-4)(y-0)=0$

$\Rightarrow$ $$x^2+y^2-4x-4y=0$$

Alter: $$(AE)^2=(DE)^2$$

$$(h-2)^2+(k-2)^2=(2\sqrt{2})^2$$

$\therefore$ $$x^2+y^2-4x-4y=0$$

858. (5) $$\sin^2\theta+\tan^2\theta=-b/a \quad ...(1)$$

$$\sin^2\theta\cdot\tan^2\theta=c/a \quad ...(2)$$

Now, Eqn. $(1)^2$ – Eqn. $(2)^2$

$$\Rightarrow \frac{b^2-c^2}{a^2}=(\sin^2\theta+\tan^2\theta)^2-(\sin^2\theta\cdot\tan^2\theta)^2$$

$$\Rightarrow \frac{b^2-c^2}{a^2}=(\sin^2\theta+\tan^2\theta+\sin^2\theta\cdot\tan^2\theta)(\sin^2\theta+\tan^2\theta-\sin^2\theta\cdot\tan^2\theta)$$

$$\Rightarrow \frac{b^2-c^2}{a^2}=(2\tan^2\theta)(2\sin^2\theta)=\frac{4c}{a}$$

$$\therefore \frac{b^2-c^2}{ac}=4 \Rightarrow \lambda=4$$

Now, $\log_4(8\sin x)<1$

$0<\sin x<1/2$

$x\in(0,\pi/6)\cup(5\pi/6,\pi)\equiv(\alpha,\beta)\cup(\gamma,\delta)$

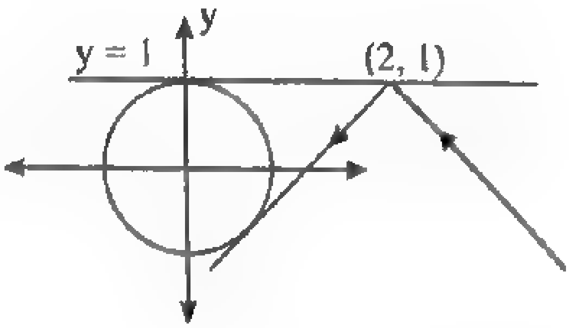
$$\therefore \frac{\gamma}{\beta}+\frac{\alpha}{\delta}=5+0=5$$

859. (7) $$SS_1=T^2$$

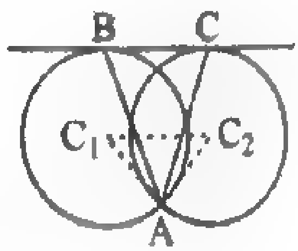
$$(x^2+y^2-1)\cdot 4=(2x+y-1)^2$$

$$\Rightarrow 3y^2-4xy+4x+2y-5=0$$

Slope of reflected ray is $\frac{4}{3}$.



860. (5) In ΔAC_1C_2



$$\cos A=\frac{r_1^2+r_2^2-(C_1C_2)^2}{2r_1r_2}=\cos 60^\circ$$

$$r_1^2+r_2^2-(C_1C_2)^2=r_1r_2 \quad ...(1)$$

and $$(C_1C_2)^2-(r_1-r_2)^2=25 \quad ...(2)$$

$\Rightarrow$ $$r_1r_2=25$$

861. (23) $y = m(x-6)$

$$mx - y - 6m = 0 \Rightarrow \left|\frac{5m+3}{\sqrt{1+m^2}}\right| = 5$$

$$(5m+3)^2 = 25 + 25m^2 \Rightarrow 9 + 30m = 25$$

$$\Rightarrow \quad \frac{p}{q} = \frac{8}{15}$$

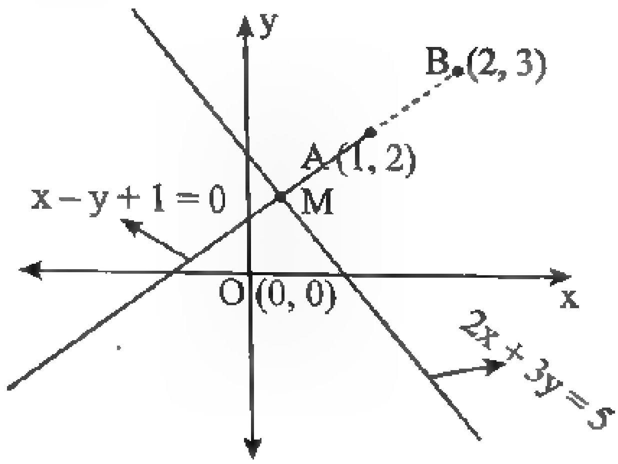
$$\therefore \quad p + q = 23$$

862. (4) As, $|MA - MB| \le AB$

$\therefore$ $|MA - MB|_{\text{maximum}} = AB$, which is possible when M is the point of intersection of line joining

$A(1, 2), B(2, 3)$ and $2x + 3y = 5$

So, $M\left(x = \frac{2}{5}, y = \frac{7}{5}\right)$



863. (10) $x^2 + 3x + 1 + \lambda(x+1) > -10$

$x^2 + (3+\lambda)x + 11 + \lambda > 0 \forall x \in R$

$D < 0 \Rightarrow (\lambda+3)^2 - 4(11+\lambda) < 0$

$\lambda^2 + 6\lambda + 9 \quad 44 - 4\lambda < 0$

$\Rightarrow \lambda^2 + 2\lambda - 35 < 0$

$\Rightarrow \lambda \in (-7, 5)$

Required sum $= 1 + 2 + 3 + 4 = 10$.

864. (35) 0, 1, 2, 2, 3, 3, 3

(1) All alike $\rightarrow 1$

(2) $2A + 1$ non-zero digit $\rightarrow {}^2C_1 \times {}^7C_1 \times \frac{3!}{2!} = 12$

(3) $2A + 1$ zero digit $\rightarrow {}^2C_1 \times 1 \times \left(\frac{3!}{2!} - 1\right) = 4$

(4) $3D$ (non-zero) $\rightarrow 1 \times 3! = 6$

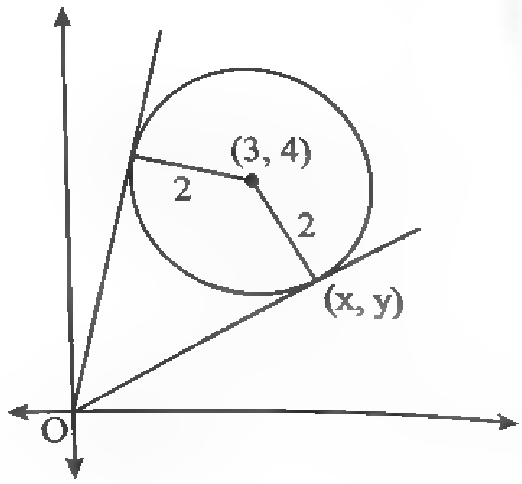
(5) $3D$ (with zero) $\rightarrow {}^3C_2 \times (3! - 2!) = 12$

Total $= 35$

865. (25) $$\frac{x}{y}=\frac{1}{m}$$

Let $y=mx$ be the tangent

Applying, $$p=r$$



$$\left|\frac{3m-4}{\sqrt{1+m^2}}\right|=2$$

$$9m^2-24m+16=4+4m^2$$

$$5m^2-24m+12=0 \quad \frac{1}{m_1},\ \frac{1}{m_2}$$

$$12m^2-24m+5=0 \quad m_1,\ m_2$$

$$(m_1+m_2)^2-2m_1m_2=4-2\left(\frac{5}{12}\right) \Rightarrow \frac{19}{6}=\frac{p}{q}$$

$\therefore$ $$p+q=25$$

866. (4) $$y|_{\max.}=\sqrt{(k-3)^2+(t-4)^2}+3=6$$

$$(k-3)^2+(t-4)^2=9$$

Put $$k-3=3\cos\theta$$

Put $$t+4=3\sin\theta$$

$$k^2+t^2=(3+3\cos\theta)^2+(-4+3\sin\theta)^2$$

$$=34+18\cos\theta-24\sin\theta$$

$\therefore$ $$k^2+t^2|_{\min.}=(34-30)=4$$

867. (25) $(\sin x+\cos x)(\tan^2 x-(1+\sqrt{3})\tan x+3+\sqrt{3})=3|\sin x+\cos x|$

Case-1: $\sin x+\cos x\geq 0$

$$(\sin x+\cos x)(\tan^2 x-(1+\sqrt{3})\tan x+\sqrt{3})=0$$

$$\tan x=-1, 1, \sqrt{3}$$

$\Rightarrow$ $$x=\frac{3\pi}{4},\frac{\pi}{4},\frac{\pi}{3},\frac{7\pi}{4}$$

Case-2: $\sin x+\cos x<0$

$$(\sin x+\cos x)\underbrace{(\tan^2 x-(1+\sqrt{3})\tan x+6+\sqrt{3})}_{\text{no solution}}=0$$

$\tan x = -1 \Rightarrow x = \frac{3\pi}{4}, \frac{7\pi}{4}$

$\therefore$ Required sum $= \frac{37\pi}{12}$

868. (32) $$b = \frac{2ac}{a+c} \Rightarrow ac = 4b$$

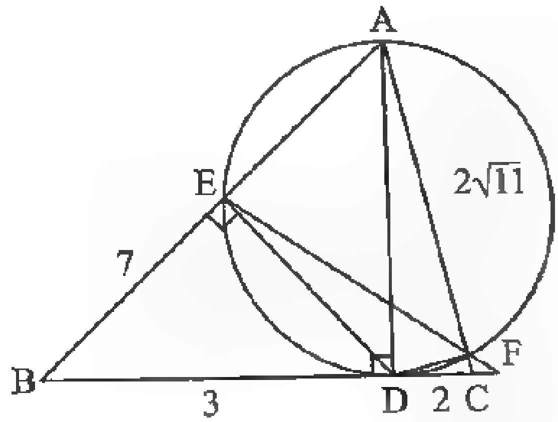
$$\frac{b}{2a-b} = 3 \Rightarrow b = 6a - 3b \Rightarrow 2b = 3a$$

$$\Rightarrow \quad ac = 2\cdot 3a \Rightarrow 6a = ac \Rightarrow c = 6, a = 2, b = 3$$

$$t_1 = \frac{1}{2}, t_2 = \frac{3}{2}, t_3 = \frac{9}{2}$$

$$t_7 = \frac{1}{2}\cdot 3^6 \Rightarrow t_7\left(\frac{2}{3}\right)^6 = 2^5$$

869. (25) $$a = 5, \cos B = \frac{3}{7}, c = 7$$



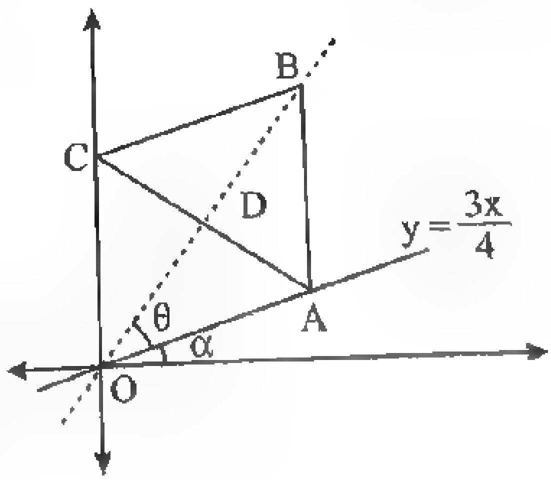
$$BE = 3\cos B = \frac{9}{7}$$

$$CF = 2\cos C = 2\times\frac{1}{\sqrt{11}}$$

$$\therefore \quad (BE)(CF) = \frac{18}{7\sqrt{11}} \equiv \frac{p}{q\sqrt{11}}$$

$$\therefore \quad p + q = 25$$

870. (20) $$2\theta = \frac{\pi}{2} - \alpha$$



$$\tan 2\theta = \cot\alpha = \frac{4}{3}$$

$$\frac{2\tan\theta}{1-\tan^2\theta} = \frac{4}{3} \Rightarrow \tan\theta = \frac{1}{2}$$

Now, slope of $OB = \tan(\theta+\alpha) = \frac{\tan\theta + \tan\alpha}{1 - \tan\theta\tan\alpha}$

$$= \frac{\frac{1}{2}+\frac{3}{4}}{1-\frac{1}{2}\left(\frac{3}{4}\right)} = 2$$

$\therefore$ Slope of $AC = \frac{-1}{2}$

Equation of $AC \Rightarrow y - 2 = -\frac{1}{2}(x-6)$

$$\Rightarrow \quad x + 2y = 10$$

$$C \equiv (0,5)$$

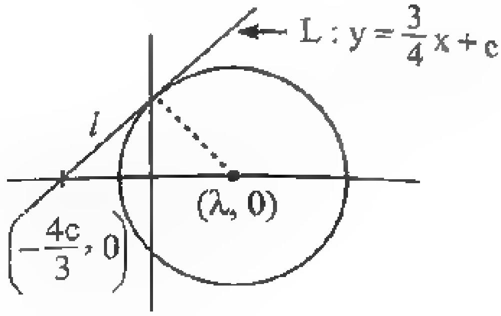
$$\text{Area of rhombus} = 2\times \text{Ar.}(\Delta OAC) = \frac{1}{2}\times 2\times 5\times \sin 2\theta = 25\times\frac{4}{5} = 20$$

871. (23)

$$l = \sqrt{\left(\lambda+\frac{4c}{3}\right)^2 - 25} = \sqrt{\left(\frac{3\lambda+4c}{3}\right)^2 - 25}$$

$$= \sqrt{\left(\frac{25}{3}\right)^2 - 25}$$

$$= \frac{20}{3} \equiv \frac{p}{q}$$



$\therefore\ p+q = 23$

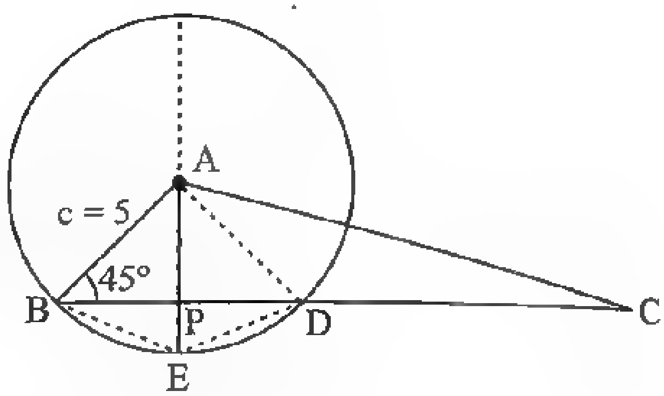
872. (27)

$$BP = \frac{5}{\sqrt{2}} = PD$$

$$PE = 5 - \frac{5}{\sqrt{2}}$$

$$\therefore \text{Ar.}(\Delta BED) = \frac{1}{2}\times 2\times\frac{5}{\sqrt{2}}\left(5-\frac{5}{\sqrt{2}}\right)$$

$$= \frac{25}{2}(\sqrt{2}\quad 1)$$



$$\Rightarrow \quad p+q - 25+2 - 27$$

873. (3)

$$(x-x_0)(x^2-qx+2) = x^3+x^2-px+q$$

$$\rightarrow \quad -x_0-q = 1,\ 2+qx_0 = -p,\ -2x_0 = q$$

$$\Rightarrow \quad x_0 = 1,\ q = -2,\ p = 0$$

$$\therefore \quad |p-q+x_0| = 3$$

874. (309)

$$^6C_5\cdot 5!(44+\ ^5C_1(9+44)) = (309)6!$$

$$\therefore \quad k = 309$$

875. (151)

$$S = 2+3\cos x+4\cos^2 x+\ldots\ldots\ldots\ldots,\infty$$

$$S\cdot\cos x = \qquad 2\cos x+3\cos^2 x+\ldots\ldots\ldots\ldots,\infty$$

$$S(1-\cos x) = 2+\cos x+\cos^2 x+\ldots\ldots\ldots\ldots\infty$$

$$= 1+\frac{1}{1-\cos x}$$

$$S = \frac{1}{1-\cos x}+\frac{1}{(1+\cos x)^2}$$

$$|5\cos x+4|+|5\cos x-2| = 6$$

$$\Rightarrow \cos x \in \left[-\frac{4}{5},\frac{2}{5}\right]$$

For the least value of S, $\cos x = -\frac{4}{5}$

$$\therefore \quad S|_{\text{least}} = \frac{1}{1+\frac{4}{5}} + \frac{1}{\left(1+\frac{4}{5}\right)^2} = \frac{5}{9} + \frac{25}{81} = \frac{70}{81} = \frac{a}{b}$$

$\therefore \quad a+b=151$

876. (14) $(2+x^3(3+x)^2)^{10}$

$$\downarrow$$

$${}^{10}C_r \cdot 2^{10-r} \cdot x^{3r}(3+x)^{2r}$$

$$\downarrow$$

$${}^{2r}C_k \cdot 3^{2r-k} \cdot x^k$$

$3r+k=8 \Rightarrow k=2, r=2$

$\therefore$ Coefficient of $x^8 = {}^{10}C_2 \cdot 2^8 \cdot {}^4C_2 \cdot 3^2 = 5 \cdot 2^9 \cdot 3^5$

877. (5) Range of $f(x)$ is $[0, 4]$

$$\therefore \sin^{-1}(\sin[f(x)]) = \sin^{-1}(\sin 0) + \sin^{-1}(\sin 1) + \sin^{-1}(\sin 2) + \sin^{-1}(\sin 3) + \sin^{-1}(\sin 4)$$

$$= 0+1+\pi-2+\pi-3+\pi-4 = 3\pi-8 \equiv a\pi+b$$

$\therefore \quad (a+b) = |3-8| = 5$

878. (0) Product of roots $= 3k^2 - k + 2 = e^{\ln 3 + \ln 2} = 6$

$\Rightarrow \quad 3k^2 - k - 4 = 0$

$\Rightarrow \quad 3k^2 - 4k + 3k - 4 = 0$

$(k+1)(3k-4) = 0$

$k = -1, k = \frac{4}{3}$

Sum of roots $= 3k+1 = 3 \cdot e^{\lambda^2 - 2\lambda + 1} + 2 \cdot e^{-(\lambda^2 - 2\lambda + 1)}$

$\because k=-1$ not satisfy given relation

So, number of integral value of k is zero.

879. (1) $g\left(\frac{1}{\alpha^2+1}\right) = \alpha \Rightarrow f(\alpha) = \frac{1}{\alpha^2+1} \Rightarrow \alpha = 2$

$$k = \frac{g(\alpha+1)}{g(g(\alpha))} = \frac{g(3)}{g(g(2))} = \frac{g(3)}{g(3)} = 1$$

880. (9) (AAAAA) (BBBBB)(CCCCC)(OOOOO)

P_1, P_2, P_3, P_4, P_5

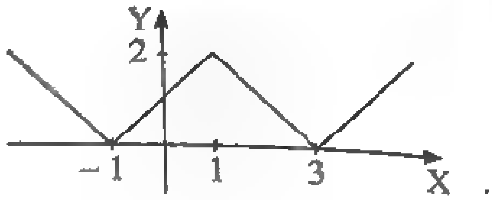
${}^5C_2 \times {}^4C_2 \times 2! \times {}^3C_2 \times 2! \times 2! = 1440$

$\therefore$ Sum of the digits $= 9$

881. (6) $f(x) = \{[x] + ||x-1|-2|\}$
$= \{||x-1|-2|\}$

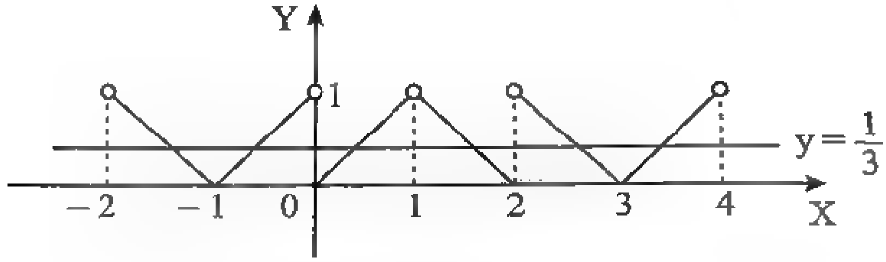
$3f(f(x)) - 1 = 0 \Rightarrow f(f(x)) = \frac{1}{3}$

Let $f(x) = t \Rightarrow f(t) = \frac{1}{3}$



Graph of ||x − 1| 2 |

Number of values of t for which $f(t) = \frac{1}{3}$

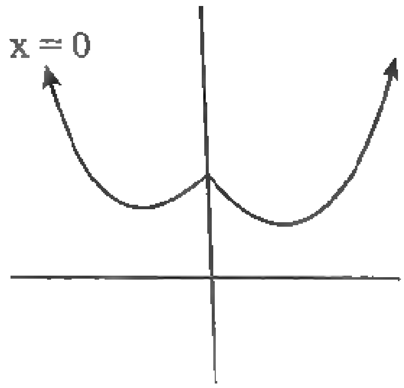


Graph of f(x) = {||x − 1| − 2|}

are 6, but exactly one value of t is lying between [0, 1).

$\therefore f(f(t)) = \frac{1}{3}$ has only 6 solutions.

882. (1)
$$f(x) = \begin{cases} \frac{ax^3 + bx^2 + cx + d}{x}, & x \neq 0 \\ 2, & x = 0 \end{cases}$$



$f(x)$ is continuous

$\Rightarrow$ $d = 0$ and $c = 2$

But a, b, c and d are in A.P.

$\Rightarrow$ $a - 6, b - 4, c = 2, d = 0$

$$f(x) = \begin{cases} 6x^2 + 4x + 2, & x \neq 0 \\ 2, & x = 0 \end{cases}$$

$$y = |f(|x|)|$$

883. (2)
$$\text{L.H.D.} = \lim_{h \to 0} (-h)^{\alpha - 1} \left(\frac{e^{-2/h} - 1}{e^{-2/h} + 1} \right)$$

$$\text{R.H.D.} = \lim_{h \to 0} (h)^{\alpha - 1} \left(\frac{1 - e^{-2/h}}{1 + e^{2/h}} \right)$$

$$\text{L.H.D.} = \text{R.H.D.}$$

$\Rightarrow$ $\alpha - 1 > 0$

$\Rightarrow$ $\alpha > 1$

884. (8) $L=\prod_{n=3}^{\infty}\left(1-\frac{4}{n^2}\right)=\prod_{n=3}^{\infty}\left(\frac{n^2-4}{n^2}\right)=\prod_{n=3}^{\infty}\left(\frac{n-2}{n}\right)\times\prod_{n=3}^{\infty}\left(\frac{n+2}{n}\right)$

$$L=\left(\frac{1}{3}\cdot\frac{2}{4}\cdot\frac{3}{5}\cdot\frac{4}{6}\cdots\cdots\frac{n-2}{n}\right)\left(\frac{5}{3}\cdot\frac{6}{4}\cdot\frac{7}{5}\cdots\cdots\frac{(n+1)}{n-1}\cdot\frac{(n+2)}{n}\right)$$

$$L=\left(\frac{1\cdot 2}{n(n-1)}\cdot\frac{(n+1)(n+2)}{3\cdot 4}\right)-\frac{n\left(1+\frac{1}{n}\right)n\left(1+\frac{2}{n}\right)}{n^2\left(1-\frac{1}{n}\right)}\times\frac{2}{3\cdot 4}=\frac{1}{6}\quad\Rightarrow\quad L=\frac{1}{6}$$

$$M=\prod_{n=2}^{\infty}\left(\frac{n^3-1}{n^3+1}\right)=M=\prod_{n=2}^{\infty}\frac{n-1}{n+1}\cdot\frac{(n^2+n+1)}{(n^2-n+1)}$$

$$=\left(\frac{1}{3}\cdot\frac{2}{4}\cdot\frac{3}{5}\cdot\frac{4}{6}\cdots\cdots\frac{n-2}{n}\cdot\frac{n-1}{n+1}\right)\times\left(\frac{7}{3}\cdot\frac{13}{7}\cdot\frac{21}{3}\cdots\cdots\frac{n^2+n+1}{n^2-n+1}\right)$$

$$=\lim_{n\to\infty}\frac{2}{n(n+1)}\cdot\frac{n^2+n+1}{3}=\frac{2}{3}\quad\Rightarrow\quad M=\frac{2}{3}$$

$$N=\prod_{n=1}^{\infty}\frac{(1+n^{-1})^2}{1+2n^{-1}}=\prod_{n=1}^{\infty}\left(\frac{n+1}{n}\right)^2\left(\frac{n}{n+2}\right)=\prod_{n=1}^{\infty}\left(\frac{(n+1)^2}{n(n+2)}\right)$$

$$N=\prod_{n=1}^{\infty}\left(\frac{n+1}{n+2}\right)\times\prod_{n=1}^{\infty}\left(\frac{n+1}{n}\right)$$

$$=\frac{2}{3}\cdot\frac{3\cdot 4\cdots\cdots\cdots(n+1)}{4\cdot 5\cdots\cdots(n+1)(n+2)}\times\frac{2\cdot 3\cdot 4\cdots\cdots\cdots n(n+1)}{1\cdot 2\cdot 3\cdots\cdots n}=\frac{2(n+1)}{n+2}$$

$$\Rightarrow\quad\lim_{n\to\infty}\frac{2(n+1)}{n+2}=2$$

$$\therefore\qquad N=2$$

$$L^{-1}+M^{-1}+N^{-1}=6+\frac{3}{2}+\frac{1}{2}=8$$

885. (3) $f(x)=\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)+\sin^{-1}\left(\frac{2x}{1+x^2}\right)$

$$f(x)=\begin{cases}-\pi-4\tan^{-1}x, & x<-1\\ 0, & -1\le x<0\\ 4\tan^{-1}x, & 0\le x<1\\ \pi, & 1\le x\end{cases}$$

$$f(\sqrt{3})+f(-\ln 2)+f(1)+f(\ln 3)=\pi+0+\pi+\pi=3\pi$$

886. (7) Notice that $x=2$ is a root of P because $2^2+(n-1)\cdot 2-2(n+1)=0$ and

$x=-1$ is a root of Q because $(n-1)\cdot(-1)^2+n\cdot(-1)+1=0$.

Therefore, we see that $P(x)=(x-2)(x+n+1)$ and $Q(x)=(x+1)((n-1)x+1)$

If $|P \cup Q| = 3$, then $P(x)$ and $Q(x)$ share a root.

Therefore, we have the three equations:

$$2 = -\left(\frac{1}{n-1}\right) \Rightarrow n = \frac{1}{2} - 1 = -n - 1 \Rightarrow n = 0$$

$$-\left(\frac{1}{n-1}\right) = -n-1 \quad \Rightarrow n = \pm\sqrt{2}$$

Hence, there are total of 7 possible values of n.

887. (3) $\because$ $$g'(f(x)) = \frac{1}{f'(x)} \Rightarrow g'(2\pi) = \frac{1}{f'\left(\frac{3\pi}{2}\right)} = \frac{1}{\left(\frac{7}{3}\right)} = \frac{3}{7}$$

And $$g''(f(x)) = \frac{-f''(x)}{(f'(x))^3}$$

$\Rightarrow$ $$g''(2\pi) = \frac{-f''\left(\frac{3\pi}{2}\right)}{\left(f'\left(\frac{3\pi}{2}\right)\right)^3} = 0$$

$\therefore$ $$7g'(2\pi) + 3g''(2\pi) = 3$$

888. (2) By Leibnitz Theorem, $\frac{d}{dx}\int_0^{x^3} k(t)\,dt = \frac{d}{dx}(x^{1+x^2})$

$$k(x^3)\ 3x^2 - 0 = (1+x^2)x^{x^2} + x^{1+x^2}(\ln x)\cdot(2x)$$

Put $x = 1 \Rightarrow k(1)\cdot 3 = 2\cdot 1 + 1\cdot 0\cdot 2$

$$k(1) = \frac{2}{3} \quad \Rightarrow \quad 3k(1) = 2$$

889. (4) $\because$ $$3 = 2\left(\frac{c}{a} - \frac{b}{a}\right)$$

$\Rightarrow$ $$3 = 2(4\alpha^2 + 5\alpha) \Rightarrow 8\alpha^2 + 10\alpha - 3 = 0$$

$\Rightarrow$ $$8\alpha^2 + 12\alpha - 2\alpha - 3 = 0 \Rightarrow (2\alpha+3)(4\alpha-1) = 0$$

$\therefore$ $$\alpha = \frac{1}{4} \Rightarrow \beta = 4\alpha = 1$$

$\therefore$ $$S = \frac{\beta}{1-\alpha} = \frac{1}{1-\frac{1}{4}} = \frac{4}{3} \Rightarrow 3S = 4$$

890. (2) $$L=\lim_{n\to\infty}\frac{1}{n^3}\sum_{k=1}^{n}k^2e^{k/n}=\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\left(\frac{k}{n}\right)^2e^{k/n}=\int_0^1 x^2e^x\,dx$$

$$L=(x^2-2x+2)e^x\Big|_0^1=e-2$$

$\Rightarrow\ e-L=2$

891. (5) $$\frac{\cos 3x}{\sin 5x}-\frac{\sin 3x}{\cos 5x}=a[\sin(2x)+\cos(2x)\cot(bx)]$$

$$\text{LHS}=\frac{\cos 3x\cos 5x-\sin 3x\sin 5x}{\cos 5x\sin 5x}$$

$$=\frac{2\cos 8x}{\sin 10x}=\frac{2(\cos 2x\cos 10x+\sin 2x\sin 10x)}{\sin 10x}$$

$$=2[\sin 2x+\cos 2x\cot 10x]$$

$\Rightarrow\qquad a=2,\ b=10\qquad\Rightarrow\qquad \dfrac{b}{a}=5$

892. (10) $$f(-x)=f(x),\ g(x)g(-x)=1$$

$$\int_0^a f(x)\,dx=10$$

$$I=\int_{-a}^{a}\frac{f(x)}{1+g(x)}\,dx\qquad\ldots(1)$$

Using King $$I=\int_{-a}^{a}\frac{f(-x)}{1+g(-x)}\,dx$$

$$\Rightarrow\qquad I=\int_{-a}^{a}\frac{g(x)f(x)\,dx}{1+g(x)}\qquad\ldots(2)$$

Eqn. (1) + Eqn. (2)

$$\Rightarrow\qquad 2I=\int_{-a}^{a}f(x)\,dx=2\int_0^a f(x)\,dx\quad\Rightarrow\quad I=\int_0^a f(x)\,dx=10$$

893. (90) $$\left(x-2+\frac{1}{x}\right)^{30}=n_0x^{30}+n_1x^{29}+\ldots\ldots+n_{29}x+n_{30}+n_{31}x^{-1}+\ldots\ldots+n_{60}x^{-30}$$

$$(x-1)^{60}=n_0x^{60}+n_1x^{59}+\ldots\ldots+n_{29}x^{31}+n_{30}x^{30}+n_{31}x^{29}+\ldots\ldots+n_{60}$$

$$\therefore n_0={}^{60}C_0,\ n_1=-{}^{60}C_1,\ n_2={}^{60}C_2,\ldots\ldots n_{30}={}^{60}C_{30}\ldots\ldots n_{60}={}^{60}C_{60}$$

$$\therefore C=n_0+n_1+n_2+\ldots\ldots+n_{60}=0$$

Hence, $C-n_{30}=-\begin{pmatrix}a\\b\end{pmatrix}$

$$\Rightarrow\qquad 0-1\cdot({}^{60}C_{30})$$

$$\therefore\qquad a=60,\ b=30$$

Hence, $a+b=90$.

894. (9) $f(1)+g(1)=9e$, $f(x)=-x^2g'(x)$ and $g(x)=-x^2f'(x)$

$$I=\int_1^4\frac{f(x)+g(x)}{x^2}dx=\int_1^4\frac{-x^2(f'(x)+g'(x))}{x^2}dx=\int_1^4-(f'(x)+g'(x))dx$$

$$=-[f(x)+g(x)]_1^4=-(f(4)+g(4)-9e)$$

Now, $f(x)+g(x)=-x^2(g'(x)+f'(x))$

$$\Rightarrow\frac{g'(x)+f'(x)}{f(x)+g(x)}=\frac{-1}{x^2}\Rightarrow f(x)+g(x)=ae^{\frac{1}{x}}\Rightarrow f(1)+g(1)=9e\Rightarrow a=9$$

$$\Rightarrow f(x)+g(x)=9e^{\frac{1}{x}}$$

$$I=-\left(9e^{\frac{1}{4}}-9e\right)=9\left(e-e^{\frac{1}{4}}\right)\equiv k\left(e-e^{\frac{1}{4}}\right)\Rightarrow k=9$$

895. (1) $$f(x)=\begin{cases}(x+1)(x+2), & x>0\\ a\sin x+b\cos x, & x\le 0\end{cases}$$

At $x=0$

LHL $=2$ and RHL $=b$ $\Rightarrow$ $b=2$

LHD $=3$ and RHD $=a$ $\Rightarrow$ $a=3$

$\Rightarrow$ $a-b=1$

896. (2) $x^2-2px+p^2-1=0$

$\Rightarrow$ $x=p\pm1$

$$\left|\frac{\alpha^2+\beta^2}{\alpha\beta}+3\right|\le5\Rightarrow-8\le\frac{\alpha^2+\beta^2}{\alpha\beta}\le2\Rightarrow-8\le\frac{\alpha^2+\beta^2}{\alpha\beta}\le2\Rightarrow-8\le\frac{2(p^2+1)}{p^2-1}\le2$$

$$-4\le\frac{p^2+1}{p^2-1}\le1\;\Rightarrow\;-4\le1+\frac{2}{p^2-1}\le1\;\Rightarrow\;-5\le\frac{2}{p^2-1}<0$$

$$\Rightarrow\quad p^2-1\le\frac{-2}{5}\;\Rightarrow\;p^2\le\frac{3}{5}\;\Rightarrow\;p\in\left[-\sqrt{\frac{3}{5}},\sqrt{\frac{3}{5}}\right]\equiv[a,b]$$

$\therefore$ $[2(a^2+b^2)]=2$

897. (9) $$d=\frac{-57-303}{30}=-12$$

$$A_r=303+r\times(-12)\ge0\;\Rightarrow\;r\le\frac{303}{12}\simeq25......$$

$$A_{25}=303+25(-2)=3$$

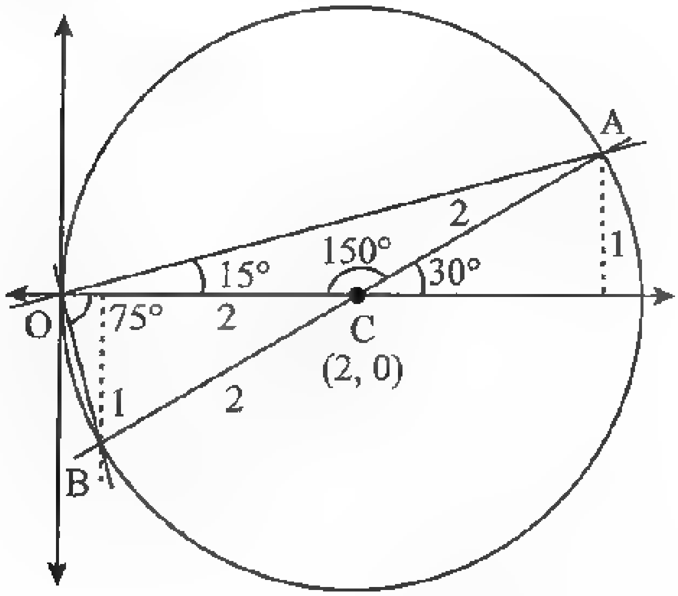
$$|A_r|_{\min.}=|A_{25}|=3$$

$$\therefore\quad\frac{S}{(A_{14}-12)|A_r|_{\min.}}=\frac{29(A_1+A_{29})}{2(A_{15})\cdot3}=\frac{29}{3}$$

$$\therefore\quad\left[\frac{S}{(A_{14}-12)|A_r|_{\min.}}\right]=9$$

898. (4)
$$I_r = 2\sqrt{\frac{1}{r^2}+\frac{1}{(r+1)^2}}$$
$$I_r^2 = \frac{4}{r^2}+\frac{4}{(r+1)^2}$$
$$I_r^2 - \frac{8}{(r+1)^2} = \frac{4}{r^2}-\frac{4}{(r+1)^2}$$
$$\sum_{r=1}^{n}\left(I_r^2-\frac{8}{(r+1)^2}\right) = 4\left(\frac{1}{1^2}-\frac{1}{(r+1)^2}\right)$$
$$r=1$$
$$\therefore \quad \lim_{n\to\infty}\sum_{r=1}^{n}\left(I_r^2 - \frac{8}{(r+1)^2}\right) = 4$$

899. (2)



$$\text{Ar.}(\Delta AOB) = \text{Ar.}(\Delta AOC)+\text{Ar.}(\Delta COB) = \frac{1}{2}\times 2\times 2\times \sin 150^\circ + \frac{1}{2}\times 2\times 2\times \sin 30^\circ = 2$$

900. (7)
$$(4\tan x+\tan^2 x+1) = 2\sqrt{2}\sin\left(x+\frac{\pi}{4}\right)(1+\tan^2 x)$$
$$(4\tan x+\sec^2 x) = 2\sqrt{2}\left(\sin x\cdot\frac{1}{2}+\cos x\cdot\frac{1}{\sqrt{2}}\right)(\sec^2 x)$$
$$\frac{4\sin x\cdot\cos x+1}{\cos^2 x} = 2\frac{(\sin x+\cos x)}{\cos^2 x}$$
$$2\sin 2x+1 = 2(\sin x+\cos x) \Rightarrow (2\sin 2x+1)^2 = 4(1+\sin 2x)$$
$$\Rightarrow \quad 4\sin^2 2x = 3 \Rightarrow \sin 2x = \pm\frac{\sqrt{3}}{2}$$
$$2x = \frac{\pi}{3},\frac{2\pi}{3},\frac{4\pi}{3},\frac{5\pi}{3} \Rightarrow x = \frac{\pi}{6},\frac{\pi}{3},\underbrace{\frac{2\pi}{3}}_{\text{rejected}},\frac{5\pi}{6}$$

$\therefore$ Sum of the solutions $= \dfrac{8\pi}{6} = \dfrac{4\pi}{3} \equiv \dfrac{p\pi}{q}$

$\therefore \quad p+q=7$

901. (2020) $\quad I = \int_0^1 (x - f(x))^{2018} dx \quad \ldots(1)$

Put $\quad x = f(t) \Rightarrow dx = f'(t)dt$

$$I = \int_1^0 (f(t) - f(f(t)))^{2018} f'(t)dt \qquad \{\because f(f(0)) = 0 \Rightarrow f(1) = 0\}$$

$$I = -\int_0^1 (f(t) - t)^{2018} f'(t)dt$$

$$I = -\int_0^1 (t - f(t))^{2018} f'(t)dt \quad \ldots(2)$$

Eqn. (1) + Eqn. (2)

$$2I = \int_0^1 (x - f(x))^{2018}(1 - f'(x))dx = \left(\frac{(x - f(x))^{2019}}{2019}\right)_0^1$$

$$= \frac{1}{2019} - \left(\frac{-1}{2019}\right)$$

$$I = \frac{1}{2019} \equiv \frac{p}{q} \Rightarrow (p+q) = 2020$$

902. (6) $\displaystyle\int_0^{1/2} \tan^{-1}\left(\frac{1}{2}\left(\sqrt{\frac{1+x}{x}} - \sqrt{\frac{x}{1+x}}\right)\right)dx = \int_0^{1/2} \tan^{-1}\left(\frac{1}{2}\left(\frac{1}{\sqrt{x(1+x)}}\right)\right)dx$

$$= \int_0^{1/2} \tan^{-1}\left(\frac{1}{\sqrt{4x^2+4x+1-1}}\right)dx = \int_0^{1/2} \tan^{-1}\left(\frac{1}{\sqrt{(2x+1)^2-1}}\right)dx$$

Put $2x+1 = \sec\theta \quad \Rightarrow \quad dx = \dfrac{\sec\theta\tan\theta}{2}d\theta$

$$I = \int_0^{\frac{\pi}{3}} \tan^{-1}(\cos\theta)\cdot\frac{\sec\theta\tan\theta}{2}d\theta = \int_0^{\frac{\pi}{3}} \left(\frac{\pi}{2} - \theta\right)\cdot\frac{\sec\theta\tan\theta}{2}d\theta$$

$$I = \frac{\pi}{4}\cdot(\sec\theta)_0^{\pi/3} - \frac{1}{2}(\theta\cdot\sec\theta - \ln(\sec\theta + \tan\theta))_0^{\pi/3}$$

$$= \frac{\pi}{4}(2-1) - \frac{1}{2}\left(\frac{\pi}{2}\cdot 2 - \ln(2+\sqrt{3})\right) = \frac{-\pi}{12} + \frac{\ln(2+\sqrt{3})}{2} \equiv p\ln(2+\sqrt{3}) - \frac{\pi}{q}$$

$\therefore \quad pq = 6$

903. (6)
$$g(x) = x^2 + ax + b$$
$$h(x) = cx - x^2$$
$$1 + a + b = 0 \quad \text{and} \quad 1 - c = 0$$
$$\Rightarrow \quad c = 1$$
$$g'(1) = m_1 = 2 + a$$
$$h'(1) = m_2 = c - 2 = -1$$
$$g'(1) = h'(1) \quad \Rightarrow \quad a = -3$$
$$\Rightarrow \quad b = 2$$

904. (45) $f(x)$ is non differentiable at $x = 0$

and $[2x]$ is discontinuous at $x = \underbrace{\frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3}_{\text{5 points}}$

$\therefore f(x)$ should be differentiable and continuous at $x = 2$, so $k = 3$

$f'(3^+) = f'(3^-) \Rightarrow 2(3 - a) = 0, a = 3$

$f(3^+) = f(3^-) \Rightarrow (3 - a)^2 + b = 5, b = 5$

$a \cdot b \cdot k = 3 \times 5 \times 3 = 45$

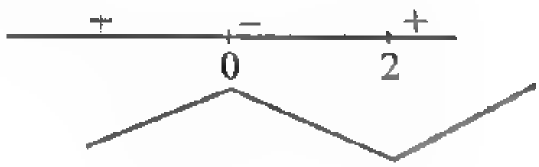
905. (6) $h(x) = \dfrac{x^2 + 4x + a}{x^2 + 6x + 2a}$ is an onto function means range 'R'.

$\alpha^2 + 6\alpha + 2a = 0 \Rightarrow \alpha^2 + 4\alpha + a = 0 \Rightarrow 2\alpha = -a \Rightarrow \alpha = \dfrac{-a}{2}$

$\Rightarrow \dfrac{a^2}{4} - 4\left(\dfrac{a}{2}\right) + a < 0 \Rightarrow a(a-4) < 0 \Rightarrow a \in (0, 4)$

$\therefore a = 1 + 2 + 3 = 6$

906. (2)
$$f'(x) = x^2 - 2x$$



$$f(x) = \frac{x^3}{3} - x^2 + c$$
$$f(2) = 0$$
$$\therefore \quad \frac{8}{3} - 4 + c = 0$$
$$\Rightarrow \quad c = \frac{4}{3}$$
$$f(x) = \frac{x^3}{3} - x^2 + \frac{4}{3}$$

Minimum ordinate will be at $x = 2$

$$\therefore \quad f(2) = \frac{8}{3} - 4 + \frac{4}{3} = 0$$

$\therefore$ Point $(a, b) = (2, 0)$

$a + 6b = 2$

907. (0) $P'(x) > 0$ and $(P(x))^2 + 4 \le 4P(x^2)$

Put $x = 0 \Rightarrow (P(0))^2 - 4P(0) + 4 \le 0$

$\therefore (P(0) - 2)^2 \le 0 \Rightarrow P(0) = 2$

|||ly put $x = 1 \Rightarrow (P(1) - 2)^2 \le 0$

$\Rightarrow P(1) = 2$

$\therefore$ Using Rolle's theorem in [0, 1] $P'(c) = 0$ for some $c \in (0, 1)$ but given $P'(x) > 0$.

Hence no polynomials exists.

908. (16) $$\frac{xf'(g(x))g'(x)}{f(g(x))} = \frac{g'(f(x))f'(x)}{g(f(x))}$$

$$x\frac{d}{dx}(\ln f(g(x))) = \frac{d}{dx}(\ln g(f(x))) \quad \ldots(1)$$

Also $$\int_0^a f(g(x))\,dx = \frac{1}{2} - \frac{e^{-2a}}{2} \;\forall\, a$$

$\therefore$ On differentiate, we get $f(g(a)) = e^{-2a} \Rightarrow f(g(x)) = e^{-2x}$ $\quad \ldots(2)$

From Eqns. (1) and (2), we get

$$x\frac{d}{dx}(\ln e^{-2x}) = \frac{d}{dx}(\ln g(f(x)))$$

$$-2x = \frac{d}{dx}(\ln g(f(x)))$$

On integrating, we get

$$\ln g(f(x)) = -x^2 + C$$

$$g(f(x)) = k \cdot e^{-x^2}$$

Given $$g(f(0)) = 1 \Rightarrow k = 1$$

$\therefore$ $$g(f(4)) = e^{-16} \Rightarrow \lambda = 16$$

909. (21) $B = adj(A) \Rightarrow B = |A|\dfrac{adjA}{|A|} \Rightarrow B = |A|A^{-1}$

$B = 3A^{-1}, \quad B^{-1} = \dfrac{1}{3}A$

$f(x) = \dfrac{1}{3}(x^3 - 6x^2 + 9x + 9)$

$f'(x) = x^2 - 4x + 3$

Globle maximum at $x = 6 \Rightarrow \dfrac{1}{3}(72 - 72 + 54 + 9) = 21$

910. (9) $L=\lim_{n\to\infty}\sum_{k=0}^{n-1}\frac{k}{n}\left(\left(\frac{k+1}{n}\right)^{\frac{1}{m}}-\left(\frac{k}{n}\right)^{\frac{1}{m}}\right)=\lim_{n\to\infty}\left[\sum_{k=0}^{n-1}\frac{k+1-1}{n}\left(\frac{k+1}{n}\right)^{\frac{1}{m}}-\sum_{k=0}^{n-1}\frac{k}{n}\left(\frac{k}{n}\right)^{\frac{1}{m}}\right]$

$$=\lim_{n\to\infty}\left[\sum_{k=0}^{n-1}\frac{k+1}{n}\left(\frac{k+1}{n}\right)^{\frac{1}{m}}-\sum_{k=0}^{n-1}\left(\frac{k}{n}\right)^{1+\frac{1}{m}}-\sum_{k=0}^{n-1}\frac{1}{n}\left(\frac{k+1}{n}\right)^{\frac{1}{m}}\right]$$

If $k+1=k \Rightarrow$ Limit will become $1\to n$

$$=\lim_{n\to\infty}\left[\sum_{k=1}^{n}\frac{k}{n}\left(\frac{k}{n}\right)^{\frac{1}{m}}-\sum_{k=0}^{n-1}\left(\frac{k}{n}\right)^{1+\frac{1}{m}}-\sum_{k=0}^{n-1}\frac{1}{n}\left(\frac{k+1}{n}\right)^{\frac{1}{m}}\right]$$

$$=\lim_{n\to\infty}(T_n-T_0)-\int_0^1 x^{1/m}\,dx=1-\frac{m}{m+1}=\frac{1}{m+1}=\frac{1}{10}\Rightarrow m=9$$

911. (168) $\qquad f(x^3+1)=t \Rightarrow f'(x^3+1)\cdot 3x^2\,dx=dt$

$$\Rightarrow \quad \frac{1}{3}\int_2^{10} g(t)\,dt-\left(\frac{3\cdot x^3}{3}\right)_0^2=0$$

$$\Rightarrow \quad \int_2^{10} g(t)\,dt=24 \Rightarrow \int_2^{10} g(x)\,dx+\int_4^{20} g^{-1}(x)\,dx=200-8$$

$$\Rightarrow \quad \int_4^{20} g^{-1}(x)\,dx=200-8-24=168$$

912. (190) $\qquad B^2=I$

$$AB=\begin{bmatrix} a & x & p\\ y & q & b\\ r & c & z\end{bmatrix}\begin{bmatrix} 0 & 0 & 1\\ 0 & 1 & 0\\ 1 & 0 & 0\end{bmatrix}=\begin{bmatrix} p & x & a\\ b & q & y\\ z & c & r\end{bmatrix}$$

$$AB=AB^3=\ldots\ldots=AB^{19}=\begin{bmatrix} p & x & a\\ b & q & y\\ z & c & r\end{bmatrix}$$

$$\text{tr.}(AB+AB^3+\ldots\ldots+AB^{19})=210$$

$\Rightarrow 10(p+q+r)=210 \Rightarrow p+q+r=21,\ p,q,r\in N$

$p'+q'+r'=18,\ p',q',r'\in W$

$\therefore$ Number of ordered triplets $(p,q,r)={}^{20}C_2=\frac{20\times 19}{2}=190$

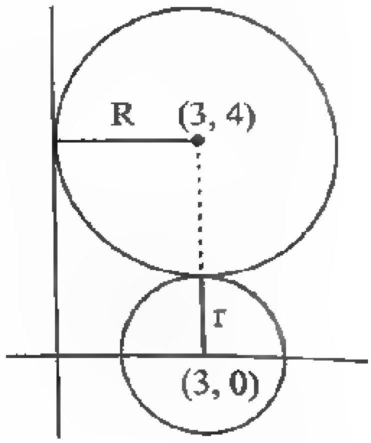
913. (3) $\qquad f(x)=(x-1)(x-2)(x-3)(c-x)+x^4-x$

Coefficient of $x^3=1$

$\therefore \quad c+1+2+3=1 \Rightarrow c=-5$

$$f(x) = (x-1)(x-2)(x-3)(-5-x)+x^4-x$$
$$f(4) = 3\cdot 2\cdot 1(-9)+4^4-4 = 198 = 2\times 3^2\times 11$$

914. (28) $xy-3x-4y+12=0$



$$(x-3)(y-4)=0 \ \Rightarrow\ x=3 \text{ and } y=4$$

$S: x^2+y^2-6x-8y+c=0$

For $R=3$, $r=1$ and 7

For $R=2$, $r=2$ and 6

For $R=1$, $r=3$ and 5

For $R=0$, $r=4$

$\therefore$ Sum of all possible values of

$$r = 1+2+3+4+5+6+7 = 28$$

915. (6) $I_n = 2n\int_0^1 \frac{3\{x\}+1}{\{3x\}+1}dx = 2n\left(\int_0^{\frac{1}{3}} \frac{3x+1}{3x+1}dx + \int_{\frac{1}{3}}^{\frac{2}{3}} \frac{3x+1}{3x-1+1}dx + \int_{\frac{2}{3}}^{1} \frac{3x+1}{3x-2+1}dx\right)$

$$= 2n\left(\frac{1}{3}+\frac{1}{3}+\frac{\ln 2}{3}+\int_{\frac{2}{3}}^{1}\left(1+\frac{2}{3x-1}\right)dx\right) = 2n\left(\frac{1}{3}+\frac{1}{3}+\frac{\ln 2}{3}+\frac{1}{3}+\frac{2}{3}(\ln(3x-1))_{\frac{2}{3}}^{1}\right)$$

$$= 2n\left(1+\frac{\ln 2}{3}+\frac{2\ln 2}{3}\right) = 2n(1+\ln 2) - n\ln(4e^2) \equiv 6\ln(4e^2)$$

$\therefore\ n-6$

916. (139) $f(-1)=0$, $f(0)=2$ and $f(2)=24$, $f'(x)=3x^2+4x+3$

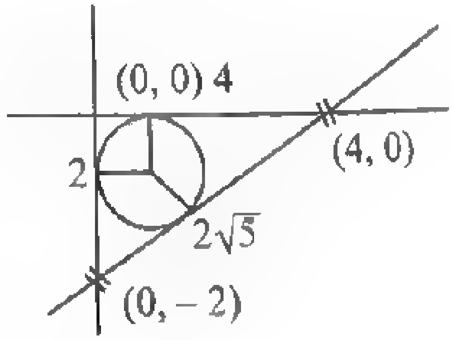
$$\frac{d}{dx}(g(g(g(x)))) = g'(g(g(x)))\cdot g'(g(x))\cdot g'(x)$$

$$\left.\frac{d}{dx}(g(g(g(x))))\right|_{x=24} = g'(g(g(24)))\cdot g'(g(24))\cdot g'(24) = g'(g(2))\cdot g'(2)\cdot \frac{1}{f'(2)}$$

$$= g'(0)\cdot\frac{1}{f'(0)}\cdot\frac{1}{f'(2)} = \frac{1}{f'(-1)}\cdot\frac{1}{f'(0)}\cdot\frac{1}{f'(2)} = \frac{1}{2}\times\frac{1}{3}\times\frac{1}{23} = \frac{1}{138} \equiv \frac{p}{q}$$

$\therefore \qquad p+q = 139$

917. (7) $x^2y-2xy^2-4xy=0 \Rightarrow xy(x-2y-4)=0$



$$r = \frac{\Delta}{s} = \frac{\frac{1}{2}\times 2\times 4}{3+\sqrt{5}} = \frac{4}{3+\sqrt{5}}$$

$$\text{Required sum} = \frac{2}{r} = \frac{2(3+\sqrt{5})}{4} = \frac{6+2\sqrt{5}}{4} = \frac{(\sqrt{5}+1)^2}{4}$$

$$= 4\cos^2\frac{\pi}{5} = 2\left(1+\cos\frac{2\pi}{p}\right) \equiv k\left(1+\cos\frac{2\pi}{p}\right)$$

$\therefore \qquad k+p = 2+5 = 7$

918. (7) $(x^2-5x+10)(x^2-5x+2)+12=0$

$(x^2-5x+10)(x^2-5x+10-8)+12=0$

$(x^2-5x+10)^2-8(x^2-5x+10)+12=0$

$x^2-5x+10=2,6$

$$\therefore \sum\frac{1}{a^2-5a+10} = \frac{2}{2}+\frac{2}{6} = \frac{4}{3} - \frac{p}{q}$$

$\therefore \qquad p+q=7$

919. (3) $$I = \int \frac{(e^x-1)(\sin x-\cos x)+x\cos x}{\sin^2 x\left(1+\left(\frac{e^x-1-x}{\sin x}\right)^2\right)}dx$$

Put, $$\frac{e^x-1\ \ x}{\sin x} = t \Rightarrow \frac{(\sin x)(e^x-1)-(e^x-1-x)\cos x}{\sin^2 x}dx - dt$$

$$\Rightarrow \frac{(e^x-1)(\sin x-\cos x)+x\cos x}{\sin^2 x}dx = dt$$

$$\therefore \qquad I = \int\frac{dt}{1+t^2} = \tan^{-1}\left(\frac{e^x-1-x}{\sin x}\right)+C$$

$$\therefore \qquad f(x) = \frac{e^x-1-x}{\sin x}$$

$$\lim_{x\to 0}\frac{f(x)}{x} = \lim_{x\to 0}\frac{e^x-1-x}{x\left(\frac{\sin x}{x}\right)x} = \frac{1}{2} \equiv \frac{p}{q}$$

$\therefore (p+q)|_{\text{least}} = 3$

920. (7) $\vec{v}_1\cdot\vec{v}_2 = 0 \Rightarrow 2\sin\theta-2\cos\theta-a=0 \Rightarrow a = 2(\sin\theta-\cos\theta)$

$a|_{\max} = 2\sqrt{2}$ when $\theta = 3\pi/4$

$$\sqrt{\begin{vmatrix} \vec{v}_1\cdot\vec{v}_1 & \vec{v}_1\cdot\vec{v}_2 & \vec{v}_1\cdot\vec{v}_3 \\ \vec{v}_2\cdot\vec{v}_1 & \vec{v}_2\cdot\vec{v}_2 & \vec{v}_2\cdot\vec{v}_3 \\ \vec{v}_3\cdot\vec{v}_1 & \vec{v}_3\cdot\vec{v}_2 & \vec{v}_3\cdot\vec{v}_3 \end{vmatrix}} = [\vec{v}_1 \quad \vec{v}_2 \quad \vec{v}_3]$$

$$[\vec{v}_1 \quad \vec{v}_2 \quad \vec{v}_3] = \begin{vmatrix} 1/\sqrt{2} & -2 & 2\sqrt{2} \\ 2 & 1/\sqrt{2} & -1 \\ 1 & 0 & 1 \end{vmatrix} = 1(2+2)+1\left(\frac{-1}{2}+4\right) = \frac{15}{2} = \frac{p}{q}$$

$$\therefore \qquad |p-4q| = |15-8| = 7$$

921. (125) $A+B=3I \Rightarrow AA^T+BA^T=3A^T \Rightarrow AA^T=3A^T-BA^T \Rightarrow 4I=3A^T-BA^T$

$\Rightarrow 12A^{-1}-BA^T+I=5I$

$\therefore \det(12A^{-1}-BA^T+I)=125$

922. (7) $\qquad f(x)=\sin^{-1}x+\cos^{-1}x+\tan^{-1}x$

Range of $f(x)=\left[\frac{\pi}{4},\frac{3\pi}{4}\right] \Rightarrow a=\frac{\pi}{4}$ and $b=\frac{3\pi}{4}$

Applying, $\cos A=\dfrac{b^2+c^2-a^2}{2bc}$

$$\frac{3}{\sqrt{10}}=\frac{\frac{9\pi^2}{16}+c^2-\frac{\pi^2}{16}}{2\cdot\left(\frac{3\pi}{4}\right)\cdot c} \Rightarrow c=\frac{\sqrt{10}\pi}{4},\ \frac{\pi}{\sqrt{10}} \text{ (rejected)}$$

Here, $\qquad a^2+b^2=c^2 \Rightarrow \angle C=90^\circ$

$$r=(s-c)\tan\frac{C}{2}=\left(\frac{\pi+\frac{\sqrt{10}\pi}{4}}{2}-\frac{\sqrt{10}\pi}{4}\right)\times 1=\frac{\pi(4-\sqrt{10})}{8}=\frac{6\pi}{8(4-\sqrt{10})}=\frac{p\pi}{q(q+\sqrt{10})}$$

$\therefore\ p=3$ and $q=4 \Rightarrow p+q=7$

923. (3) $\qquad x^2dy+y^2dx=0$

$$\Rightarrow \qquad \frac{dy}{y^2}+\frac{dx}{x^2}=0 \Rightarrow \frac{-1}{y}-\frac{1}{x}=C$$

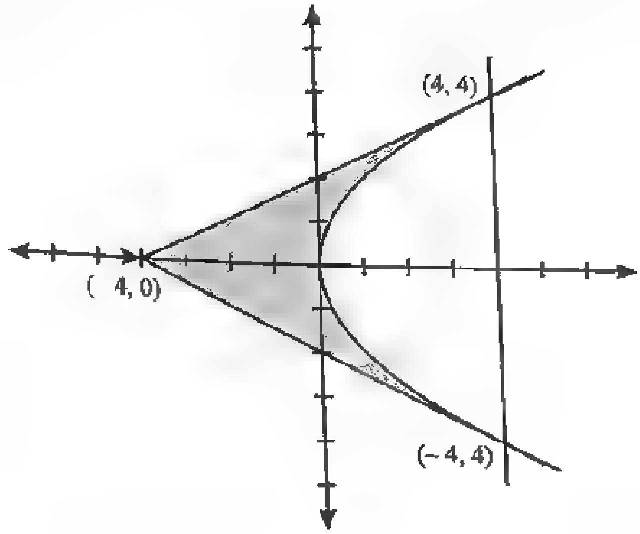
$(2,2) \Rightarrow C=1$

$$\therefore \qquad \frac{1}{y}+\frac{1}{x}=1 \Rightarrow y=\frac{x}{x-1}$$

Required area $=\displaystyle\int_2^3 \frac{x}{x-1}dx=\int_2^3\left(1+\frac{1}{x-1}\right)dx \Rightarrow (x+\ln(x-1))_2^3=1+\ln 2=a+\ln b$

Hence, $a+b=3$

924. (10) Area of the shaded region is



$$= \frac{1}{2} \times 8 \times 8 - \frac{2}{3} \times 32$$

$$= \frac{1}{3} \times 32$$

$$\Rightarrow \quad \frac{32}{3} = \Delta$$

$$\therefore \quad [\Delta] = 10$$

925. (2)

$$\int_{-\alpha}^{0} f(x)\,dx + \int_{2}^{0} f^{-1}(x)\,dx = 2\alpha$$

$$\therefore \quad \int_{0}^{2} f^{-1}(x)\,dx = 1 - 2\alpha$$

$$\int_{0}^{\beta} f(x)\,dx + \int_{0}^{-4} f^{-1}(x)\,dx = -4\beta$$

$$\int_{0}^{-4} f^{-1}(x)\,dx = -4\beta + 3 \quad \Rightarrow \quad \int_{-4}^{0} f^{-1}(x)\,dx = 4\beta - 3$$

$$\therefore \quad \text{Required area} = \left|\int_{0}^{2} f^{-1}(x)\,dx\right| + \left|\int_{-4}^{0} f^{-1}(x)\,dx\right|$$

$$= 2\alpha - 1 + 4\beta - 3 = 2\alpha + 4\beta - 4 \equiv p\alpha + q\beta + r$$

$$\therefore \quad p + q + r = 2 + 4 - 4 = 2$$

926. (13) $B \Rightarrow 2, 4, 6, 8, 10, 12, 14, 16$

$C \Rightarrow 3, 6, 9, 12, 15$

$$C - B \Rightarrow \begin{array}{ccc} 3, & 9, & 15 \\ \big| & \big| & \big| \\ 2, & 8, & 2 \end{array}$$

$$A \Rightarrow \begin{array}{ccccc} 10, & 11, & 12, & \ldots\ldots. & 16 \\ \big| & \big| & \big| & & \big| \\ 7, & 6, & 5, & \ldots\ldots. & 1 \end{array}$$

$$\therefore \quad n(A) = 28$$

$$P\left(\frac{C-B}{A}\right) = \frac{2}{28} = \frac{1}{14} \equiv \frac{a}{b}$$

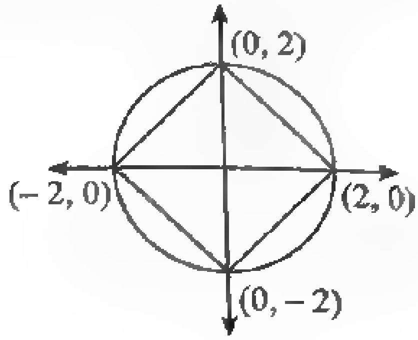
$$\therefore \quad |a - b| = |1 - 14| = 13$$

927. (8) $A=\{z\,||z|=2\}$

$B=\{z\,||z-\bar{z}|+|z+\bar{z}|\le 4\}$

$B=\{z\,||x|+|y|\le 2\}$

Area of the square is = 8 sq. units.



928. (20) $c_{22}c_{33}-c_{23}c_{32}=\det.(A^{20})=2^{20}=2^m$

$\therefore\ m=20$

929. (16) $k=\lim\limits_{x\to 0}\left(\dfrac{e^x(e^{nx}-1)}{e^x-1}+e^x x^3\right)=n$

$f(x)=e^x+e^{2x}+e^{3x}+\ldots\ldots+e^{nx}+e^x\cdot x^3$

$f'(x)=e^x+2e^{2x}+3e^{3x}+\ldots\ldots+ne^{nx}+e^x\cdot x^3+e^x\cdot 3x^2$

$f''(x)=e^x+2^2e^{2x}+3^2e^{3x}+\ldots\ldots+n^2e^{nx}+e^x\cdot x^3+6e^x\cdot x^2+e^x\cdot 6x$

$f'''(x)=e^x+2^3e^{2x}+3^3e^{3x}+\ldots\ldots+n^3e^{nx}+e^x\cdot x^3+9e^x\cdot x^2+e^x\cdot 18x+6e^x$

$f'''(0)=1^3+2^3+3^3+\ldots\ldots+n^3+6=1302$

$\left(\dfrac{n(n+1)}{2}\right)^2=1296\Rightarrow n=8$

$\therefore\quad k+n=16$

930. (41) Let $P(E_1)=a, P(E_2)=b$ and $P(E_3)=c$

$3a(1-b)(1-c)=(1-a)b(1-c)=9(1-a)(1-b)c=3(1-a)(1-b)(1-c)$

$\dfrac{3a}{1-a}=\dfrac{b}{1-b}=\dfrac{9c}{1-c}=3\ \Rightarrow\ a=\dfrac{1}{2},\ b=\dfrac{3}{4},\ c=\dfrac{1}{4}$

Now, $\begin{vmatrix}1/2 & 3/4 & 1/4\\ 3/4 & 1/4 & 1/2\\ 1/4 & 1/2 & 3/4\end{vmatrix}=\dfrac{1}{64}\begin{vmatrix}2 & 3 & 1\\ 3 & 1 & 2\\ 1 & 2 & 3\end{vmatrix}=-\dfrac{9}{32}$

$\therefore\ \dfrac{a}{b}=\dfrac{9}{32}\Rightarrow a+b=41$

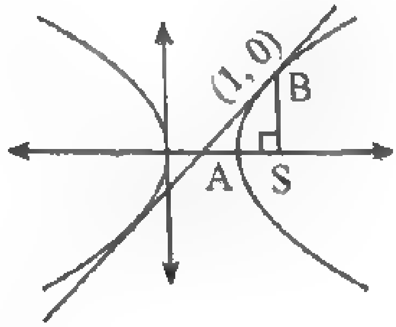
931. (28) Tangent to the parabola

$$y=mx-\frac{a}{4m}$$

$(1,0)\Rightarrow a=4m^2$

(1, 0) lies on the directrix of the hyperbola

$$\frac{2}{e}=1\Rightarrow e=2$$



$$\therefore \quad 1+\frac{b^2}{4}=4 \Rightarrow b^2=12$$

Tangent to the hyperbola

$$y=mx\pm\sqrt{4m^2-b^2}$$

$$\therefore \quad \frac{a^2}{16m^2}=4m^2-b^2$$

$$m^2=4m^2-b^2 \Rightarrow 3m^2=12 \Rightarrow m^2=4$$

$$\Rightarrow \quad a=16$$

$$\therefore \quad a+b^2=28$$

932. (7) $$\sum_{r=2}^{\infty}\tan^{-1}\left(\frac{(r-2)-(r-3)}{1+(r-3)(r-2)}\right)=\sum_{r=2}^{\infty}(\tan^{-1}(r-2)-\tan^{-1}(r-3))$$

$$=\tan^{-1}0-\tan^{-1}(-1)$$

$$\tan^{-1}1-\tan^{-1}0$$

$$\tan^{-1}2-\tan^{-1}1$$

$$\vdots$$

$$\vdots$$

$$\tan^{-1}(n-2)-\tan^{-1}(n-3)$$

$$S_n=\tan^{-1}(n-2)+\frac{\pi}{4}$$

$$\therefore \quad S_\infty=\frac{\pi}{2}+\frac{\pi}{4}=\frac{3\pi}{4}$$

933. (6) $f(6)=(x-1)(x-2)(x-3)(x-4)(x-5)+x$

934. (2) $$-1\le\frac{2-x}{2x}\le 1$$

$$\therefore \quad \frac{2-x}{2x}+1\ge 0 \quad \text{and} \quad \frac{2-x}{2x}-1\le 0$$

$$\Rightarrow \quad \frac{2+x}{2x}\ge 0 \quad \text{and} \quad \frac{2-3x}{2x}\le 0$$

$$\therefore \quad x\in(-\infty,-2]\cup\left[\frac{2}{3},\infty\right)$$

$$\therefore \quad x\ne\{-1,0\}$$

935. (2) Let $$x=\frac{\pi}{2}-h$$

$$\therefore \quad \lim_{h\to 0}\left((\pi-2h)\cot h-\frac{\pi}{\sin h}\right)=\lim_{h\to 0}\left(\frac{\pi\cos h}{\sin h}-\frac{\pi}{\sin h}-\frac{2h}{\tan h}\right)=-2$$

936. (25) Since $f(x)$ is onto hence range of $f(x)$ equals co-domain

Now range of $4x^2+3x$ in $\left[\frac{-1}{2}, 0\right]$ is $\left[\frac{-9}{16}, 0\right]$

Hence, range of $f(x)=\cos^{-1}(4x^2+3x)$ is $\left[\frac{\pi}{2}, \pi-\cos^{-1}\frac{9}{16}\right]$

937. (9) Let $\left(\frac{x}{2}, \frac{x}{2}, \frac{y}{3}, \frac{y}{3}, \frac{y}{3}, \frac{z}{4}, \frac{z}{4}, \frac{z}{4}, \frac{z}{4}\right)$

$$\therefore \quad \text{GM} \le \text{AM}$$

$$\Rightarrow \left(\frac{x^2 \cdot y^3 \cdot z^4}{3^3 \cdot 2^{10}}\right)^{\frac{1}{9}} \le 3$$

$$\Rightarrow \quad x^2 y^3 z^4 \le 3^{12} \cdot 2^{10} \to 9 \cdot 6^{10}$$

938. (6) $\frac{1}{3} \le \sin x < \frac{1}{2}$ is the only interval.

$$\sin^{-1}\left(\frac{1}{3}\right) \le x < \frac{\pi}{6}$$

$$\therefore \quad \alpha - \sin^{-1}\left(\frac{1}{3}\right);\ \beta = \frac{\pi}{6}$$

$$\therefore \quad \cos(\alpha+\beta) = \frac{2\sqrt{2}}{3}\left(\frac{\sqrt{3}}{2}\right) - \frac{1}{3}\left(\frac{1}{2}\right) = \frac{\sqrt{6}}{3} - \frac{1}{6} \equiv \frac{\sqrt{a}}{3} - \frac{1}{a}$$

$$\therefore \quad a = 6$$

939. (1)

$$\frac{dx}{dy} = \frac{\frac{dx}{dt}}{\frac{dy}{dt}} = \frac{12t^2}{12t^3} = \frac{1}{t}$$

$$\frac{d^2x}{dy^2} = \frac{d}{dt}\left(\frac{1}{t}\right) \times \left(\frac{dt}{dy}\right) = \frac{-1}{t^2} \times \frac{1}{12t^3} = \frac{-1}{12t^5}$$

$$\frac{\left(\frac{d^2x}{dy^2}\right)}{\left(\frac{dx}{dy}\right)^n} = \text{constant}$$

$$\therefore \quad n = 5$$

$$\therefore \quad \text{Sum} = \frac{\frac{4}{5}}{1-\frac{1}{n}} = \frac{\frac{4}{5}}{1-\frac{1}{5}} = 1$$

940. (4) Differentiating w.r.t. y keeping x constant, we get

$$f'(x+y)\cdot 1 = f'(y)$$

Putting $y=0$, $f'(x)=1 \Rightarrow f(x)=x+C$

Putting $x=y=0$ in given relation, $f(0)=0$

$$\therefore \quad C=0$$

$$\therefore \quad f(x)=x$$

$$A = \lim_{x\to 0}\frac{2^{\tan x}-2^{\sin x}}{x^3} = \lim_{x\to 0}\frac{2^{\sin x}(2^{\tan x-\sin x}-1)}{(\tan x-\sin x)}\times\left(\frac{\tan x-\sin x}{x^3}\right)$$

$$= 1\times \ln 2\times \lim_{x\to 0}\frac{\tan x(1-\cos x)}{x^3} = \frac{\ln 2}{2}$$

$$\therefore \quad 4A = 2\ln 2 = \ln 4$$

$$\therefore \quad e^{4A} = 4$$

941. (6)

$$I = \int_{\frac{-\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}}\sqrt{\frac{1-x}{1+x}}\sin^{-1}x\,dx = \int_{\frac{-\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}}\frac{1-x}{\sqrt{1-x^2}}\sin^{-1}x\,dx$$

Put $x=\sin\theta$

$$I = \int_{\frac{-\pi}{3}}^{\frac{\pi}{3}}(1-\sin\theta)\theta\,d\theta = \underbrace{\int_{\frac{-\pi}{3}}^{\frac{\pi}{3}}\theta\,d\theta}_{\text{odd},\,\therefore\,0} - \int_{\frac{-\pi}{3}}^{\frac{\pi}{3}}\theta\sin\theta\,d\theta = -\int_{0}^{\frac{\pi}{3}}\theta\sin\theta\,d\theta$$

$$I = -2\left[-\theta\cos\theta\Big|_0^{\pi/3} + \int_0^{\frac{\pi}{3}}\cos\theta\,d\theta\right]$$

$$I = -2\left[\frac{-\pi}{6}+\frac{\sqrt{3}}{2}\right] = \frac{\pi}{3}-\sqrt{3} = \frac{\pi}{M}-\sqrt{N}$$

$$\therefore \quad (M+N)=6$$

942. (4) Let $P_1 = (t_1^2, \sqrt{a}\,t_1^3)$ and $P_2 = (t_2^2, \sqrt{a}\,t_2^3)$

$$\because\ 2y\frac{dy}{dx} = 3ax^2 \Rightarrow \frac{dy}{dx} = \frac{3ax^2}{2y}$$

Slope of tangent at $P_1 = \dfrac{3\sqrt{a}t_1}{2} = \dfrac{\sqrt{a}(t_1^3-t_2^3)}{(t_1^2-t_2^2)}$

$$\Rightarrow 3(t_1)(t_1+t_2) = 2(t_1^2+t_2^2+2t_1t_2)$$

$$\Rightarrow t_1^2-t_2^2+t_1t_2 = 0 \Rightarrow (t_1+2t_2)(t_1-t_2)=0$$

$t_1 = -2t_2$

$\therefore$ Abscissae, $t_1^2, t_2^2, t_3^2, \ldots \Rightarrow t_1^2, \frac{t_1^2}{4}, \frac{t_1^2}{16}, \ldots$ will be G.P. with common ratio $\frac{1}{4}$ for $\forall$ a.

$\therefore$ Sum of abscissae $= \dfrac{x_1}{1-\frac{1}{4}} = \dfrac{3}{3/4} = 4$

943. (4) $$y = \frac{x^2-x+c}{x^2+x+c} \le 1 - \frac{2}{1-2\sqrt{c}} = \frac{5}{3}$$

$\Rightarrow$ $c = 4$

944. (5) $$\frac{x_1+2x_2+3x_3+4x_4+5x_5}{15} \ge (x_1 \cdot x_2^2 \cdot x_3^3 \cdot x_4^4 \cdot x_5^5)^{\frac{1}{15}}$$

$\Rightarrow$ $x_1 = x_2 = x_3 = x_4 = x_5 = 1$

945. (6) $$f'(x) = \frac{f(x)}{x} + \ln x \Rightarrow f(x)\cdot\frac{1}{x} = \int \frac{\ln x}{x}dx + C$$

$$\Rightarrow \frac{f(x)}{x} = \frac{(\ln x)^2}{2} + C \Rightarrow f(x) = \frac{x(\ln x)^2}{2}$$

$$L = \lim_{x\to 1}\frac{f(x)}{\sin^2 \pi x} = \lim_{x\to 1}\frac{(\ln(1+(x-1)))^2}{2\pi^2(x-1)^2} = \frac{1}{2\pi^2}$$

$$\Rightarrow \left[\frac{1}{\pi L}\right] = 2\pi$$

946. (4) $x+2y^2 = 0$ and $x+3y^2 = 1 \Rightarrow$ point of intersection is $(-2, 1)$ and $(-2, \quad 1)$.

$$A = \left|\int_{-1}^{1}(5y^2-1)dy\right| = \frac{4}{3}$$

947. (6) $f(x) \ge -3$

$\Rightarrow x^2+2px+4p+f(x) \ge x^2+2px+4p-3 > 0 \; \forall \; x \in R$

$\Rightarrow p^2-4p+3 < 0 \Rightarrow 1 \le p \le 3$

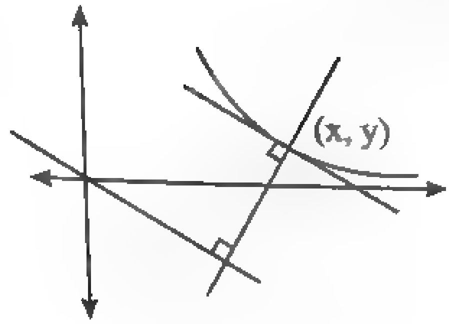
948. (15) $(ax_1+by_1+c)+(ax_2+by_2+c)+(ax_3+by_3+c) = 0$

$$\Rightarrow a\left(\frac{x_1+x_2+x_3}{3}\right)+b\left(\frac{y_1+y_2+y_3}{3}\right)+c = 0$$

949. (107) $y = f(x)$ is onto

$$\Rightarrow 1 < c < \frac{215}{2} \quad \Rightarrow 2 \le [c] \le 107$$

950. (6) Equation of normal at $P(x, y)$



$$Y - y = \frac{-1}{m}(X - x)$$

$$mY - my = -X + x$$

$$x + my - (x + my) = 0$$

$$\left|\frac{my + x}{\sqrt{1+m^2}}\right| = y$$

$\Rightarrow$ $$m^2y^2 + x^2 + 2xmy = m^2y^2 + y^2 \Rightarrow m = \frac{y^2 - x^2}{2xy}$$

$$2xy\frac{dy}{dx} = y^2 - x^2$$

$y^2 = t \Rightarrow 2y\frac{dy}{dx} = \frac{dt}{dx} \Rightarrow x\frac{dt}{dx} = t - x^2$

$\Rightarrow \frac{dt}{dx} - \frac{t}{x} = -x$ which is linear differential equation

$\text{I.F.} = e^{-\int \frac{dx}{x}} = \frac{1}{x}$

$\therefore$ $$t\left(\frac{1}{x}\right) = -\int 1 \cdot dx + C \Rightarrow \frac{y^2}{x} = -x + C$$

$(1, 1) \Rightarrow C = -2$

$$y^2 = -x^2 + 2x \Rightarrow x^2 + y^2 - 2x = 0$$

$\Rightarrow \quad (x-1)^2 + y^2 = 1$

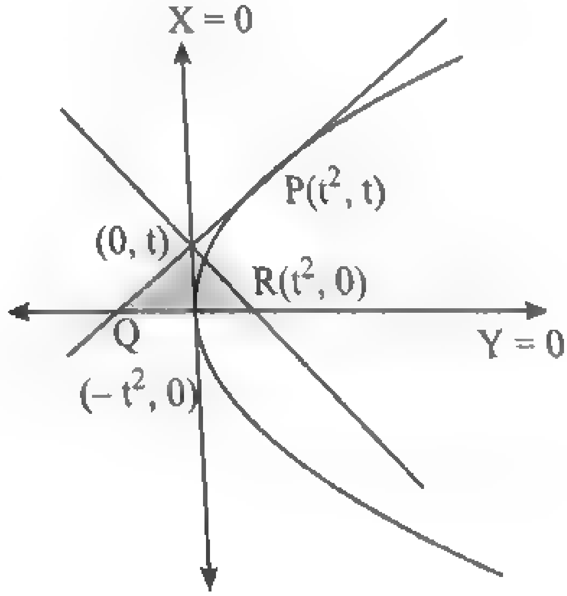
$\therefore$ Area bounded by the curve, $A = \pi$

$\therefore$ $$[2A] = [2\pi] = 6$$

951. (3) $m = 2$ and $n = 1$

952. (17) $$y^2 = 4x$$

$$= P(at^2, 2at)$$



Tangent at P.

$$tY = X + t^2$$

$$Q \equiv (-t^2, 0), R \equiv (t^2, 0)$$

Area of ΔPQR, $\frac{1}{2} \times 2t^2 \times t = \pm 64$

$\Rightarrow$ $$t = \pm 4$$

Now, abscissa of point $P \Rightarrow x - 1 = 16$

$\Rightarrow$ $$x = 17$$

953. (13) $\log_{10}(x^2-2x+2)-1<0 \Rightarrow x^2-2x+2>10$

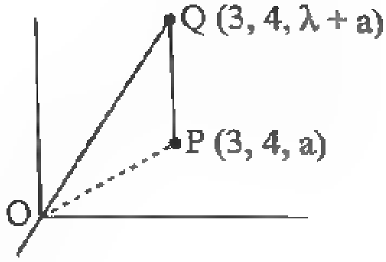
$\Rightarrow x^2-2x-8<0 \Rightarrow x\in(-2,4)$

$2x \notin I \Rightarrow x \neq \frac{-3}{2}, -1, \frac{-1}{2}, 0, \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3, \frac{7}{2}$

$\therefore x\in(-2,4)-\left\{\pm\frac{3}{2}, \pm\frac{1}{2}, \pm 1, 0, 2, \frac{5}{2}, 3, \frac{7}{2}\right\}$

$\therefore \left(a+b+\sum_{i=1}^{n} p_i\right) = -2+4+5+6=13$

954. (29) $Q=(3,4,\lambda+a)$



$3-8+2(\lambda+a)=a^2+4a+1$

$2\lambda=a^2+2a+6 \Rightarrow \lambda=\frac{1}{2}(a^2+2a+6)$

$\text{Ar}(\Delta OPQ)=\frac{1}{2}\times 5\times\frac{1}{2}(a^2+2a+6)=\frac{5}{4}[(a+1)^2+5]$

$\therefore$ Least area $=\frac{25}{4}=\frac{p}{q}$

$\Rightarrow p+q=29$

955. (197) $I=98\int_0^1(\sin 2)e^x(\tan x+\sec^2 x)dx$

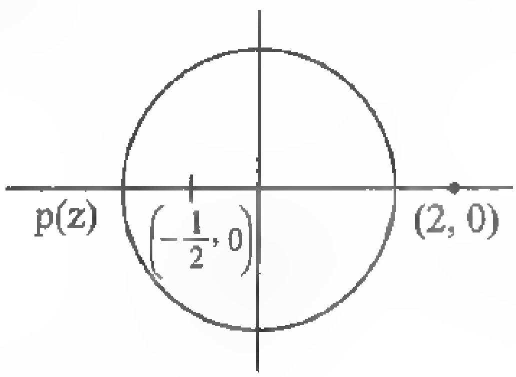
$=98(\sin 2)\cdot[e^x \tan x]_0^1$

$=98\times 2(\sin 1)(\cos 1)e(\tan 1)$

$=196\cdot e\cdot\sin^2 1 = p\cdot e\cdot \sin^2 q \rightarrow p+q=197$

956. (36) Locus of z is $x^2+y^2=1$

From the figure, it is clear that maximum value of



$2\left(|z_1-2|+\left|z_2+\frac{1}{2}\right|\right)=2\times\frac{9}{2}=\lambda$

$\therefore \quad 4\lambda=36$

957. (19) $f(1)=5, f(2)=8, f'(1)=3$ and $f''(1)=0$

$f(x)=(x-1)^3(x-2)+3x+2$

$f(3)=8+9+2=19$

958. (8) Given $a_1 e_1 = a_2 e_2$

$$PF_1^2 + PF_2^2 = 4a_1^2 e_1^2 \quad \text{...(1) Using Pythagorus theorem}$$

$$PF_1 + PF_2 = 2a_1 \quad \text{...(2)}$$

$$[PF_1 + PF_2] = 2a_2 \quad \text{...(3)}$$

$$(2)^2 + (3)^2 \Rightarrow 2[PF_1^2 + PF_2^2] = 24(a_1^2 + a_2^2) \quad \text{...(4)}$$

From Eqns. (1) and (4)

$$2(a_1^2 + a_2^2) = 4a_1^2 e_1^2$$

$$a_1^2 + a_2^2 = 2(a_1^2 e_1^2)$$

$$\therefore \quad 1 + \left(\frac{a_2}{a_1}\right)^2 = 2e_1^2$$

$$1 + \left(\frac{e_1}{e_2}\right)^2 = 2e_1^2$$

$$\therefore \quad \frac{1}{e_1^2} + \frac{1}{e_2^2} = 2$$

Let $\frac{1}{e_1} = \sqrt{2}\cos\theta, \ \frac{1}{e_2} = \sqrt{2}\sin\theta$

$$\therefore \quad E = 9e_1^2 + e_2^2 = \frac{9}{2} \times \sec^2\theta + \frac{1}{2}\operatorname{cosec}^2\theta$$

$$= \frac{1}{2}[9(1 + \tan^2\theta) + 1 + \cot^2\theta]$$

$$= \frac{1}{2}[10 + 9\tan^2\theta + \cot^2\theta] \text{ minimum value} = 6 \text{ using AM} \geq \text{GM}$$

$$\therefore \quad E_{\min} = 8$$

959. (7)

$$f(x+1) = 2f(x) \quad x \in R \quad \text{...(1)}$$

$$f(x) = x^2 - x \quad x \in (0, 1] \quad \text{...(2)}$$

$$f(x) \geq \frac{-8}{9} (\approx -0.8) \forall x \in (-\infty, m) \quad \text{...(3)}$$

$$\Rightarrow \quad 3m = 7$$

$x \to x + 1$ in Eqn. (1)

$$f(x+2) = 2f(x+1)2^2 f(x)$$

$$f(x+3) = 2^n f(x) = 2^n (x^2 - x)$$

$x + n = t$

$$f(t) = 2^n[(t-n)^2 - (t-n)] \text{ where } t \in (n, n+1]$$

$$f(t) = 2n[t^2 - t(2n+1) + n^2 + n]$$

$$2^n\left[\left(t-\frac{2n+1}{2}\right)^2-\frac{1}{4}\right]$$

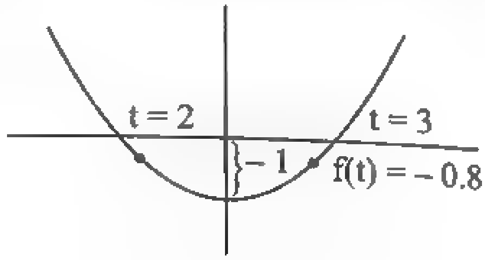
$\therefore \quad f(t) \geq -2^{n-2}$

Where $\quad n=1\ f(t)\geq -\frac{1}{2} \quad t\in(1,2]$

$\quad n=2\ f(t)\geq -1 \quad t\in(1,2]$

-0.8 will come in this region we will get two values of t and we will take smaller value of t as $x\in(-\infty, m)$.

$\therefore \quad f(T)=\frac{-8}{9}$



For $n=2$

$$4[t^2-5t+6]=\frac{-8}{9}$$

$$t=\frac{8}{3},\frac{7}{3}$$

$\therefore \quad m=\frac{7}{3}$

960. (351) $A=\{1,2,3,4,5,6,7\}$

Case-1: All elements of set A satisfy $f(x)=x$

In this case number of functions $=1$

Case-2: 4 elements of set A satisfy $f(x)=x$

Total number of functions $= {}^7C_4\cdot 2=70$

e.g., : $f(4)=4, f(5)=5, f(6)=6, f(7)=7$

Now for elements $\{1,2,3\}$ we have two options of mapping.

Case-3: 1 elements of set A satisfy $f(x)=x$

For remaining six elements make groups of (3, 3)

Hence, total functions $= {}^7C_1\times\frac{6!}{3!\cdot 3!\cdot 2!}\times 2\times 2=280$

Hence, total functions are 351.

961. (26) $A=\{1,2,3,4,5\}$

Case-1: All elements of set A maps to itself *i.e.*, satisfying $f(x)=x$

In this case number of functions $=1$

Case-2: 3 elements of set A maps to itself *i.e.*, satisfying $f(x)=x$

Total number of functions $= {}^5C_3\cdot 1=10$

Now for remaining two elements $\{1,2\}$ we have only one option of mapping (Dearrangement).

Case-3: 1 elements of set A maps to itself *i.e.*, satisfying $f(x)=x$

For remaining four elements make groups of (2, 2)

Hence, total functions $= {}^5C_1 \times \frac{4!}{2!\cdot 2!\cdot 2!} \times 1 \times 1 = 15$

Hence, total functions are 26.

962. (6) To show that this is the maximum, let the roots of $f(f(x))=0$ be a, b, c.

then, we must have $f(x)=a$ or $f(x)=b$ or $f(x)=c$.

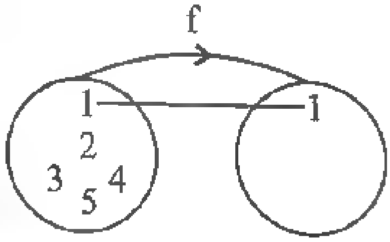
Since f is a quadratic equation, each of these equations can only have at most 2 roots.

This means, the number of roots to $f(f(f(x)))=0$ is less than or equal to 6.

963. (196) $f(f(x))=f(x);\ f(x)=y \Rightarrow f(y)=y$

Case-1: Range contains exactly one element it can be done in ${}^5C_1$ ways say 1

remaining 4 elements *i.e.*, 2, 3, 4, 5 can be mapped only in one ways $\Rightarrow$ total $= {}^5C_1 \cdot 1 = 5$

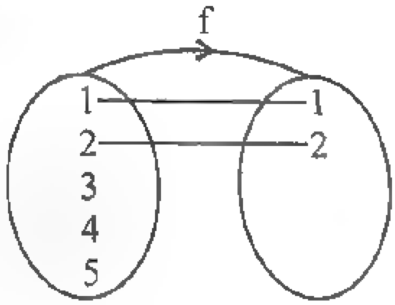


Case-2: Range contains two elements this can be done in ${}^5C_2$ ways say 1, 2

$\therefore\ f(1)=1;\ f(2)=2$

Remaining 3 elements *i.e.*, 3, 4 and 5 each can be mapped in 2 ways

Total $= {}^5C_2 \cdot 2^3 = 80$

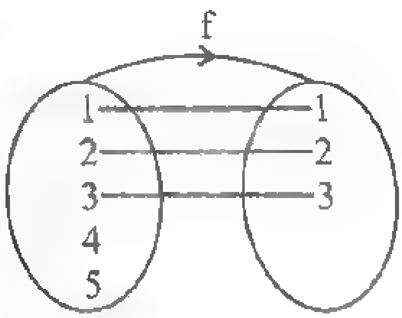


Case-3: Range contains 3 elements which can be done in ${}^5C_3$ ways say 1, 2, 3

$\therefore\ f(1)=1;\ f(2)=2$ and $f(3)=3$

Now, remaining 4 and 5 can be mapped only in 3 ways

Total $= {}^5C_3 \cdot 3^2 = 90$ ways

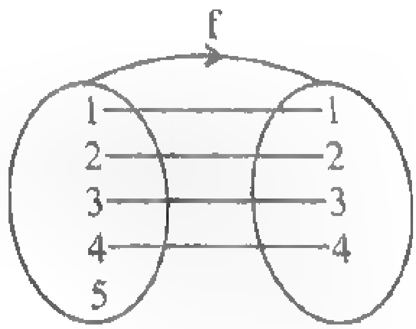


Case-4: Range contains all 4 elements which can be done in ${}^5C_4$ ways say 1, 2, 3, 4

$f(1)=1;\ f(2)=2;\ f(3)=3;\ f(4)=4$

Now remaining 5 can be mapped in 4 ways

Total $= {}^5C_4 \cdot 4 = 20$

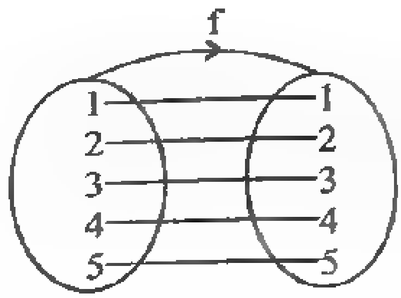


Case-5: Range contains all 5 elements which can be done in ${}^5C_5$ ways say 1, 2, 3, 4, 5

$f(1)=1$; $f(2)=2$; $f(3)=3$; $f(4)=4$; $f(5)=5$

Only 1 way

Total $= 5 + 80 + 90 + 20 + 1 = 196$



964. $(\frac{1}{\sqrt{3}})$ Do yourself.

965. (2) Since the equation is even on both sides,we only need to consider $x \geq d$ to find the positive solution x_2 and then $x_1 = -x_2$. For $0 \leq x \leq 1$,

$$2\left(x^2+\frac{1}{x^2}\right)+1-x^2 = 4\left(\frac{3}{2}-2^{x^2-1}-\frac{1}{2^{x^2-1}}\right)$$

$$x^2+\frac{2}{x^2}+1 = 6-2^{x^2-1}-\frac{1}{2^{x^2-1}}$$

$$x^2+\frac{2}{x^2}+2^{x^2-1}+\frac{1}{2^{x^2\ 1}} = 5$$

By inspection, the solution is $x^2 = 1$ or $x_1 = -1$ and $x^2 = 1$, then

$$I = \int_0^4 \left\{\frac{x}{4}\right\}\left(1+\left[\tan\left(\frac{\{x\}}{1+\{x\}}\right)\right]\right)dx \qquad \text{Note that } \left[\tan\left(\frac{\{x\}}{1+\{x\}}\right)\right] = 0$$

$$= \int_0^4 \left\{\frac{x}{4}\right\} dx = \int_0^4 \frac{x}{4}\, dx = \left.\frac{x^2}{8}\right|_0^4 = 2$$

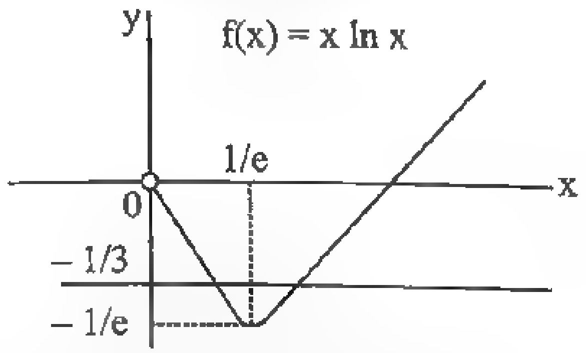
966. (3) **Case-I:** If $a < 0$, then $e^x - a > 0$ and $3ax + 1$ is sometimes + ve/ −ve both

$\therefore$ $f(x)$ cannot be positive always because of second bracket. Hence in this case no possible values of a.

Case-II: If $a = 0$ then $f(x) = e^x$ which is positive $\forall\ x \in R$ either both brackets must be positive or both brackets must be negative.Hence both the critical points must coincide.

$$\therefore \quad e^x = a \Rightarrow x = \ln a \text{ and } 3ax+1 = 0 \Rightarrow x = \frac{-1}{3a}$$

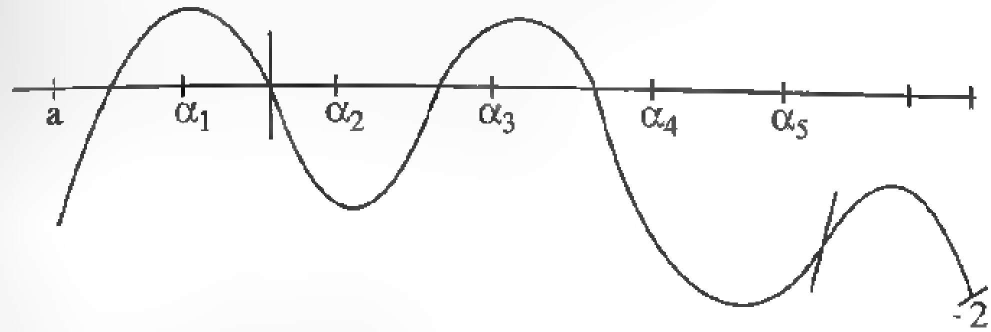
$$\therefore \ln a = \frac{-1}{3a} \Rightarrow a \ln a = \frac{-1}{3}$$



No. of values of a satisfying above equation is 2.

Hence total number of possible values of a are 3.

967. (4)



$f(x)$ and $f''(x)$ may concide

$\therefore$ $f(x)f''(x)$ has 5 distinct minimum roots

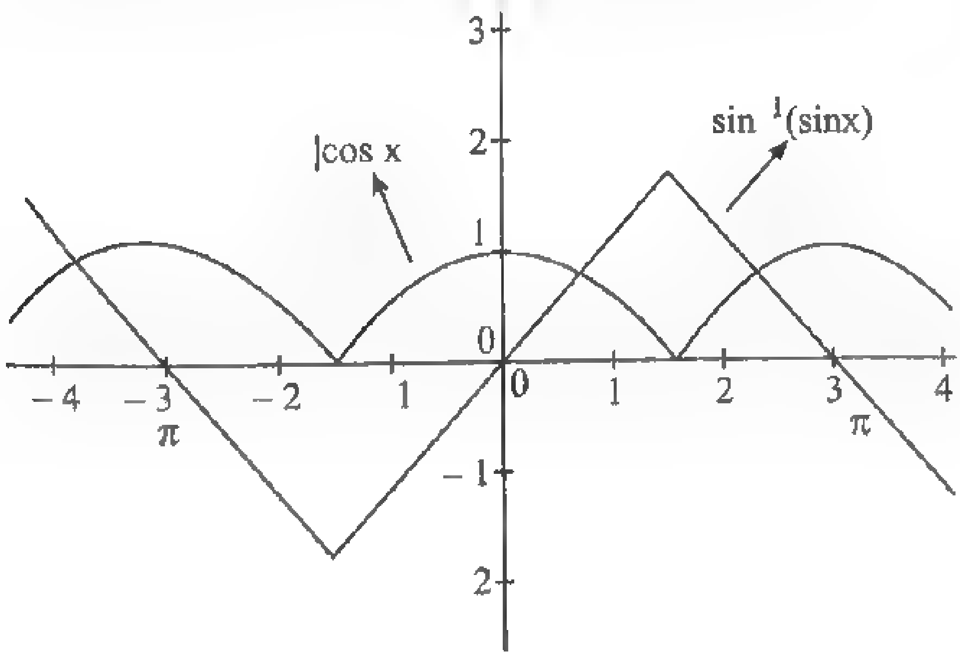
$\therefore$ $\frac{d}{dx}(f(x)f''(x))$ has minimum 4 distinct roots.

968. (2) $\sqrt{1+\cos 2x} = \sqrt{2}\sin^{-1}(\sin x)$

$\Rightarrow$ $\sqrt{2}\,|\cos x = \sqrt{2}\sin^{-1}(\sin x)$

$\Rightarrow$ $|\cos x| = \sin^{-1}(\sin x)$

When we draw the graph both functions (shown below) we can actually see that they intersect only at two points $\forall\, x \in -\pi \le x \le \pi$.



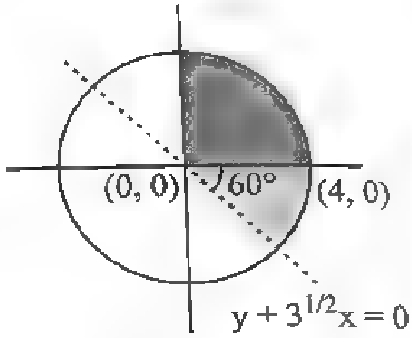
969. (23) Let $z = x + iy$

S_1 denotes the interior of circle of radius 4 units

S_3 denotes $x > 0$

$$S_2 = \text{Im}\left(\frac{(x-1+i(y+\sqrt{3}))(1+i\sqrt{3})}{4}\right) > 0$$

$$S_2 = i(y + \sqrt{3}x) > 0$$

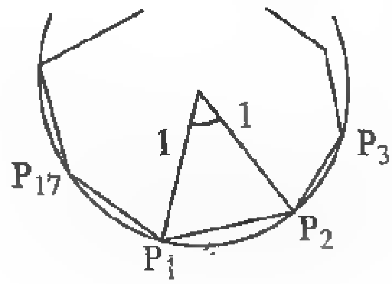


Now the shaded region represents the required area

Required area = Area of quarter of circle + Area of sector

$$= \frac{\pi r^2}{4} + \frac{\pi r^2 \theta}{360^\circ} = 4\pi + \frac{8}{3}\pi \Rightarrow \frac{20}{3}\pi$$

970. (17) $\theta = \dfrac{2\pi}{n} = n - 17$



$(P_1P_2)(P_1P_3)\ldots(P_1P_{17})$

Using $2\sin\dfrac{\theta}{2} \times 2\sin\dfrac{2\theta}{2} \ldots\ldots 2\sin\dfrac{16\theta}{2}$

$$2^{16}\sin\frac{\theta}{2}\sin\frac{2\theta}{2}\ldots\ldots\sin\frac{16\theta}{2} - 17$$

$$2^{n-1}\sin\frac{\theta}{2}\sin\frac{2\theta}{2}\ldots\ldots\sin(n-1)\frac{\theta}{2} = n$$

971. (6) Do yourself.

972. (14) Solving H and L we get

$$2x^2 + 2x - 1 = \begin{cases} \alpha \\ \beta \end{cases}$$

$$\alpha + \beta = -1 \quad \ldots(1)$$

$$\alpha\beta = \frac{-1}{2} \quad \ldots(2)$$

Eqn. of family of circles passing through A and B is $S_D + \lambda_L = 0$

$(x - \alpha)(x - \beta) + (y + \alpha + 1)(y + \beta + 1) + \lambda(x + y + 1) = 0$

$x^2 + y^2 - (\alpha + \beta)x + \alpha\beta + (\alpha + \beta + 2)y + (\alpha + 1)(\beta + 1) + \lambda(x + y + 1) = 0$

$$x^2 + y^2 + x - \frac{1}{2} + y - \frac{1}{2} + 1(x + y + 1) = 0$$

$\therefore$ Finally of circles through A and B is

$$x^2 + y^2 + x(\lambda + 1) + y(\lambda + 1) + \lambda - 1 = 0$$

Now, $\therefore$ this circle touches at $A(\alpha - \alpha - 1)$

$$\therefore \quad \left.\frac{dy}{dx}\right]_A \times m_{AC} = -1$$

Diff. hyperbola at A

$$2x+2y\frac{dy}{dx}+4\left[\frac{xdy}{dx}+y\right]+8+8\frac{dy}{dx}=0$$

$$\frac{dy}{dx}[y+2x+4]+x+2y+4=0$$

$$\frac{dy}{dx}=\frac{-(x+2y+4)}{2x+y+4}$$

Put $x=\alpha \quad y=-\alpha-1$

$$\left.\frac{dy}{dx}\right]_A=\frac{\alpha-2}{\alpha+3}$$

Let centre of circle is $\left\{\frac{-(\lambda+1)}{2}, \frac{-(\lambda+1)}{2}\right\}=\{k,k\}$

$$m_{AC}=\frac{k+\alpha+1}{k-\alpha}$$

$$\therefore \quad m_1m_2=-1$$

$$\frac{\alpha-2}{\alpha+3}\times\frac{k+\alpha+1}{k-\alpha}=-1$$

$$\frac{\alpha-2}{\alpha+3}=\frac{-(k-\alpha)}{k+\alpha+1}$$

$$\frac{1+2\alpha}{-5}=\frac{-k+\alpha+k+\alpha+1}{-k+\alpha-k-\alpha-1}$$

$$\frac{2\alpha+1}{-5}=\frac{2\alpha+1}{-2k-1}$$

$$\therefore \quad \alpha\neq-\frac{1}{2} \qquad \text{[From Eqn. (1) \& (2)]}$$

$$2k+1=5$$

$$k=2$$

$$\therefore \quad \frac{-(\lambda+1)}{2}=2 \Rightarrow \lambda=-5$$

$\therefore$ Eqn. of circle is $x^2+y^2-4x-4y-6=0$

$$R^2=4+4+6=14$$

973. (129) $(a^2\cot 9^\circ+d^2\tan 9^\circ)+(b^2\cot 27^\circ+c^2\tan 27^\circ)$ ...(1)

Given $\quad a+b+d=5$

Let $\quad a+d=k_1$

$$b+c=k_2$$

$\therefore$ $k_1+k_2=5$

Let $E_1=a^2\cot 9^\circ+d^2\tan 9^\circ$

$$E_1=a^2\cot 9^\circ+(k_1-a)^2\tan 9^\circ \quad \text{...(2)}$$

$$E_1=a^2(\cot 9^\circ+\tan 9^\circ)-ak_1\tan 9^\circ+k_1^2\tan 9^\circ$$

This is quadratic in a whose minimum value is obtained at $x=\dfrac{-B}{2A}$

i.e., $a=\dfrac{2k_1\tan 9^\circ}{2(\cot 9^\circ+\tan 9^\circ)}=k_1\sin^2 9^\circ$

Let $E_2=b^2\cot 27^\circ+c^2\tan 27^\circ$

$$b+c=k_2$$

$$E_2=b^2\cot 27^\circ+(k^2-b)^2\tan 27^\circ \quad \text{...(3)}$$

Make quadratic in b

E_2 min is obtained

$$b=k_2\sin^2 27^\circ$$

From Eqn. (2)

$\therefore$ $E_{1\,\min}=k_1^2\sin^4 9^\circ\cot 9^\circ+(k_1\cos^2 9)^2\tan 9^\circ$

$$=k_1^2\sin 9^\circ\cos 9^\circ(\sin^2 9^\circ+\cos^2 9^\circ)$$

$$=k_1^2\sin 9^\circ\cos 9^\circ$$

Similarly $E_2\min=k_2^2\sin 27^\circ\cos 27^\circ$

$\therefore$ $E_{\min}=E_{1\,\min}+E_{2\,\min}$

$k_1^2\sin 9^\circ\cos 9^\circ+k_2^2\sin 27^\circ\cos 27^\circ$

$$=\frac{1}{2}[k_1^2\sin 18^\circ+(5-k_1)^2\sin 54^\circ]$$

$$=\frac{1}{2}[k_1^2(\sin 18^\circ+\cos 36^\circ)10k_1\cos 36^\circ+25\cos 36^\circ]$$

$$E_{\min}=\frac{-D}{4A}\quad(\text{quadratic in } k_1)$$

$$=+\frac{25}{2}\left[\frac{\cos 36^\circ\sin 18^\circ}{\sin 18^\circ+\cos 36^\circ}\right]$$

$$=\frac{25}{4\sqrt{5}}=\frac{5\sqrt{5}}{4}=\frac{\sqrt{125}}{4}=\frac{\sqrt{x}}{4}$$

$$x+y=129$$

974. (40) $E=(3\sqrt{5-4\cos x}+\sqrt{13-12\sin x})$

$$=(\sqrt{45-36\cos x}+\sqrt{13-12\sin x})$$

$$=\left(\sqrt{(6-3\cos x)^2+(3\sin x)^2}+\sqrt{(2-3\sin x)^2+(3\cos x)^2}\right)$$

$$=PA+PB\big|_{\text{minimum}}=AB=\sqrt{40}$$

Where $P = 3\cos x,\ 3\sin x$

$A = (6, 0)$ and $B = (0, 2)$

Hence, minimum value of $E^2 = 40$

975. (9)
$$\frac{3BC - AB}{4BC} = \sin^2 A$$

$$\Rightarrow \quad \frac{3\sin A - \sin C}{4\sin C} = \sin^2 A$$

$$\Rightarrow \quad 3\sin A - 4\sin^3 A = \sin C$$

$$\Rightarrow \quad \sin 3A = \sin C$$

$$3A = C \text{ or } 3A = 180 - C$$

If
$$3A = 180 - C$$

$$B = 2A$$

2^{nd} Equation $\Rightarrow$
$$\frac{1}{2}\cot\frac{A}{2} = \sin A + \sin 2A + \sin 3A = \frac{\sin(3A/2)\times\sin 2A}{\sin(A/2)}$$

$$\Rightarrow \quad \cos\left(\frac{7A}{2}\right) = 0$$

$$\Rightarrow \quad A = \frac{\pi}{7}$$

$$\Rightarrow \quad B = 2A = \frac{2\pi}{7}$$

$$\Rightarrow \quad C = \pi - 3A = \frac{4\pi}{7}$$

3^{rd} Equation $\Rightarrow \cos^2 A + \cos^2 B + \cos^2 C = p$

$$\Rightarrow \quad 1 - 2\cos A\cos B\cos C = p$$

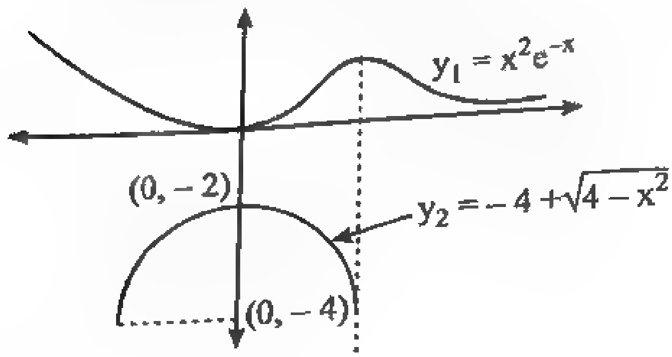
$$\Rightarrow \quad 1 - 2\cos A\cos 2A\cos 4A = p$$

Where
$$\angle A = \frac{\pi}{7}$$

$$\therefore \quad 1 - 2\times\left(-\frac{1}{8}\right) = p$$

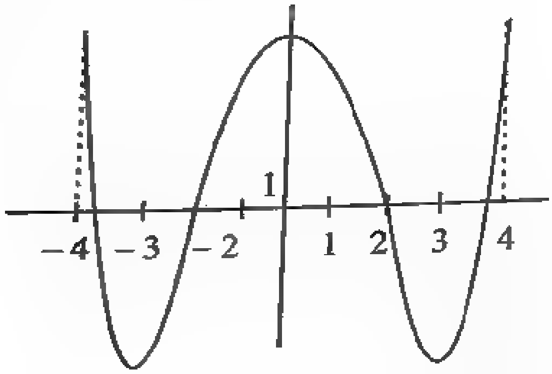
$$p = \frac{5}{4} = \frac{m}{n} \Rightarrow (m+n) = 9$$

976. (2)
$$y = \underset{\displaystyle y_1}{\underset{\downarrow}{x^2 e^{-x}}} - \underset{\displaystyle y_2}{\underset{\downarrow}{(-4 + \sqrt{4 - x^2})}}$$



Clearly $y_1 - y_2|_{\min} = 2$

977. (2)



Clearly, $f''(x) = 0$ at least at two points.

978. (3) $$f_n(x) = \lim_{t \to x} \frac{\sin^{-1} nt}{2t} = \frac{\sin^{-1} nx}{2x}$$

Now, $$\lim_{x \to 0}\left(\left[\frac{\sin^2 2x}{2x}\right] + \left[\frac{\sin^{-1} 4x}{2x}\right]\right) = 1 + 2 = 3$$

979. (2) $$\int_0^1 \cos\left(\frac{\pi}{2}x\right)dx \cdot \int_0^1 \cos^2\left(\frac{\pi}{2}x\right)dx \cdot \int_0^1 \cos^3\left(\frac{\pi}{2}x\right)dx \cdot \int_0^1 \cos^4\left(\frac{\pi}{2}x\right)dx$$

$$= \left(\frac{2}{\pi}\right)^4 \left(\int_0^{\pi/2} \cos^2 t\,dt\right)\left(\int_0^{\pi/2} \cos^3 t\,dt\right)\left(\int_0^{\pi/2} \cos^4 t\,dt\right) = \left(\frac{2}{\pi}\right)^4 1 \cdot \frac{\pi}{4} \cdot \frac{2}{3} \cdot \frac{3\pi}{16} = \frac{1}{2\pi^2} = \frac{k}{\pi^2}$$

$\therefore \frac{1}{k} = 2$

980. (2) $$C_1C_2 = r_1 + r_2$$

$$\sqrt{a^2 + b^2} = 2 \pm \sqrt{a^2 + b^2 - 2}$$

$$a^2 + b^2 = 4 + a^2 + b^2 \pm 4\sqrt{a^2 + b^2 - 2}$$

$$4\sqrt{a^2 + b^2 - 2} = 2$$

$$\sqrt{a^2 + b^2 - 2} = \frac{1}{2} = \frac{r}{2}$$

$\therefore$ $$4r_2 = 2$$

981. (5) $T_{r+1} = {}^6C_r \cdot (x^2)^{6-r} \cdot \left(\frac{-3}{x}\right)^r$

$$12-3r=0$$

$$\Rightarrow \quad r=4$$

$$T_5 = {}^6C_4 \cdot (-3)^4 \Rightarrow 5\cdot 3^5$$

982. (7) Required number of words = number of words in which M's are separated – number of words in which M's are separated by I's are together

$$= \frac{4!}{2!} \times {}^5C_2 - 3! \times {}^4C_2$$

$$= 120 - 36 = 84 = 12 \times 7$$

983. (2) $\log_{\frac{7}{6}}\left(\dfrac{k^2+\dfrac{k^2}{4}+\dfrac{r^2}{9}}{\dfrac{k^2}{2}+\dfrac{k^2}{6}+\dfrac{k^2}{3}}\right) = \log_{\frac{7}{6}}\left(\frac{49}{36}\right) = 2$

984. (16) $x=[256, 257)$ and $y=\dfrac{1}{256}$

985. (8) Since $f(x)$ is periodic with period a

$\therefore f(-1) = f(a-1) \Rightarrow 15 = (a-1)^2 - 6(a-1)+8 \rightarrow a=0$ or 8

at $a=0$ $f(x)$ becomes point function which is continuous.

986. (4) $y=\sqrt{t}+\sqrt{(\pi/2)-t}$ where $t=\sin^{-1}x \in [0, \pi/2]$

Now $y_{\min} = 1$ at $x=0$ and $y_{\max} = \sqrt{2}$ at $t=\dfrac{1}{2}$

987. (29) $L = \lim\limits_{n\to\infty}\left(\dfrac{1}{\sqrt{n}\sqrt{n+1}}+\dfrac{1}{\sqrt{n}\sqrt{n+2}}+\ldots+\dfrac{1}{\sqrt{n}\sqrt{n+n}}\right)$

$$= \lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{\sqrt{n}\sqrt{n+r}} = \lim_{n\to\infty}\sum_{r=1}^{n}\left(\frac{1}{\sqrt{n}}\cdot\frac{1}{\sqrt{n\left(1+\frac{r}{n}\right)}}\right) = \lim_{n\to\infty}\sum_{r=1}^{n}\left(\frac{1}{n}\cdot\frac{1}{\sqrt{1+\frac{r}{n}}}\right)$$

$$= \int_0^1 \frac{1}{\sqrt{1+x}}\,dx = 2\sqrt{2}-2$$

Hence $a=2,\ b=2,\ c=2,\ d=1$

$\therefore a^4+b^3+c^2+d=29$

988. (30) Given $A+C=2B$ and $A+B+C=180°$

$\therefore B=60°$

Now, let $A=60°-d, B=60°$ and $C=60°+d$

Given $\sin A+\sin C = 2\sin^2 B$

$$\Rightarrow 2\sin\left(\frac{A+C}{2}\right)\cos\left(\frac{A-C}{2}\right)=2\left(\frac{\sqrt{3}}{2}\right)^2$$

Hence, $\sin 60°+\cos d=\frac{3}{4} \Rightarrow \cos d=\frac{\sqrt{3}}{2}$

$\therefore\ d=30°$

Hence $A=30°$, $B=60°$ and $C=90°$

$\Rightarrow |A-B|=30°$

989. (1395) $\quad T(n)=\cos^2(30°-n°)-\cos(30°-n°)\cos(30°+n°)+\cos^2(30°+n°)$

$$=\frac{1}{2}(1+\cos(60°+2n°))-\frac{1}{2}(\cos(2n°)+\cos 60°)+\frac{1}{2}(1+\cos(60°+2n°))$$

$$=\frac{1}{2}+\frac{1}{4}\cos(2n°)+\frac{\sqrt{3}}{4}\sin(2n°)-\frac{1}{2}+\frac{1}{2}+\frac{1}{4}\cos(2n°)-\frac{\sqrt{3}}{4}\sin(2n°)$$

$$=\frac{3}{4}$$

$$\therefore\ 4\sum_{n=1}^{30} nT(n)=4\sum_{n=1}^{30}\frac{3n}{4}=3\sum_{n=1}^{30} n=1395$$

990. (10) $\quad \int_1^{xy} f(t)dt = y\int_1^x f(t)dt+x\int_1^y f(t)dt\ \forall\ x, y,\in (R)-\{0\}$ and $f(1)=1$

Differentiate both sides w.r.t. x

$$yf(xy)=yf(x)+\int_1^y f(t)dt$$

Put $x=1$, we get

$$yf(y)=y+\int_1^y f(t)dt \qquad (\because f(1)=1)$$

Again differentiate w.r.t. y

$$f(y)+yf'(y)=1+f(y)$$

$$\Rightarrow \quad yf'(y)=1$$

$$\Rightarrow \quad f'(y)=\frac{1}{y}$$

Hence, $f(y)=1+\ln y$

$$\therefore \quad g(x)=-\left(x^2+\frac{1}{x^2}\right)$$

Now, we have to find

$$I=\int_0^{\infty} e^{-\left(x^2+\frac{1}{x^2}\right)}dx \qquad \ldots(1)$$

Replace $x \to \frac{1}{x}$

$$I = \int_0^\infty e^{-\left(x^2+\frac{1}{x^2}\right)} \frac{1}{x^2}\, dx \qquad \ldots(2)$$

$$2I = \int_0^\infty e^{-\left(x^2+\frac{1}{x^2}\right)}\left(1+\frac{1}{x^2}\right) dx$$

$$2I = \int_0^\infty e^{-\left(x-\frac{1}{x}\right)^2-2}\left(1+\frac{1}{x^2}\right) dx$$

$$\Rightarrow \quad 2e^2 I = \int_0^\infty e^{-\left(x-\frac{1}{x}\right)^2}\left(1+\frac{1}{x^2}\right) dx$$

Put $\left(x-\frac{1}{x}\right) = t$

$$\Rightarrow \quad 2e^2 I = \int_{-\infty}^{\infty} e^{-t^2}\, dt = 2\int_{-\infty}^{\infty} e^{-t^2}\, dt$$

Hence, $$e^2 I = \int_{-\infty}^{\infty} e^{-t^2}\, dt = \sqrt{\pi}$$

$$\therefore \quad I = \frac{\sqrt{\pi}}{e^2} = \frac{\sqrt{a}}{b} \qquad \therefore a = \pi \text{ and } b = e^2$$

$$\Rightarrow \quad [a+b] = [\pi + e^2] = 10$$

991. (2.5) $$\int_0^1 f(x)dx = 1$$

$$\int_0^1 f(2x)dx = 3$$

Put $2x = u$

$$\frac{1}{2}\int_0^2 f(u)du = 3$$

$$\int_0^1 f(u)du + \int_1^2 f(u)du = 6$$

$$\therefore \quad \int_1^2 f(x)dx = 5$$

992. (6) $\frac{dy}{dx} + iy = 2\sin(x)$ (Linear D.E. with integrating factor e^{ix})

Hence, $ye^{ix} = \int 2\sin x e^{ix}\,dx = \int 2\sin x(\cos x + i\sin x)\,dx$

$$= \sin^2 x + i\left(x - \frac{1}{2}\sin 2x\right) + c$$

Given $y(0) = \frac{3}{2} \Rightarrow c = \frac{3}{2}$

$\therefore$ $ye^{ix} = \sin^2 x + i\left(x - \frac{1}{2}\sin 2x\right) + \frac{3}{2}$

Now put $x = \pi$

$\therefore$ $y(\pi)e^{i\pi} = 0 + i(\pi - 0) + \frac{3}{2}$

$$y(\pi)(-1) = i\pi + \frac{3}{2}$$

Hence, $y(\pi) = -\frac{3}{2} - \pi i$

Therefore $abc = 6$

993. $(a > \frac{e^2}{4})$ $f(x) = 0 \Rightarrow (e^x + 1)(e^x - ax^2) = 0$

$\therefore$ $e^x = ax^2$

$$a = \frac{e^x}{x^2}$$

Now draw the graph of $\frac{e^x}{x^2}$ and interpret for three distinct roots a ? $\frac{e^2}{4}$

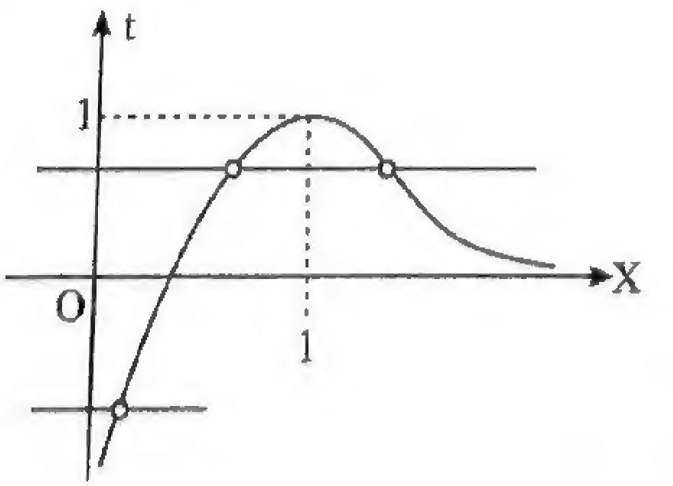
994. $(\frac{-1}{2}, 1)$ $f(x) = g(x)$

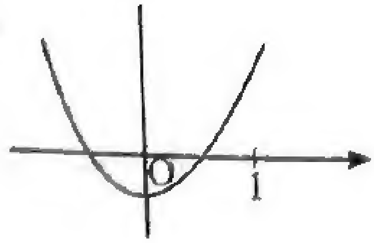
$$x^2 = \left(a + \frac{\ln x + 1}{x}\right)\left(1 + \frac{\ln x + 1}{x}\right)$$

Put $\frac{\ln x + 1}{x} = t$

$$(a + t)(1 + t) = 1$$

$$F(t) + t^2 + (a+1)t + a - 1 = 0$$





For 3 solutions, one solution in $(-\infty, 0)$ and another solution in $(0, 1)$

$F(0) < 0$ and $F(1) > 0$

$a < 1$ and $F(1) > \frac{-1}{2}$

$\Rightarrow a \in \left(\frac{-1}{2}, 1\right)$

[**Note:** If $F(t)$ has 4 solutions than that case is rejected (think)]

995. (5) Let $\quad z = x + iy \quad \therefore \quad \bar{z} = x - iy$

$\therefore \quad (2iy)^2 = 12(x^2 + y^2) - 4 \Rightarrow 12x^2 + 16y^2 = 4$

$3x^2 + 4y^2 = 1 \quad \Rightarrow \quad \frac{x^2}{\frac{1}{3}} + \frac{y^2}{\frac{1}{4}} = 1$

$\therefore \quad x = \sqrt{\frac{1}{3}}\cos\theta, \; y = \sqrt{\frac{1}{4}}\sin\theta$

$\therefore \quad 3\sqrt{3}\,\text{Re}(z) + 8\,\text{Im}(z) = 3\cos\theta + 4\sin\theta$

$\therefore \quad \max = 5$

996. (5) Let $\quad P = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$P\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} a-b \\ c-d \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

$\therefore \quad a - b = -1$ ...(1)

$c - d = 2$ ...(2)

and $\quad P \cdot P\begin{bmatrix} 1 \\ -1 \end{bmatrix} = P\begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$\Rightarrow \quad -a + 2b = 1$ ...(3)

and $\quad -c + 2d = 0$ ...(4)

From (1) and (3), $\quad b = 0$ and $a = -1$

From (2) and (4), $\quad d = 2$ and $c = 4$

$\therefore \quad P = \begin{bmatrix} -1 & 0 \\ 4 & 2 \end{bmatrix}$

$\therefore \quad |P = xI| = \begin{bmatrix} -1-x & 0 \\ 4 & 2-x \end{bmatrix} = 0$

$\Rightarrow \quad (x+1)(x-2) = 0 \quad x = -1, 2$

$\therefore \quad x_1^2 + x_2^2 = 5$

997. (2) $\because \quad \frac{f(x)f(y)}{xy} = \frac{f(x)}{x} + \frac{f(y)}{y}$

Let $\quad F(x) = \frac{f(x)}{x}$

$\therefore \quad F(0) = \lim_{x\to 0}\frac{f(x)}{x} = \lim_{x\to 0}\frac{f'(x)}{1} = 2$

$\therefore \quad F(x)F(y) = F(x)+F(y)$

Putting $\quad y=0,\ 2F(x)=F(x)+2 \Rightarrow F(x)=2$

$\therefore \quad f(x) = 2x$

$\therefore \quad \lim_{x\to 0}\left[\frac{2x}{\sin x}\right] = 2$

998. (5) Both circles are orthogonal to each other and $C(0,4)$ and $D(3,0)$ are centres and CD will diameter of circumcircle $= \sqrt{3^2+4^2} = 5$.

999. (2) $\quad f(xf(y)) = x^p y^4$, put $x = \frac{1}{f(y)}$

$\therefore \quad f(1) = \left(\frac{1}{f(y)}\right)^p y^4 = \frac{y^4}{(f(y))^p}$

For $\quad y=1, \quad f(1) = \frac{1}{(f(1))^p} \Rightarrow f(1)=1$

$\therefore \quad f(y) = y^{4/p}$ ...(1)

$\therefore \quad f(xy^{4/p}) = x^p y^4$

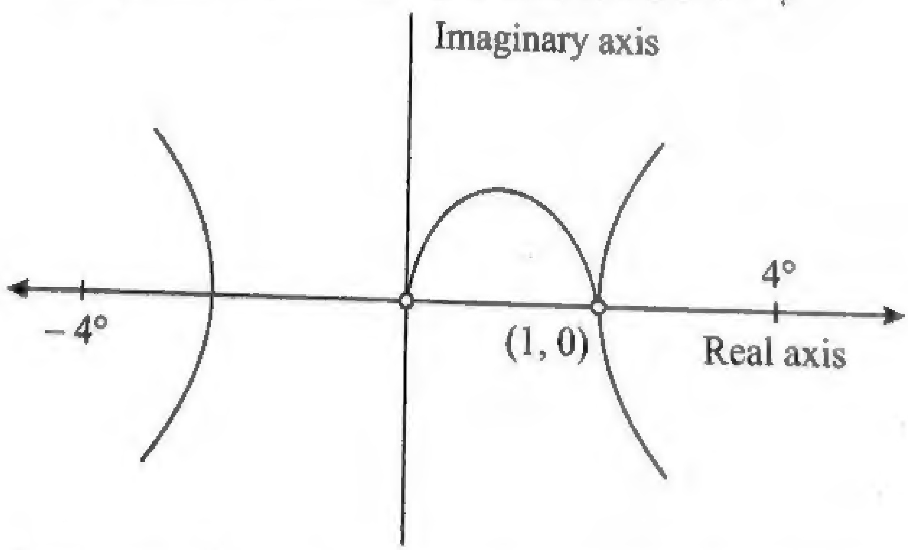
Put $\quad y = z^{p/4}$

$\therefore \quad f(xz) = x^p z^p \Rightarrow f(x) = x^p$ ...(2)

From Eqns. (1) and (2)

$$\frac{4}{p} = p \quad \Rightarrow \quad p=2$$

1000. (3) A, B and C represented geometrically as clear $A \cup B \cup C = \phi$



Clearly S represents the set of complex number lying on the circle $|z|=1,\ z\neq -1$

$|z_1+z_2|^2+|z_2+z_3|^2+|z_3+z_1|^2 = 3+(z_1+z_2+z_3)$

$= 3+|z_1+z_2+z_3|^2 \geq 3$

□□□